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Optimal Control of Meningococcal Meningitis Transmission Dynamics: A Case Study of Nigeria.

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Abstract: *Meningococcal Meningitis disease outbreak is a common phenomenon in the African Meningitis belt. The monumental death tolls resulting from the recurring outbreaks call for public health concern. Consequently, a deterministic model for the transmission dynamics of the disease which incorporates vaccination of the susceptibles and timely treatment of the infectives as control measures is considered. The problem is formulated as an optimal control problem with the goal of minimizing the annual incidence of the disease as well as the cost of implementing the control measures. Based on Pontryagin's Maximum Principle (PMP), the optimality system to the optimal control problem is derived and it is solved numerically using Runge-Kunta of order four scheme with the forward-backward sweep approach. The numerical result is then simulated for different scenarios of the disease outbreaks and the findings from our simulations are discussed.*

Keywords: *Constraint equations, Meningococcal Meningitis, Objective functional, Optimality system, Pontryagin's Maximum Principle.*

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I. Introduction

Meningitis is derived from the Greek word "Meninx" which means membrane and the medical suffix "-itis" which implies inflammation [12]. Thus, Meningococcal Meningitis is a bacterial form of Meningitis causing the inflammation of the thin lining surrounding the brain and the spinal cord. It could results into severe brain damage and death in about 50% of untreated cases [20]. It is on record that the first Meningococcal disease epidemic occurred in the sub-Saharan Africa around late 19th century, although in 2015 alone, 8.7 million cases of the disease were recorded globally [9]. These cases resulted in 379,000 deaths which was significantly lower that the casualties of 464,000 deaths recorded in 1990. However, the reduction in death tolls resulting from the 1990 and 2015 meningitis outbreaks could be attributed to the successful vaccine campaign embarked upon. Nevertheless, it is important to note that about 10% to 20% of any given population are carriers of the Meningitis bacteria while this proportion may increase to as high as 25% of the population in epidemic situation [20]. Therefore, the likelihood of Meningitis outbreak is very high, particularly in an area of sub-Saharan Africa called the Meningitis belt, if routine control measures are not put in place to contain the epidemic [20].

Unfortunately, Nigeria falls within the Meningitis belt, hence, it cannot but be affected by the recurring outbreaks. For instance, Nigeria experienced Meningitis epidemic for the three successive years ending in the year 1979 [15]. Although, this recurring epidemic was due to the vaccine supplies short- age. Similarly, in 2017, Nigeria suffered another epidemic with a total of 1407 suspected cases reported and 211 deaths (15%) cutting across five different states in the country [15]. These scenarios show that the recurring meningitis threat calls for public health concern, hence effective control measures should be implemented to forestall frequent recurrence and minimize attendant casualties from any of such outbreak.

Over the years, mathematical models have been deployed to inform effective health policies. Essentially, mathematical modelling has been adopted to provide guidelines on measures to be taken to curtail the spread of infectious diseases. Some of earlier works in this regards are by Ross, Bernoulli, McKendrick and Kermack, just to mention a few [5, 7, 19]. In particular, series of research works on modelling Meningitis transmission dynamics and its control have been carried out by different scholars, though their works have different emphases and interests [1, 4, 6, 14, 16, 23]. For example, Blyuss proposed and analyzed a deterministic model for the spread and control of Meningitis [2]. Based on the finding from their work, he pointed out the crucial factors influencing the Meningitis transmission dynamics. Also, he found that the level of temporary immunity enjoyed by individuals in the community is very vital in disease surveillance and measuring vaccine efficiency.

In another related study, Martinez et al proposed a model for the dynamics of Meningitis based on cellular automata theory. Their simulation results agree with the empirical ones in terms of the role played by the carriers in the disease transmission dynamics [13]. Also, Vareen assessed the impact of vaccination program on

the spread of the disease in countries prone to Meningitis epidemic. His findings show that countries with high Meningitis transmission rate should scale up their vaccination coverage as appropriate to mitigate the alarming spread of the disease [17]. It is important to point out here that most of the models on Meningitis dynamics often assume total population as a constant or use constant rates for the control measures while outcomes from such approaches are usually sub-optimal. However, these shortcomings could be addressed with applications of optimal control theory in the management of such epidemic situations. Thus, the disease scenario in the Meningitis belt is formulated as an optimal control problem in order to solve the problem optimally.

In this paper, a mathematical model for the transmission dynamics and control of Meningococcal Meningitis is considered. The meningitis epidemic situation is formulated as an optimal control problem with a goal to remarkably reduce the incidence and prevalence of the disease in a cost effective manner within a specified time interval. The paper is arranged in the following order: section 2 contains the proposed Meningococcal Meningitis model which is shown to be well-posed; in section 3, the modeled situation is formulated an optimal control problem subject to the proposed model dynamics, the optimal controls are characterized, and the optimality system are derived using Pontryagin's maximum principle (PMP); in section 4, the resulting optimality system is solved numerically and the results are discussed.

II. The Mathematical Model

Considering the different stages involved in the transmission and progression of Meningitis disease, a model qfor the disease dynamics across a given population is considered. The model divides the population of interest into four mutually exclusive compartments, namely: S(t) - Susceptibles, C(t) - Carriers, I(t) - Infected individuals, and R(t) - Recovered individuals. Here, S(t), C(t), I(t), R(t) represents the number of individuals in each of the respective compartments per unit time, while the total population N(t) = S(t) + C(t) + I(t) + R(t). Thus, the model is a system of non-linear ordinary differential equation below:

$$\begin{aligned}
\dot{S} &= \Lambda - \frac{\beta S(C+I)}{N} - (u_1+\mu)S + \sigma R,\\
\dot{C} &= \frac{\beta S(C+I)}{N} - (\rho+\gamma+\mu)C,\\
\dot{I} &= \rho C - (u_2+\mu+\delta)I,\\
\dot{R} &= u_1 S + \gamma C + u_2 I - (\sigma+\mu)R. \qquad (2.1)
\end{aligned}$$

where $\dot{N} = \dot{S} + \dot{C} + \dot{I} + \dot{R}$.

The model schematic diagram is as below:

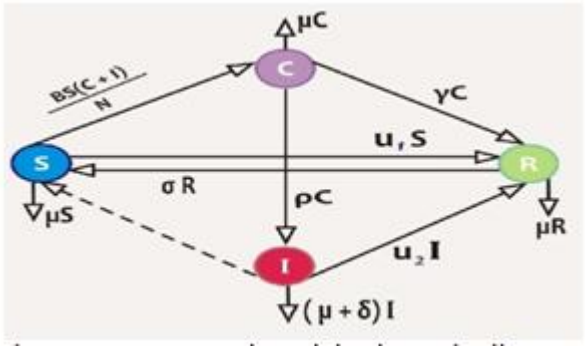

Figure 2.1: Proposed model schematic diagram.

while the description of the model parameters are given in the table that follows:

Table 2.1: Description of the model parameters.

Parameter	Parameter Description
Λ	Constant recruitment into the S class per unit time
β	Transmission Rate
u_1	Vaccine Rate
σ	Waning Rate (Immunity Lost)

ρ	Progression Rate (Disease Carrier)
γ	Spontaneous Recovery Rate
u_2	Treatment Recovery Rate
δ	Disease Induced Death Rate
μ	Natural Death Rates

It is imperative to note that $\lim_{t\to\infty} N(t) \le \frac{\Lambda}{\mu}$. In addition, the region Ω defined by:

$$\Omega = \left\{(S,C,I,R) \in \Re_+^4 \right\} \le \frac{\Lambda}{\mu}$$

is positively invariant with respect to the dynamics described for the model (2.1).
Lemma 1: The region is positively invariant for the model (2.1).

Proof: Let $t_1 = \sup\{t > 0 | S \ge 0,\ C \ge 0,\ I \ge 0,\ R \ge 0,\ \in [0,t]\}$. From 2.1, we have

$$\frac{dS}{dt} = \Pi(t) - (\kappa(t) + \mu + u_1(t))S, \qquad where \quad \kappa(t) = \frac{\beta(C+I)}{N} \qquad and \qquad \Pi(t) = \Lambda + \sigma R.$$

This is same as

$$\frac{dS}{dt} + (\kappa(t) + \mu + u_1(t))S = \Pi(t).$$

This implies that

$$\frac{d}{dt}\left(S(t)\exp\{\mu t + \int_0^t (\kappa(\tau) + u_1(\tau))d\tau\}\right) = \Pi(t)\exp\{\mu t + \int_0^t (\kappa(\tau) + u_1(\tau))d\tau\}.$$

Therefore,

$$S(t_1)\exp\{\mu t + \int_0^{t_1} (\kappa(\tau) + u_1(\tau))d\tau\} - S(0) = \int_0^{t_1} \Pi(\psi)\exp\{\mu\psi + \int_0^{\psi} (\kappa(\varepsilon) + u_1(\varepsilon))d\psi\}.$$

So,

$$\begin{aligned} S(t_1) \ &= S(0)\exp\{-\mu t - \int_0^{t_1} (\kappa(\tau) + u_1(\tau))d\tau\} \\ &\quad + (\exp\{-\mu t - \int_0^{t_1} (\kappa(\tau) + u_1(\tau))d\tau\}) \times \int_0^{t_1} \Pi(\psi)\exp\{\mu\psi + \int_0^{\psi} (\kappa(\varepsilon) + u_1(\varepsilon))d\psi\} \\ &\ge 0. \end{aligned}$$

Similarly, we can show that $C(t) \ge 0,\ I(t) \ge 0,\ and\ R(t) \ge 0.$ This completes the proof.

It is It is salient to mention that this lemma ensures that the model predicts only positive values for each of the model variables. This is sensible because number of individuals in each of the compartments can not be negative.

Lemma 2 : The region Ω is an attractor and it attracts all solutions starting in the interior of the positive orthant $\Re_+^4$.

Proof:

Using the non-negativity of the model state variables established in the preceeding lemma and the fact that

$$\dot{N} = \Lambda - \mu N - \delta A;$$

for initial conditions in $\Re_+^4$ and $t \ge 0,$ by standard comparism theorem, we have

$$\dot{N} \le \Lambda - \mu N.$$

This implies that

$$\frac{d}{dt}(Ne^{\mu t}) \le \Lambda e^{\mu t}.$$

$$\Rightarrow Ne^{\mu t} - N(0) \le \frac{\Lambda}{\mu}(e^{\mu t} - 1) \le \frac{\Lambda}{\mu} e^{\mu t}.$$

So, for all $t \geq 0$,

$$N(t) \leq N(0)e^{-\mu t} + \frac{\Lambda}{\mu}.$$

If $(S^*,\ C^*, I^*,\ R^*)$ is in Ω limit point of an orbit in $\Re^4_+$, then there is a subsequence $t_i \to \infty$ such that $\lim_{i\to\infty} \big(S(t_i), C(t_i), I(t_i), R(t_i)\big) = (S^*,\ C^*, I^*,\ R^*).$

Hence, $\lim_{i\to\infty} N(t_i) = N^* = S^* + C^* + I^* + R^*$. From equation (2.3), it follows that $N^* \leq \frac{\Lambda}{\mu}$; while $(S^*,\ C^*,\ I^*,\ R^*) \in \Omega$.

Consequently, for any initial starting point (S_0, C_0, I_0, R_0) $\in \Re^4_+$, the trajectory lies in Ω. Thus, the system is well-posed. Moreover, the basic reproduction number $\Re_0$ of the disease with re- spect to the proposed model is obtained using the Next–Generation Matrix approach by Van Driessche and Watmough (See [5]) as expressed below:

$$\Re_0 = \frac{\beta(\sigma+\mu)}{(u_1+\sigma+\mu)(\rho+\gamma+\mu)} + \frac{\beta(\sigma+\mu)\rho}{(u_1+\sigma+\mu)(\rho+\gamma+\mu)(u_2+\mu+\delta)}$$

This threshold parameter $\Re_0$, is an essential quantity in epidemiological studies because it provides an indicator on disease spread pattern and factors that should be addressed to curtail the alarming spread of an infectious dis- ease [23]. Conventionally, $\Re_0$ is defined as the average number of secondary infections produced by a single infected individual when introduced into an host population where everyone is susceptible [5].

III. Formulation Of Optimal Control Problem

Here, an optimal control problem is formulated with the goal of minimizing new cases and prevalence of the disease as well as the cost of implementing the deployed control measures. The objective functional is defined as below:

$$J = min_{u_1,u_2} \int_0^T [\varpi_1 C(t) + \varpi_2 I(t) + \frac{1}{2}\varpi_3 u_1^2(t) + \frac{1}{2}\varpi_4 u_2^2(t)]dt \qquad (3.1)$$

while the admissible control set U is Lebesgue measurable and it is defined as:

$$\mathbb{U} = \{(u_1(t), u_2(t)) \mid 0 \leq u_1 \leq u_{1max} \leq 1, 0 \leq u_2 \leq u_{2max} \leq 1, t \in [0,T]\}.$$

Thus, the target is to find $(u_1^*, u_2^*) \in U$ that minimizes the associative cost of the vaccination and treatment over the specified time interval as well as the total number of infected individuals. In addition, $w_1 > 0$ and $w_2 > 0$ are weight constants to maintain balance in the size of C(t) and I(t) while $w_3 > 0$ and $w_4 > 0$ are weights associated with cost of vaccination (u_1) and treatment (u_2) respectively.

3.1 Existence of the Optimal Controls

The sufficient condition for the existence of a solution to the optimal control problem will be established following the approach adopted in the work by Yusuf and Benyah [22].

Theorem 1 There exist an optimal control pair $u_1^*(t), u_2^*(t)$ with a corre- sponding solution (S*, C*, I*, R*) to the system (2.1) such that

$$J(u_1^*(t), u_1^*(t)) = min_{u_1,u_2 \in \mathbb{U}} J(u_1(t), u_2(t))$$

Proof 1 According to Flemming and Rishel, the existence of the optimal controls is guaranteed by the compactness of the control and the state space and the convexity in the problem [8]. Nevertheless, the non-trivial requirements from Fleming and Rishel's theorem need to be stated and verified. These requirements are as follows:

i. The set of all solutions to the proposed model equations (2.1) and the associated initial conditions together with the corresponding control func- tions in U is non-empty.

ii. The state variables dynamical system can be written as a linear func- tion of the control variables with coefficient dependent on time and state variables.
iii. The integrand E in the objective functional (3.1)

$$\varpi_1 C(t) + \varpi_2 I(t) + \frac{1}{2}\varpi_3 u_1^2(t) + \frac{1}{2}\varpi_4 u_2^2(t)$$

is convex on the control set U and satisfies :

$$J(u_1(t), u_2(t)) \geq \varphi_1(|u_1|^2 + |u_2|^2)^{\frac{\Theta}{2}} - \varphi_2.$$

Based on the theorem proposed by Picard-Lindelof in [3], if the solutions to the state equations are a priori bounded and if the state equations are continuous and Lipschitz in the state variables, then there exists a unique so- lution corresponding to every admissible control set in U. In addition, using the fact that for all $(S,C,I,R) \in \Omega$ all the model states are bounded below and above, then the solutions to the state equations are bounded. Also, the boundedness of the partial derivatives with respect to each of the state variables in the system can be shown directly, hence the system is Lipschitz with respect to the state variables. Therefore, the first requirement is met. Also, the state equations are linearly dependent on the controls $u_1(t)$ and $u_2(t)$; this is obvious by inspection. Thus, the second requirement is fulfilled too. Since, the objective functional is quadratic in the control variables $u_1(t)$ and $u_2(t)$; then it is convex. So, the only left to be satisfied is to show that there is no bound on $\mathbb{E}$. This is shown below:
Therefore, the above establishes the required bound on $\mathbb{E}$. Thus, there exists a unique solution to the optimality system for small time intervals due to the opposite time orientations of the state equations and the adjoint equations. Also, the uniqueness of the solution of the optimality system guarantees the uniqueness of the optimal control if it exists.

3.2: Optimal control Variable characterization.

In order to determine the optimal levels of each of the control measures (u_1^*) and (u_2^*), and the associated state variables (S^*, C^* , I^*, R^*) that would yield optimal value for the objective functional, the Pontryagin's maximum principle (PMP) is applied [8, 11]. Thus, an Hamiltonian (H) is constructed as below:

$$\begin{aligned}\mathbb{H}(S,C,I,R,u_1,u_2,t) &= \varpi_1 C(t) + \varpi_2 I(t) + \tfrac{1}{2}\varpi_3 u_1^2(t) + \tfrac{1}{2}\varpi_4 u_2^2(t) \\ &+ \lambda_1\left[\wedge - \tfrac{\beta S(C+I)}{N} - (u_1+\mu)S + \sigma R\right] \\ &+ \lambda_2\left[\tfrac{\beta S(C+I)}{N} - (\rho+\gamma+\mu)C\right] \\ &+ \lambda_3[\rho C - (u_2+\mu+\delta)I] \\ &+ \lambda_4[u_1 S + \gamma C + u_2 I - (\sigma+\mu)R]\end{aligned} \tag{3.3}$$

Thereafter, the dynamics of the adjoint variables are obtained using the following relations:

$$\frac{d\lambda_1}{dt} = -\frac{\partial \mathbb{H}}{\partial S}, \quad \frac{d\lambda_2}{dt} = -\frac{\partial \mathbb{H}}{\partial C}, \quad \frac{d\lambda_3}{dt} = -\frac{\partial \mathbb{H}}{\partial I}, \quad \frac{d\lambda_4}{dt} = -\frac{\partial \mathbb{H}}{\partial R}.$$

These give

$$\begin{aligned}\frac{d\lambda_1}{dt} &= \lambda_1\left[\left(\frac{\beta(C+I)}{N} - \frac{\beta S(C+I)}{N^2}\right) + (u_1+\mu)\right] + \lambda_2\left(\frac{\beta S(C+I)}{N^2} - \frac{\beta(C+I)}{N}\right) - \lambda_4 u_1 \\ \frac{d\lambda_2}{dt} &= -\varpi_1 + \lambda_1\left(\frac{\beta S}{N} - \frac{\beta S(C+I)}{N^2}\right) + \lambda_2\left(\frac{\beta S(C+I)}{N^2} - \frac{\beta S}{N}\right) \\ &\quad + \lambda_2(\rho+\gamma+\mu) - \lambda_3\rho - \lambda_4\gamma \\ \frac{d\lambda_3}{dt} &= -\varpi_2 + \lambda_1\left(\frac{\beta S}{N} - \frac{\beta S(C+I)}{N^2}\right) + \lambda_2\left(\frac{\beta S(C+I)}{N^2} - \frac{\beta S}{N}\right) \\ &\quad + \lambda_3(u_2+\mu+\delta) - \lambda_4 u_2 \\ \frac{d\lambda_4}{dt} &= \lambda_4(\sigma+\mu) - \lambda_1\sigma\end{aligned} \tag{3.4}$$

Based on the transversality condition, we have $\lambda_1(T) = 0$; $\lambda_2(T) = 0$; $\lambda_3(T) = 0$; $\lambda_4(T) = 0$.
In addition, the optimal controls (set) $U = (u_1, u_2) \in U$ are obtained by invoking the optimality condition. This is achieved by differentiating the Hamiltonian ($\mathbb{H}$) in (3.3) with respect to each of the controls u_1 and u_2. Thus,

we have

$$\frac{\partial \mathbb{H}}{\partial u_1} = \varpi_3 u_1 - \lambda_1 S + \lambda_4 S = 0, \qquad (3.5)$$

and

$$\frac{\partial \mathbb{H}}{\partial u_2} = \varpi_4 u_2 - \lambda_3 I + \lambda_4 I = 0. \qquad (3.6)$$

Therefore, the optimal control pair (u_1^*, u_2^*) is given by

$$\begin{aligned} u_1^* &= min\left\{max\left(0, \frac{1}{\varpi_3}(\lambda_1 - \lambda_4)S\right), u_{1max}\right\} \\ u_2^* &= min\left\{max\left(0, \frac{1}{\varpi_4}(\lambda_3 - \lambda_4)I\right), u_{2max}\right\} \end{aligned} \qquad (3.7)$$

Since the control measures (u_1^*, u_2^*) are bounded with lower bound as zero and upper bound as $u_{i\max}$ *where* $i = 1, 2$. Then, we have

$$u_1^*(t) = \left\{ \begin{array}{ccc} 0, & if \frac{1}{\varpi_3}(\lambda_1 - \lambda_4)S & \leq 0 \\ \frac{1}{\varpi_3}(\lambda_1 - \lambda_4)S, & if 0 < \frac{1}{\varpi_3}(\lambda_1 - \lambda_4)S & < u_{1max} \\ u_{1max}, & if \frac{1}{\varpi_3}(\lambda_1 - \lambda_4)S & \geq u_{1max} \end{array} \right\} \in \mathbb{U} \qquad (3.8)$$

and

$$u_2^*(t) = \left\{ \begin{array}{ccc} 0, & if \frac{1}{\varpi_4}(\lambda_3 - \lambda_4)I & \leq 0 \\ \frac{1}{\varpi_3}(\lambda_1 - \lambda_4)I, & if 0 < \frac{1}{\varpi_4}(\lambda_3 - \lambda_4)I & < u_{2max} \\ u_{2max}, & if \frac{1}{\varpi_4}(\lambda_3 - \lambda_4)I & \geq u_{2max} \end{array} \right\} \in \mathbb{U} \qquad (3.9)$$

Hence, the resulting optimality system follows:

$$\begin{aligned} \dot{S} &= \wedge - \frac{\beta S(C+I)}{N} + \sigma R - (u_1^* + \mu)S \\ \dot{C} &= \frac{\beta S(C+I)}{N} - (\rho + \gamma + \mu)C \\ \dot{I} &= \rho C - (u_2^* + \mu + \delta)I \\ \dot{R} &= u_1^* S + \gamma C + u_2^* I - (\sigma + \mu) \\ S(0) &= S_0,\ C(0) = C_0,\ I(0) = I_0,\ R(0) = R_0 \end{aligned}$$

$$\begin{aligned} \frac{d\lambda_1}{dt} &= \lambda_1\left[\left(\frac{\beta(C+I)}{N} - \frac{\beta S(C+I)}{N^2}\right) + (u_1^* + \mu)\right] \\ &\quad + \lambda_2\left(\frac{\beta S(C+I)}{N^2} - \frac{\beta(C+I)}{N}\right) - \lambda_4(u_1^*) \\ \frac{d\lambda_2}{dt} &= -\varpi_1 + \lambda_1\left(\frac{\beta S}{N} - \frac{\beta S(C+I)}{N^2}\right) + \lambda_2\left(\frac{\beta S(C+I)}{N^2} - \frac{\beta S}{N}\right) \\ &\quad + \lambda_2(\rho + \gamma + \mu) - \lambda_3\rho - \lambda_4\gamma \\ \frac{d\lambda_3}{dt} &= -\varpi_2 + \lambda_1\left(\frac{\beta S}{N} - \frac{\beta S(C+I)}{N^2}\right) + \lambda_2\left(\frac{\beta S(C+I)}{N^2} - \frac{\beta S}{N}\right) \\ &\quad + \lambda_3(u_2^* + \mu + \delta) - \lambda_4(u_2^*) \\ \frac{d\lambda_4}{dt} &= \lambda_4(\sigma + \mu) - \lambda_1\sigma \end{aligned} \qquad (3.10$$

$$\lambda_1(T) = 0,\ \lambda_2(T) = 0,\ \lambda_3(T) = 0, \lambda_4(T) = 0.$$

IV. Numerical Results And Discussion

4.1 Estimation of model parameters

The model parameter values were estimated using the Nigeria demographic data and the disease epidemiological data gotten from reliable sources (where possible) while those that could not be estimated were obtained from re- search articles. For instance, the Meningitis disease induced death rate was estimated based on the information that the disease is fatal in 50% of the untreated cases, hence, the disease induced death rate (δ) of the infected individuals was taken to be 0.5 [20]. Also, there is effective treatment for Meningococcal Meningitis; thus, it was assumed that a patient who undergoes early treatment has 50% chance of survival. Hence, the disease treatment success rate (u_{2max}) is taken to be 0.5. Conservatively, it was assumed that not more than 50% of the population can be vaccinated. Hence, $u_{1max} = 0.5$. In addition, the constant recruitment per unit time ($\wedge$) into the susceptible class was estimated using the annual increase in the Nigeria's population from year 2000 - 2015 and the average of the annual population increase was estimated to be 3.922 million per year [18], while the average life expectancy of a Nigerian at birth is taken to be $\frac{53+56}{2} = 54.5$ *years* [20].

Thus, the natural death rate (μ) is estimated $\frac{1}{54.5} = 0.018$. However, the remaining parameter values were sourced from [2, 10] as presented in Table 4.1

Table 4.1: Model Parameter Values

Parameter	Value	Source
$\wedge$	3.922	[18] and Estimate
β	2.5	Estimate
u_{1max}	0.5	Estimate
σ	0.1	[2, 10]
ρ	0.8	[2, 10]
γ	0.25	Estimate
u_{2max}	0.5	Estimate
δ	0.5	[20]and Estimate
μ	0.018	[20] and Estimate

Table 4.2: Model Variables Initial Conditions [23]

Variables (Population in Million)	Value
Susceptible S(0)	136.5
Carrier C(0)	27.3
Infected I(0)	9.1
Recovery R(0)	9.1

4.2 Numerical Results.

The optimality system is solved using Runge-Kunta of order four scheme with forward-backward sweep approach. Starting with an initial guess for the control variables $u_1(t)$ and $u_2(t)$, the state equations are solved forward in time with initial conditions given in table 4.2 while the results are then used to solve the adjoint variables backward in time with terminal conditions $\lambda_i(t) = 0$, for i = 1 4 . Thereafter, the control variables are updated based on their characterization in equations (3.7). This procedure is repeated until results for each of the variables converge sufficiently. Moreover, this entire is coded in computer program written and executed with MATLAB software. The results are simulated for two different scenarios of the epidemic; taking the weights constants in the optimal control problem as $w_1 = 1$, $w_2 = 1000$, $w_3 = 10$, and $w_4 = 1000$ while the parameters values are as given in Table (4.2). The first scenario depicts a situation with parameter set resulting in $\Re_0 < 1$ and the second scenario depicts a situation with parameter set resulting in $\Re_0 > 1$.

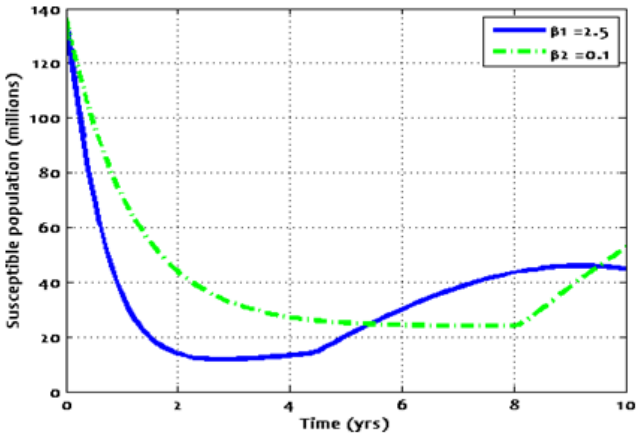

Figure 4.1: Population of S(t) for different values of β

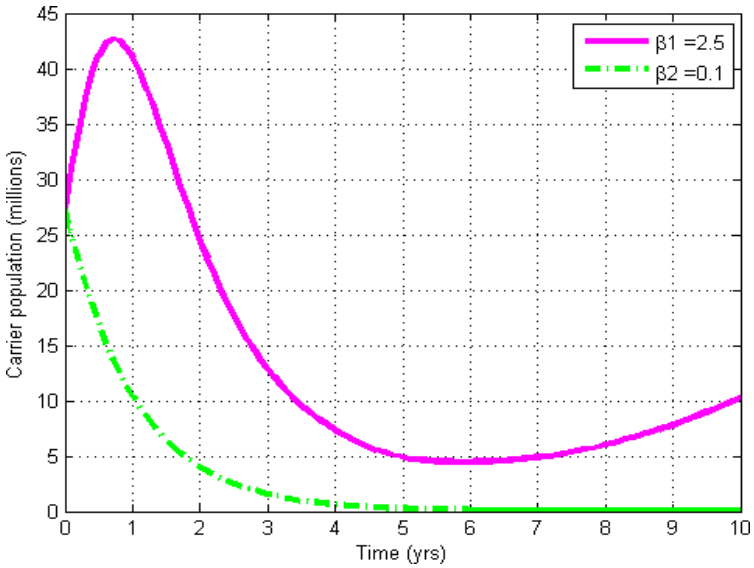

Figure 4.2: Population of C(t) for different values of β

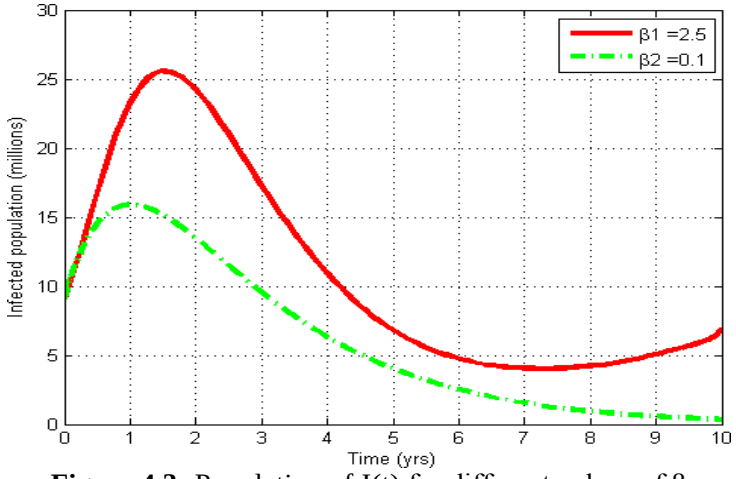

Figure 4.3: Population of I(t) for different values of β

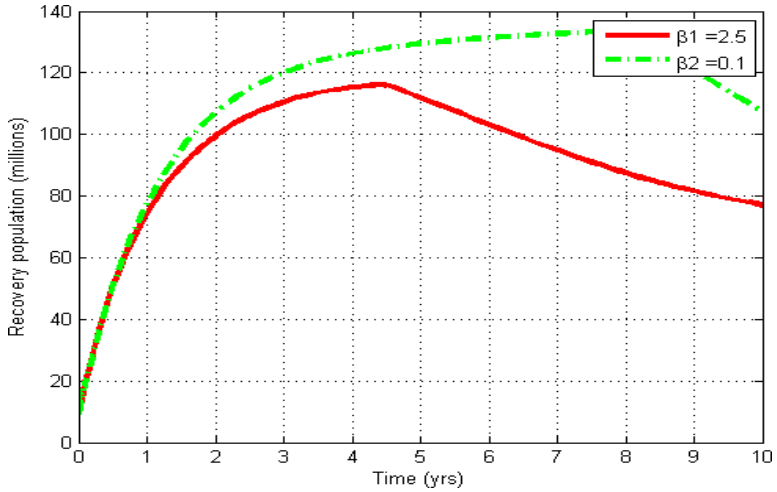

Figure 4.4: Population of R(t) for different values of β

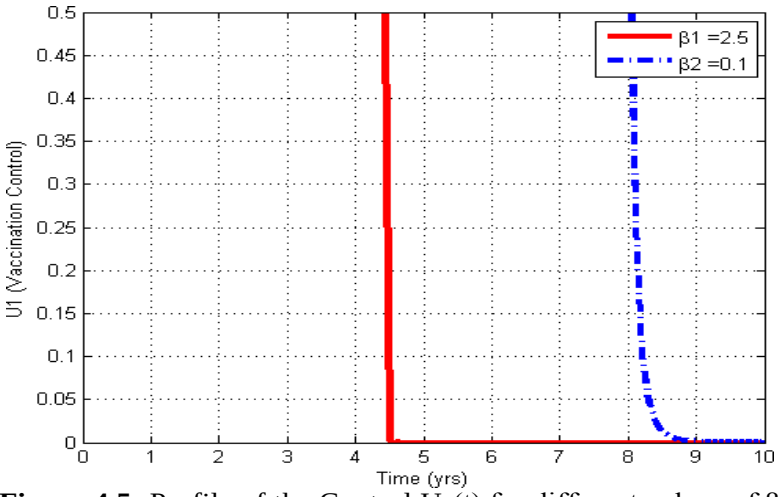

Figure 4.5: Profile of the Control $U_1(t)$ for different values of β

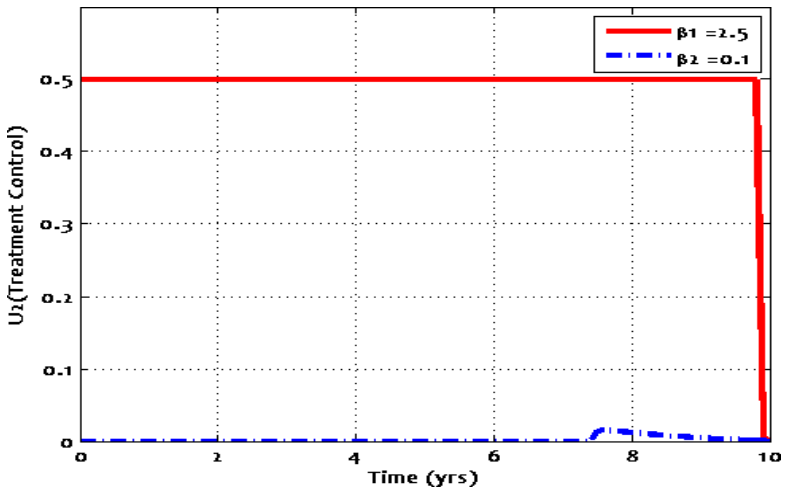

Figure 4.6: Profile of the Control $U_1(t)$ for different values of β

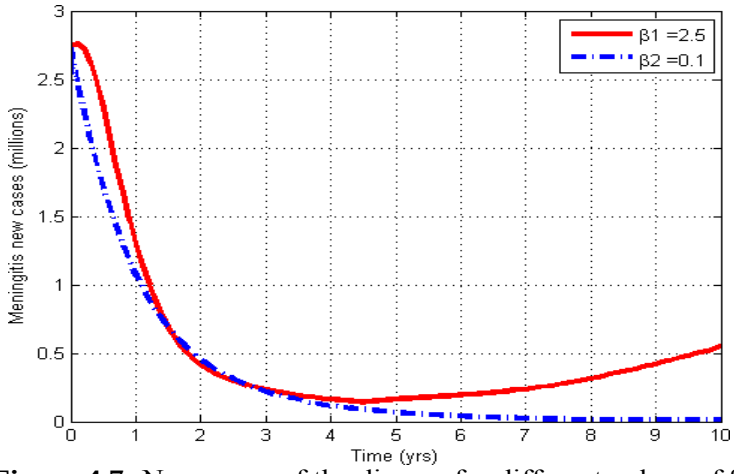

Figure 4.7: New cases of the disease for different values of β

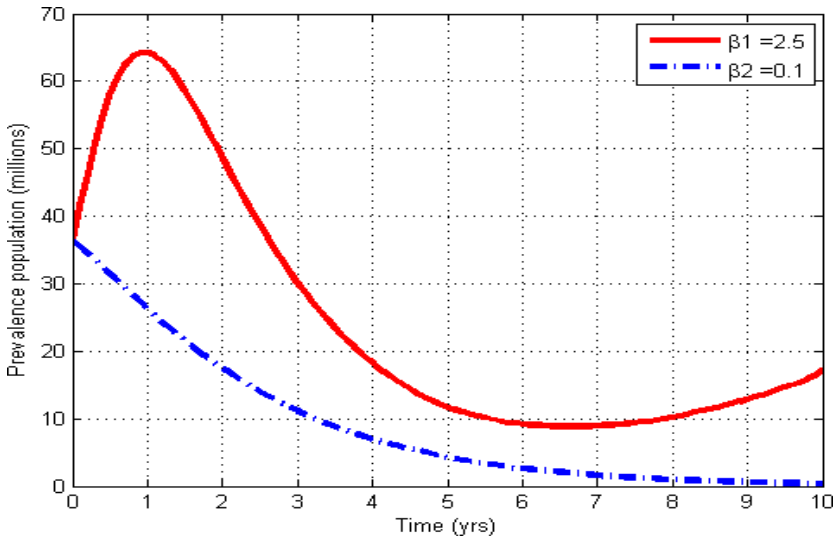

Figure 4.8: Prevalence of the disease for different values of β

Figures 4.1 - 4.4 show the population profiles for each of the four compart- ments with respect to the two scenarios. Also, Figures 4.5, 4.6, 4.7, and 4.8 give the profile of the vaccination control , treatment control, the new cases, and prevalence of disease as regards the two scenarios. These graphs portray that if appropriate control measures can be put in place to reduce the disease transmission rate such that $\Re_0 < 1$; the disease epidemic can be driven towards eradication in about ten years. However, in order to achieve this feat, the vaccination control has to be deployed continuously at the maximum effective level for about four and a half years while the treatment control have to be deployed maximally over the ten year period. Nevertheless, some of the control measures that could be adopted to reduce the transmission rate (β) is to embark on public educational and enlightenment campaign which would equip people with the right information about how the disease spread and what can be done to prevent infection. This would enable people to take necessary precaution against getting infected with the disease. Moreover, timely detection of infected individuals could also help to reduce the transmission rate if identified infected individuals are isolated to forestall further spread of the disease while possible contacts of the infected individuals should be screened immediately to ascertain their infection status with appropriate medical steps taking.

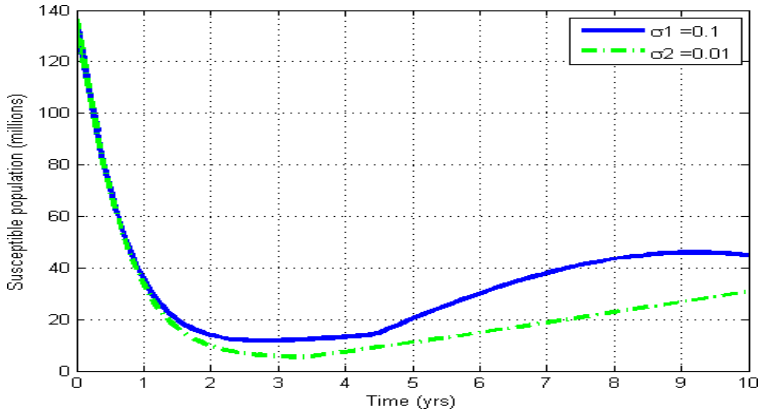

Figure 4.9: Population of S(t) for different values of σ

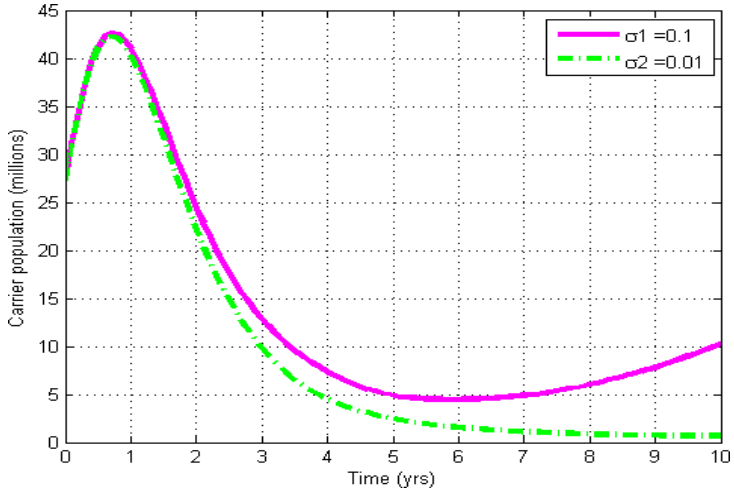

Figure 4.10: Population of C(t) for different values of σ

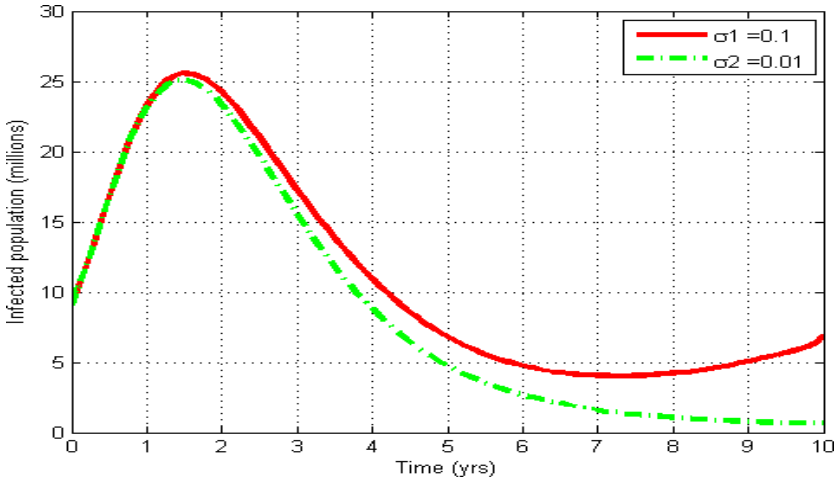

Figure 4.11: Population of I(t) for different values of σ

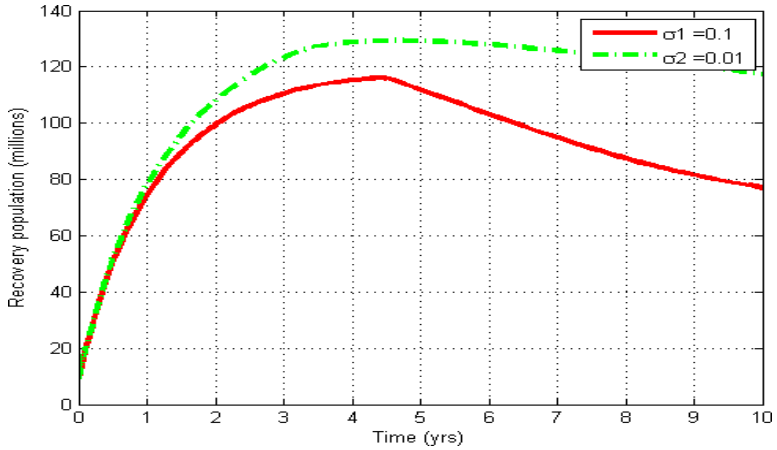

Figure 4.12: Population of R(t) for different values of σ

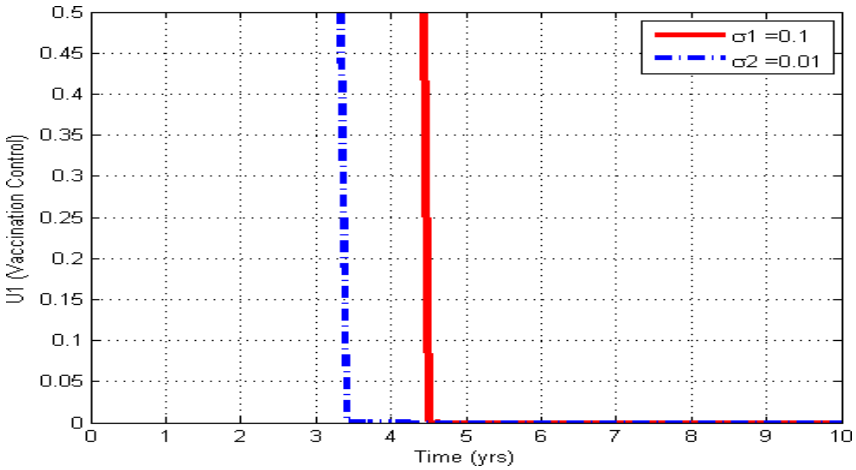

Figure 4.13: Profile of the Control $U_1(t)$ for different values of σ

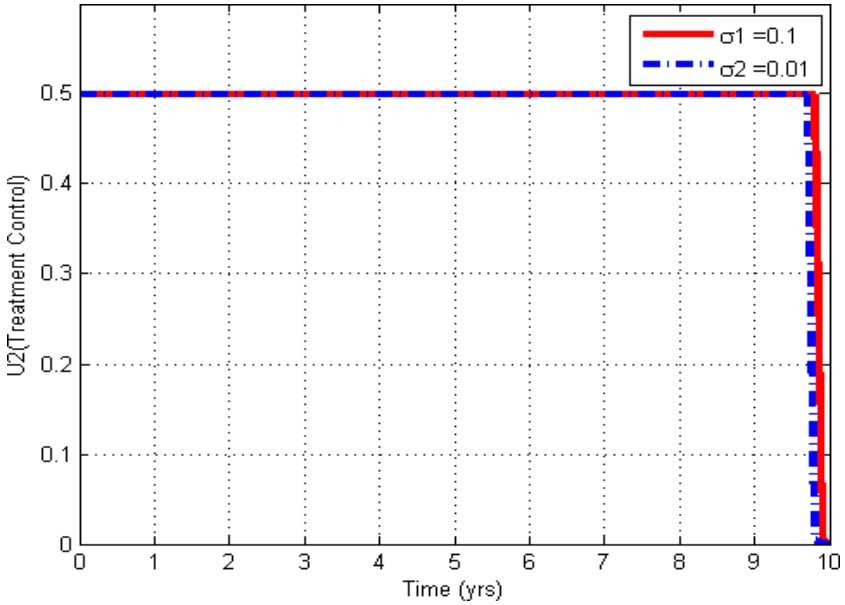

Figure 4.14: Profile of the Control $U_1(t)$ for different values of σ

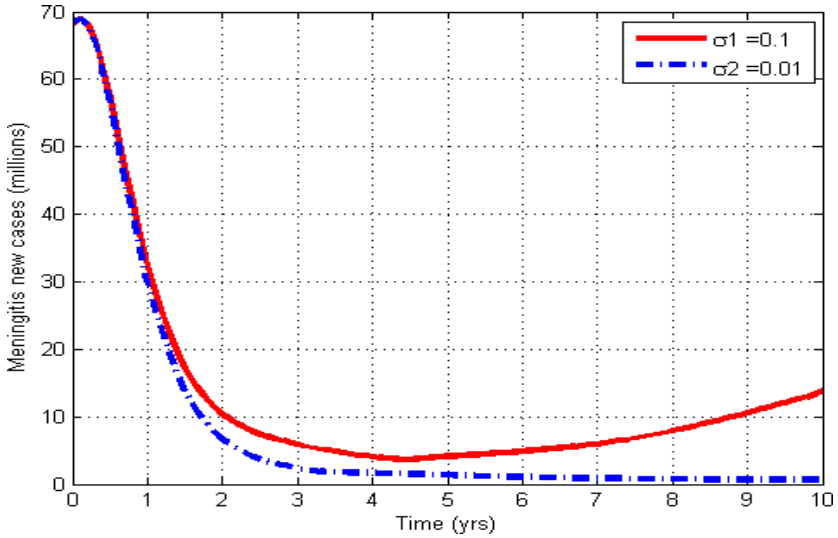

Figure 4.15: New cases of the disease for different values of σ

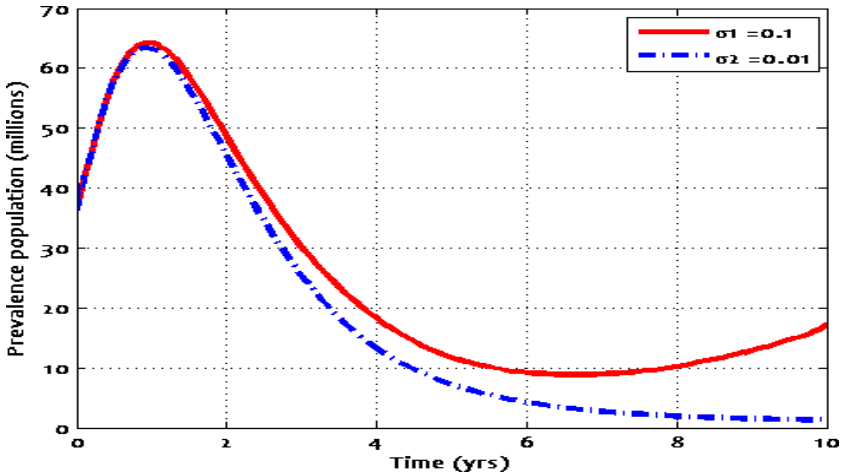

Figure 4.16: Prevalence of the disease for different values of σ

Similarly, Figures 4.9 - 4.12 show the population profile of the four compartments under the two scenarios with varying values of the immunity waning rate. Moreover, Figures 4.13, 4.14, 4.15, and 4.16 give the profile of the vaccination control, treatment control, the new cases , and prevalences of the disease respectively with respect to the two scenarios. Just as in the instances with the change in the transmission rate, the epidemic situation gradually dies out in the scenario with The implication of this is that if research can be intensified to develop a meningitis vaccine that would confer lifelong immunity on vaccinated individuals, the disease would eventually be wiped out in no distant time. Although, there may be need to scale up the vaccination coverage appropriately so that it can confer herd immunity on the populace at that time. It is important to mention that in order to minimize the incidence and prevalence of the disease as depicted in the graphs the treatment control has to be deployed at its maximal effective level all through while the vaccination control is utilized at its maximal effective level for about 4.5 years before it is discontinued.

V. Concluding Remarks

In this paper, a deterministic model of an SCIR Meningococcal Meningitis transmission dynamics was considered. This model was used as the constraint equations for the optimal control problem (OCP) formulated to depict the Meningitis epidemic situation in the Meningitis belt. The goal of the optimal control problem was to determine the optimal levels of each of the control measures that should be deployed to minimize the incidence and prevalence of the disease together with the cost of adopted control measures within a specified time frame. Thereafter, the appropriate optimality system to the OCP was derived using the Pontryagin's Maximum Principle (PMP) and the resulting optimality system was solved numerically. The simulation results show that control measures that can reduce the disease transmission rate and vaccination immunity waning rate as well as enhance the treatment success rate would be useful in the strive towards the eradication of the recurring epidemic. Also,

findings from the simulations indicate that the disease would persist as long as the control measures do not bring down the disease basic reproduction number below unity. In general, the simulation results demonstrate how modelling and simulation could help provide guidelines for the implementation of disease control measures in a cost-effective way without jeopardizing the set epidemiological target.

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