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Exact Analytical Solutions of Simples Electrical Circuit's Equations Using Maple Software and Lambert W Function.

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Abstract: *In this paper, we use Maple software to determine the exact analytical solutions for the current flows through a simple electrical circuit of a diode, a resistor and generators with four different types of electrical signals. We derive exact analytical expressions for the voltages at the terminals of all elements in the circuit. Then, we calculate the diode dynamical resistances. The proposed analytical solutions are all expressed using the Lambert W function. We highlight the influence for different intervals of the resistance and the four types of applied electrical signals on the expressions of the current intensity through the circuit and those of voltage across all circuit components. Finally, we show the influence of saturation current intensities, ideality factor values and temperature.*

Key words: *Exact analytical solution of an electrical circuit's equation, Lambert W function, ideality factor, dynamic resistance of a diode.*

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I. Introduction

We considered an electrical circuit (Fig.1) constituted by a voltage generator, a resistor (R) and a diode (D). In the first section, we determined the analytical expressions of the current intensity (I) (fig.1), voltages across the resistance (V_{AB}) and the terminals of the diode (V_{BC}) and so that of the dynamic resistance (R_d). In the second section, we consider four different types of signals: the sinusoidal signal, sawtooth, square and continuous. Using the Maple software, we represent the traces of the voltage and current in the circuit as a function of time. We study the influence of R, saturation current (I_S), temperature (T) and ideality factor (η) on I(t), V_{AB}(t), V_{BC}(t) et R_d(t).

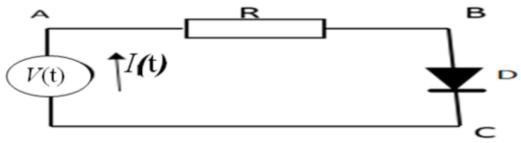


Figure 1: diagram of the electronic circuit with a simple diode.

Analytical expressions for I(t),V_{AB}(t), V_{BC}(t) et R_d(t)

The equation current-voltage of the electrical circuit comprising a simple diode (Fig.1) is written [1]:

$$I(t) = I_s\left(e^{a(V(t)-R.I(t))} - 1\right) \ (1); \ \ with\ V_{AB}(t) = R.I(t), V(t) = V_{AB}(t) + V_{BC}(t)\ and\ a = \frac{q}{\eta k_B T}$$,W

here k_B is the Boltzmann constant and q is the electric charge of an electron. By combining the above relations, and using the formalism for the Lambert W function [2], we obtain the following analytical expressions [3-4] :

$$I(t) = \frac{1}{aR} LambertW\left(aRI_s e^{a(V(t)+RI_s)}\right) - I_s \ (2)\ ; \ V_{BC}(t) = V(t) - \frac{1}{a} LambertW\left(aRI_s e^{a(V(t)+RI_s)}\right) + RI_s \ (3);$$

$$V_{AB}(t) = \frac{1}{a} LambertW\left(aRI_s e^{a(V(t)+RI_s)}\right) - RI_s \qquad (4)$$

Where Lambert W is the inverse function of the function f(x)= x exp(x), defined on [-1/exp(-1),+∞[.

The dynamic resistance of the diode is given by dV_{BC}/dI, therefore:

$$\frac{dV_{BC}(t)}{dI(t)} = \frac{1}{(Is + I(t))a} \;=\; R_d(\text{t}) \quad (5)$$

Influence for R, I_S, T and η in I(t),V_{AB}(t), V_{BC}(t) et R_d(t)

- **graphical representations**

In this part, we limit ourselves to representations to best illustrate the degree of influence of the parameters R, I_S, T, and η, for each type of signal.

Influence of R in the case of sinusoidal signal

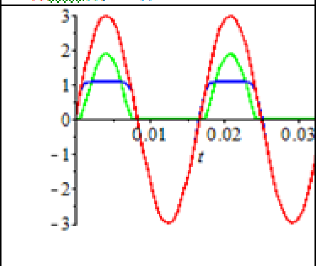
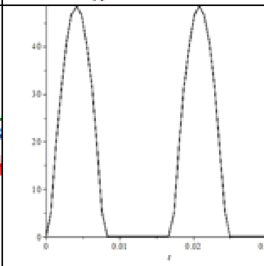
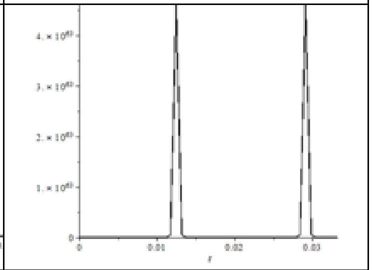
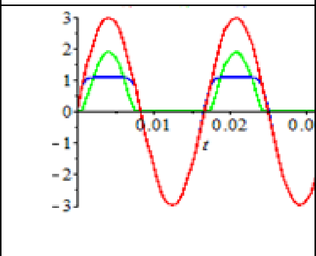
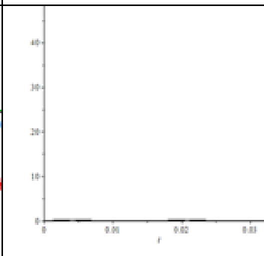
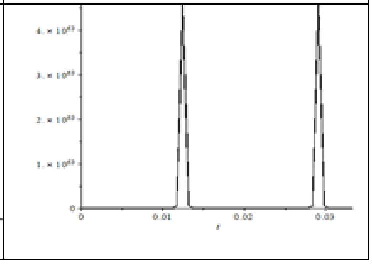
	V(t),V_{AB}(t),V_{BC}(t)	Courant I(t)	dynamic resistance R_d(t)
R=0.1 Ω			
R=100 Ω			

Table 1: The graph of V(t),V_{AB}(t), V_{BC}(t), I(t) and R_d(t) for R=0.1Ω and R=100Ω.

Influence of I_S in the case of sawtooth signal

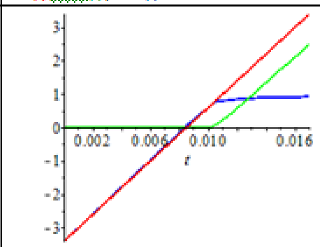
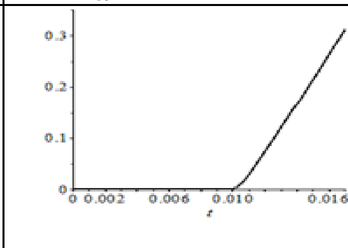
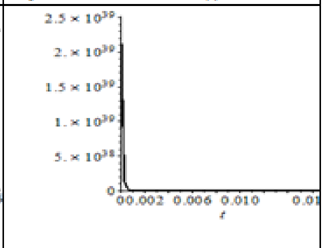
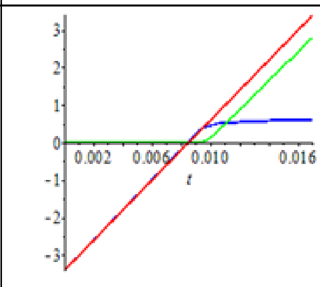
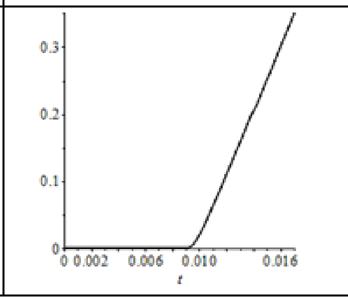
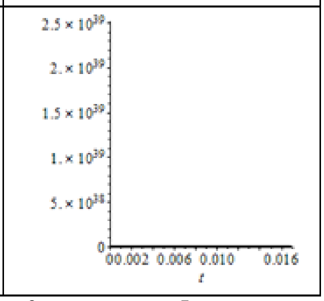
	V(t),V_{AB}(t),V_{BC}(t)	Courant I(t)	dynamic resistance Rd(t)
I_S= 10^{-9}A			
I_S= 10^{-7}A			

Table 2: The graph of V(t),V_{AB}(t), V_{BC}(t), I(t) and R_d(t) for I_S=10^{-9}A and I_S= 10^{-7}A.

Influence of T in the case of continuous signal

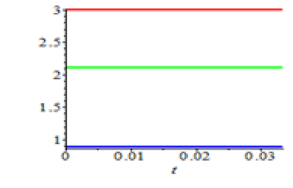
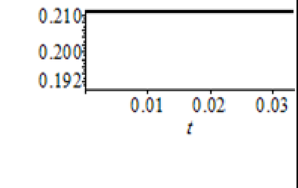
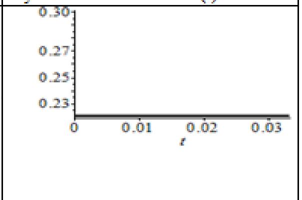
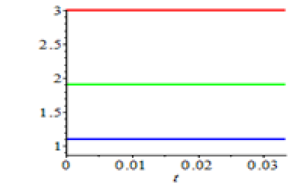
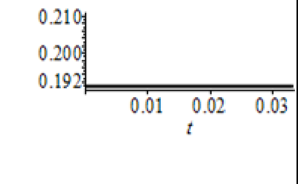
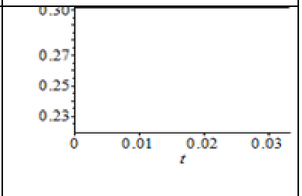
	V(t),V_{AB}(t),V_{BC}(t)	Courant I(t)	dynamic resistance Rd(t)
T= 300 k			
T= 370 k			

Table 3: The graph of V(t),V_{AB}(t), V_{BC}(t), I(t) and R_d(t) for T=300K and T=370K.

Influence of η in the case of square signal

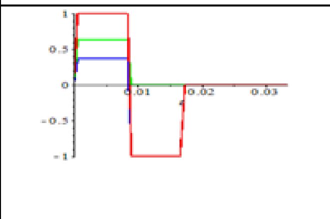
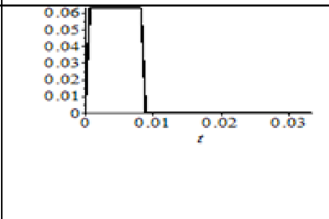
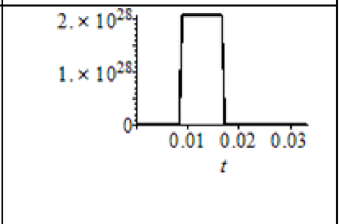
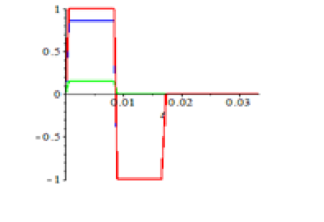
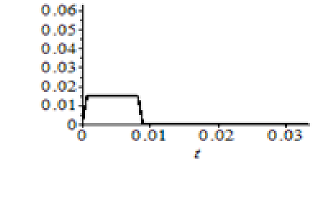
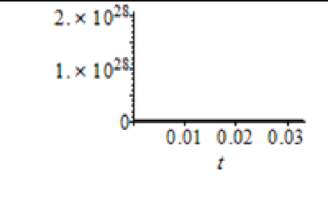
	V(t), VAB(t), VBC(t)	Courant I(t)	dynamic resistance Rd(t)
η=0,8			
η= 2			

Table 4 : The graph of V(t),V_{AB}(t), V_{BC}(t), I(t) and R_d(t) for η=0.8 and η=2.

The evaluation of the influence of four parameters R, I_S, T and η in the case of the four types of electrical signals considered is transferred to the four tables below:

Table 5: R takes values between 0.1 and 100 Ω.

	Sinusoidal signal	sawtooth signal	continues Signal	Square signal
V_{AB}(t)	grow	grow	grow	grow
V_{BC}(t)	decrease	decrease	decrease	decrease
Courant I(t)	decrease	decrease	decrease	decrease
R_d(t)	invariable	invariable	grow	invariable

Table 6: Is takes values between 10^{-9} and 10^{-7}A.

	Sinusoidal signal	sawtoot signal	continues Signal	Square signal
V_{AB}(t)	grow	grow	grow	grow
V_{BC}(t)	decrease	decrease	decrease	decrease
Courant I(t)	grow	grow	grow	grow
R_d(t)	decrease	decrease	decrease	decrease

Table 7: T takes values between 300 et 370K.

	Sinusoidal signal	sawtooth signal	continues Signal	Square signal
V_{AB}(t)	decrease	decrease	decrease	decrease
V_{BC}(t)	grow	grow	grow	grow
Courant I(t)	decrease	decrease	decrease	decrease
R_d(t)	decrease	decrease	grow	decrease

Table 8 : η takes values between 0.8 et 2

	Sinusoidal signal	sawtooth signal	continues Signal	Square signal
V_{AB}(t)	decrease	decrease	decrease	decrease
V_{BC}(t)	grow	grow	grow	grow
Courant I(t)	decrease	decrease	decrease	decrease
R_d(t)	decrease	decrease	grow	decrease

The observation of all of these four tables allows concluded that:

❖ The variations from V_{AB}(t), V_{BC}(t) and I(t) presents similar trends for the four types of signals.

❖ By against, The variation from R_d(t) in the case of continuous electric signal does not have the same pattern as compared to other three signals when R, T and η vary.

II. Conclusion

From a pedagogical point of view, Integration of software Computer Algebra such Maple help to solve of problems and overcome difficulties in teaching mathematics and physical.

In the intervals of variation of R, Is, T, and η which we have chosen, we noticed similar trends $V_{AB}(t)$, $V_{BC}(t)$, I(t) et $R_d(t)$, it will be interesting to extend the intervals of these parameters and see their influences. The type of continuous signal has an influence on the variations of $R_d(t)$ but it has no influence on variations in $V_{AB}(t)$, $V_{BC}(t)$ and I(t).

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