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Area of Outer Napoleon In The Parallelogram and Area of Outer Semi Napoleon in The Kite

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Abstract:*Outer theorem Napoleon in the parallelogram, on each side of the parallelogram is constructed a square in outside direction, then each midpoint square if connected it will be shape square. This square is called the outer Napoleon quadrilateral . While outer semi Napoleon's theorem on a kite that is on each side of a kite is constructed square in outside direction, then each midpoint square if connected it will produce a rectangle that is not a square. This quadrilateral is called the outer semi Napoleon quadrilateral. In this paper, the area of the outer Napoleon will be discussed in the parallelogram and the area of outer semi Napoleon's of kites. The process of proof is done in a simple way, namely by using trigonometric concepts and using congruence.*

Keywords: *Parallelogram, Kites, Napoleon Theorem, Semi Napoleon Theorem*

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I. Introduction To The

Napoleon Theorem invented by Napoleon Bonaparte (1769-1821) in (Gergiev & Mushkarov, 2017). After four years of Napoleon's death, this theorem was first published by W. Rutherford at the New Mathematical Question in The Ladies Diary in 1825 in the case of an equilateral triangle built outward [1, 4, 5 and 6], then developed by Wetzel in the case of built in equilateral triangles leading to [2,5,6 and 13]. Figure 1 is an illustration of Napoleon's theorem on triangles [3].

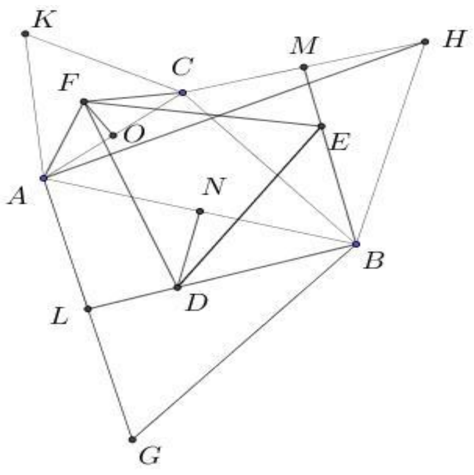

Figure 1: Napoleon theorem on the triangle

Napoleon's theorem states that if an equilateral triangle is built on each side of an arbitrary triangle, either built inward or outward, then from the three central points of the equilateral triangle a new equilateral triangle is formed. [1,2,3,7,8,9,11,12 and 14]. The following opinion according to Georgiev and Mushkarov is the new equilateral triangle called the Napoleon triangle [5,6 and 13]. Napoleon's theorem on triangles can be proved by using congruence [5] and trigonometric concepts [1,4, and 12]. Some developments of the Napoleon theorem are discussed in various textbooks and journals, among others [7,8,11,12 and 14].

One of them is the development of the Napoleon theorem on the quadrilateral written by [8,11 and 12], namely by starting construction from a quadrilateral that has two pairs of parallel sides and a rectangle that does not have two pairs of parallel sides. Quadrilateral that has two pairs of parallel sides such as parallelogram, rhombus, square and rectangle, when a square is built on each side of the rectangle pointing outward, then to the four square center points connected in a square, the square formed is what is called the outer Napoleon quadrilateral.

outer semi Napoleon's theorem in a quadrilateral that does not have two pairs of parallel sides such as a trapezoid, a kite and an arbitrary quadrilateral, when a square is built on each side of the rectangle outward, then the four square center points when connected to a trapezoid are legs forming a kite , the kites form an isosceles trapezoid, and in any quadrilateral form any quadrilateral [7,8,11 and 12]. So that it is assumed that not all quadrilaterals on each side are square and then from the four central points are square, this formed quadrilateral is what is meant by outer semi Napoleon's quadrilateral.

Based on the description above, the author discusses about determining the area of the rectangular outer regions of Napoleon and the quadrilateral of the Napoleon spring, in the case of a special quadrilateral, including parallelogram and kite. The concept used is a simple trigonometric concept and congruence.

In this writing Geogebra application is used which is very helpful in constructing points and lines. In learning mathematics, Geogebra can be used as a demonstration and visualization medium, construction aids, tools for finding mathematical concepts and preparing teaching materials. Figure 2 is one of the results that the author has done with the Geogebra application.

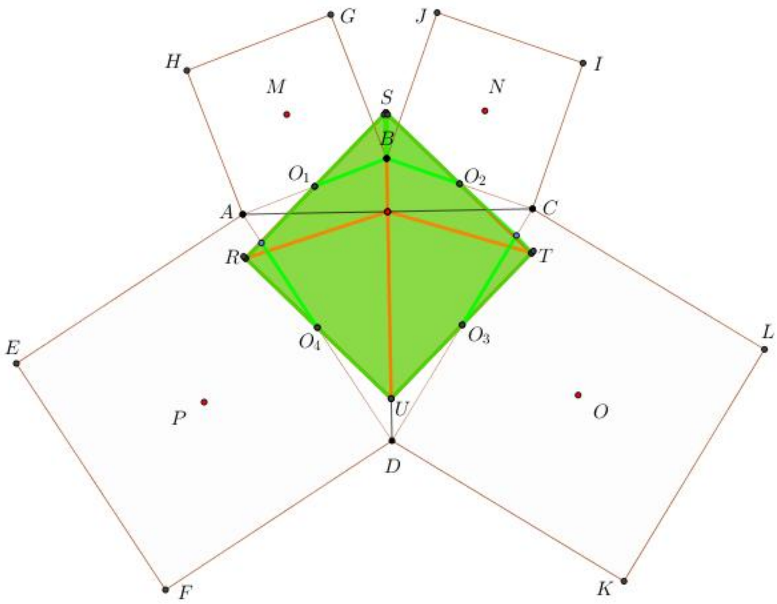

Figure 2: Extensive illustration □RSTU in the outer semi Napoleon

II. Outer Napoleon Theorem and Outer Semi Napoleon Theorem

Napoleon theorem in the parallelogram consists of two cases, namely case 1 describes a square built on each side of the parallelogram leads into inside [2,7 and 13], then each square center point is connected and produces a rectangle called the iner Napoleon quadrilateral, and case 2 describes the square built on each side of the parallelogram leads out [3,8 and 11] then each square center point is connected and produce a quadrilateral called the outer Napoleon quadrilateral.

Then semi Napoleon's theorem on the kite also consists of two cases, namely case 1 describes a square built on each side of the kite leads into inside [7], then each square center point is connected and produces a quadrilateral called the iner semi Napoleon quadrilateral and case 2 describes a square built on each side of the kite leads out [8 and 11], then each square center point is connected and produces a rectangle called semi outer Napoleon's quadrilateral. The following are given outer Napoleon theorem and outer semi Napoleon theorems in parallelogram and kite [7,8,11 and 12]. Theorem 2.1, 2.2 theorem, and the following 2.3 theorem, the references can be seen in [1,8,10,11,12 and 13].

Theorem 2.1. Provide rectangle in shaping ABCD parallelogram. On each side is constructed ABHG square , ADEF square, CDKL square, and BCIJ square in outside direction. For example M, N, O, and P are midpoints of square that constructed in outside direction. If the four midpoints are connected they will be shape MNOP square.

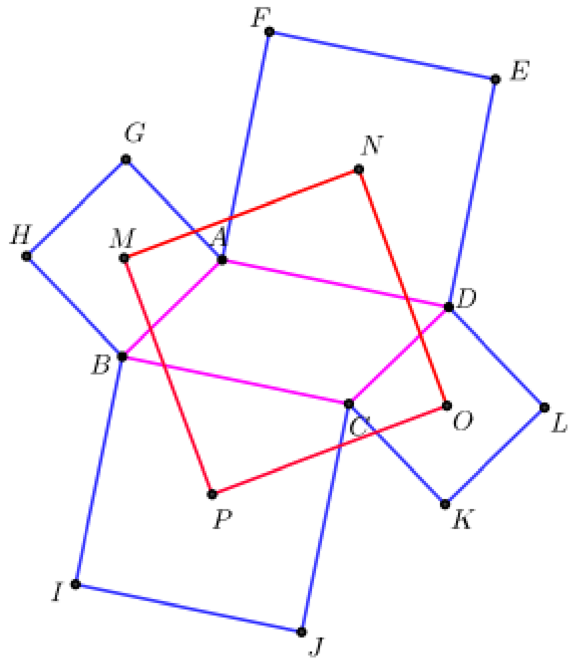

Figure 3: Illustration of theorem 2.1

Because in theorem 2.1 □MNOP is a square, the above theorem is called the outer Napoleon theorem.

Theorem 2.2. . Provide rectangle in shaping ABCD kite. On each side is constructed ABHG square, BCIJ square, CDKL square, and ADFE square in outside direction. For example M, N, O, and P are midpoint of square that construction in outside direction. If the four midpoints are connected they will be MNOP shape isosceles trapezoid.

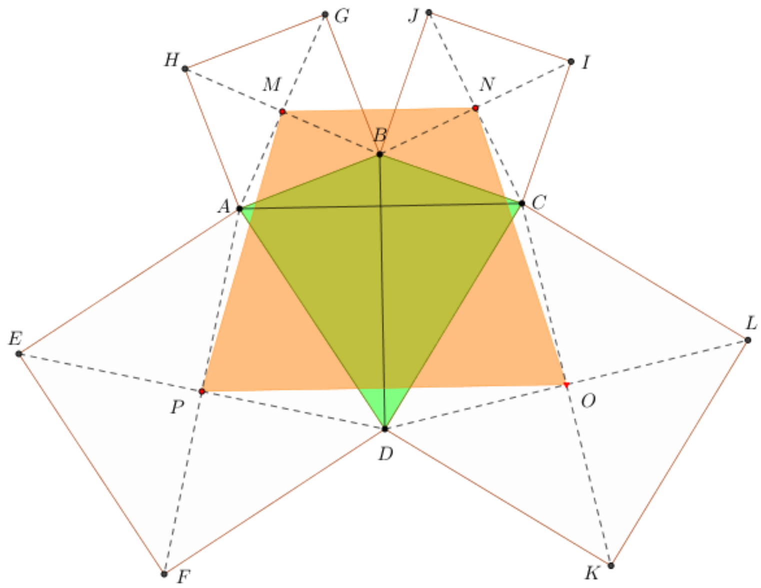

Figure 4: Illustration of theorem 2.2

Because in 2.2 theorem □MNOP does not produce a square, the above theorem is called outer semi Napoleon's theorem.

Theorem 2.3. Let ABCD denote any quadrilateral, then construct the midpoint of each side of the quadrilateral ABCD, let P, Q, R, and S. Connect the four midpoints of each side of the quadrilateral so that it will form a new quadrilateral, then it will form quadrilateral PQRS and have area $\frac{1}{2}$L□ ABCD.

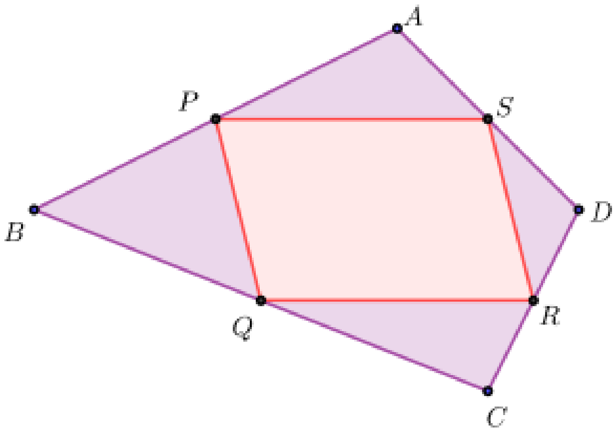

Figure 5: Illustration of theorem 2.3

III. Area of Outer Napoleon Quadrilateral and Outer Semi Napoleon Quadrangle

Area of rectangular outer Napoleon MNOP in parallelogram for cases of outside direction as in Figure 6 can be determined by determine one of the square MNOP sides. Furthermore, for the area of outer semi Napoleon MNOP quadrilateral on a kite as shown in Figure 8 can be determined by determining the diagonal length of PN and MO. The following is given a theorem about the area of the outer Napoleon quadrilateral in the parallelogram and the area of Napoleon's semi-outer outer quarter in the kite.

3.1 Theorem Suppose □MNOP as in 2.1 theorem then area □MNOP $= \frac{1}{2}a^2 + \frac{1}{2}b^2 + L$□ABCD.

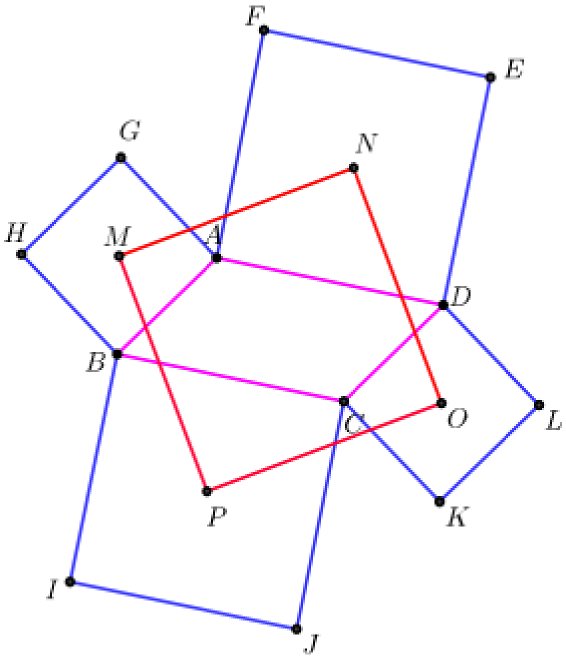

Figure 6: Illustration □MNOP is a quadrilateral in the Napoleon

Evidence: Look at Figure 7, suppose that the sides of AB = CD = a and AD = BC = b. Then, looking at the side AB there is aline BM, AM, which is half a diagonal of □ABHG , this also applies to the other side, namely theside CD, AD and BC that thelines DO, CO, AN, DN, BP, and CP are half diagonals from □CDLK, □ADEF and □BCJI.

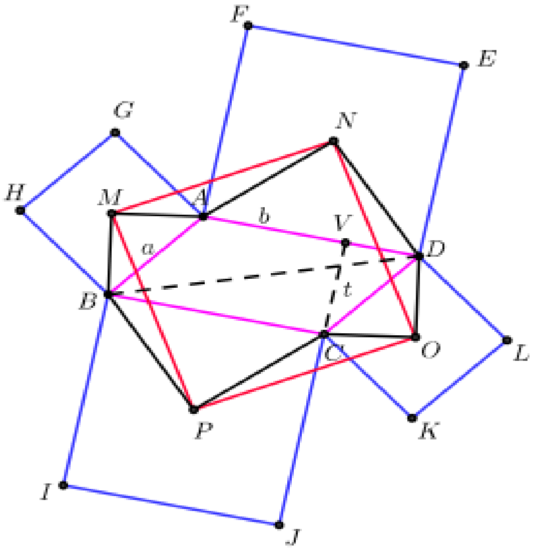

Figure 7: Diagonal illustration of PN and MO onkites MNOP

Next using the cosine rule at $\triangle AMN$ is obtained:

$$MN^2 = AM^2 + AN^2 - 2(AM)(AN)\cos\cos\angle MAN$$

$$= \left(\frac{1}{2}\sqrt{2}a^2\right) + \left(\frac{1}{2}\sqrt{2}b^2\right) - \left(2\left(\frac{1}{2}\sqrt{2}a\right)\left(\frac{1}{2}\sqrt{2}b\right)\right)\cos\cos\angle MAN$$

$= \frac{1}{2}a^2 + \frac{1}{2}b^2 - ab\cos\cos\angle MAN$

$= \frac{1}{2}a^2 + \frac{1}{2}b^2 - ab\cos\cos(270^0 - \angle BAD)$

$= \frac{1}{2}a^2 + \frac{1}{2}b^2 - ab(\cos\cos 270^0.\cos\angle BAD + \sin\sin 270^0.\sin\angle BAD)$

$= \frac{1}{2}a^2 + \frac{1}{2}b^2 + ab\sin\sin ADBAD$ (3.1)

Then using the triangle area rule for $\triangle ABD$ obtained:

$\sin\angle BAD = \frac{2.L\Delta ABD}{ab}$ (3.2)

Next by substituting equation (3.1) into equation (3.2) so that it is obtained

$MN^2 = \frac{1}{2}a^2 + \frac{1}{2}b^2 + ab\sin\sin ADBAD$

$$M\,N = \sqrt{\frac{1}{2}a^2 + \frac{1}{2}b^2 + L.ABCD}$$

MNOP is a square, so that

$$\text{Area MNOP} = \sqrt{\frac{1}{2}a^2 + \frac{1}{2}b^2 + L.ABCD} \times \sqrt{\frac{1}{2}a^2 + \frac{1}{2}b^2 + L.ABCD}$$

$$\text{Area MNOP} = \frac{1}{2}a^2 + \frac{1}{2}b^2 + L.ABCD$$

Theorem 3.2. Suppose that MNOP is an isosceles trapezoid which is outer semi Napoleon's quadrilateral on the kite ABCD, then L□ MNOP = 2L□ ABCD.

Proof: Let R be the midpoint of the side of PM, S midpoint from side MN, T midpoint from side NO, and U represent midpoint from side of PO, then connect point R, S, T and U to form □RSTU with use the theorem 2.3 then L□MNOP = 2L□ RSTU as Figure 8.

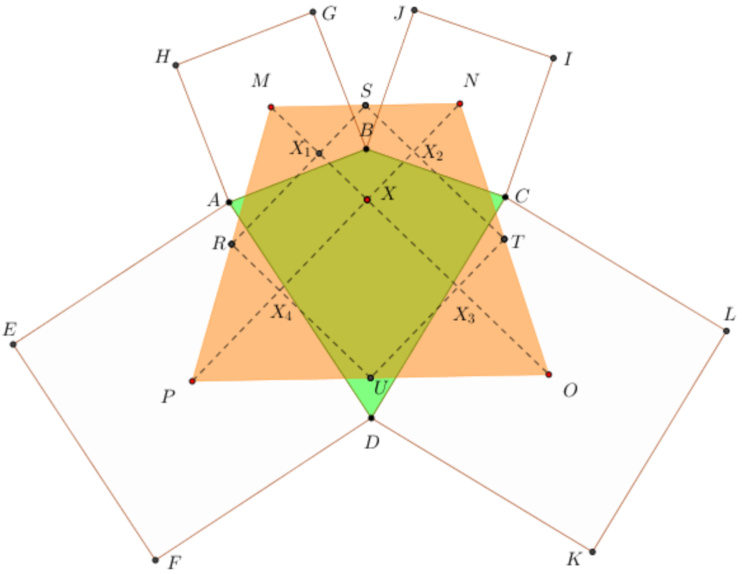

Figure 8: Illustration of Cut Point X_1, X_2, X_3, X_4

Then it will be shown L□RSTU = L□ABCD. Note Figure 9, for example O_1 is the intersection of theline RS with AB, O_2 is the intersection of the line ST with BC, O_3 is the intersection of the line CD with TU, and also O_4 is the intersection of the line AD with RU. So that obtained $\Delta DUO_3 \cong \Delta CTO_3$ and $\Delta DUO_4 \cong \Delta ARO_4$.

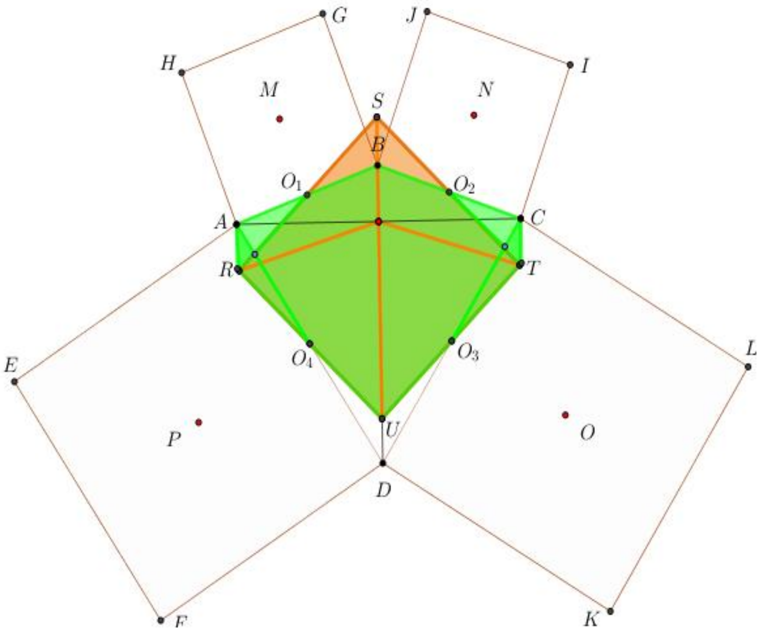

Figure 9: Illustration ΔDUO_3 And ΔDUO_4

After that connect the point TCO_2 so that it becomes ΔTCO_2, then $\Delta TCO_2 \cong SBO_2$, in the same way $\Delta\Delta ARO_1 \cong BSO_1$. Because $\Delta CTO_3 U\, DUO_3$, $\Delta ARO_4 U\, DUO_4$, $\Delta TCO_2 \cong SBO_2$, and $\Delta ARO_1 \cong BSO_1$ then the entire surface □RSTU is covered by all surfaces □ABCD like Figure 10, so based on the theorem 2.4, it is obtained that L□MNOP = 2L□ABCD.

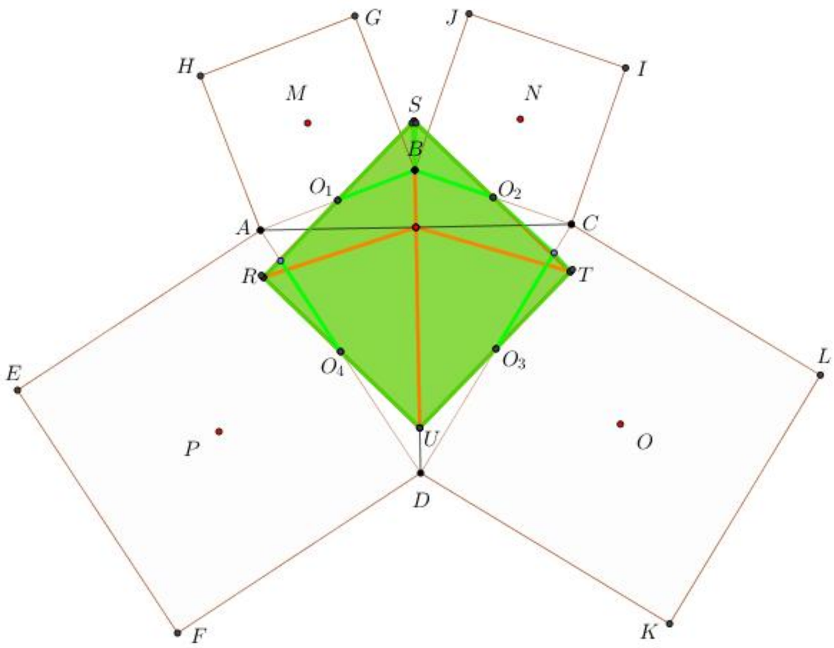

Figure 10: Illustration L□RSTU = L□ABCD

IV. Conclusion The

relationship between the area of the rectangular outer Napoleon in the parallelogram and the outer semi Napoleon quadrilateral in the kite is the area of the outer Napoleon quadrilateral and the area of outer semi Napoleon's quadrilateral is always greater than its original quadrilateral.

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