

## Evaluation of Task Bottleneck Generalized Assignment Problem in Supply Chain Capsule Optimization: Lsm Approach

Dr. V.Kalyana Chakravarthy<sup>\*</sup>, Dr. V.V.Venkata Ramana<sup>\*\*</sup>,  
Dr. C.Umashankar<sup>\*\*\*</sup>

Professor, Department of Aeronautical Engineering Surya group of institutions, vikiravandi, Villupuram  
Tamilnadu, India

Sr.Technical Director, National Informatics Centre, Vijayawada Andhra Pradesh, India

Registrar (Rtd), Rashtriya Sanskrit Vidyapeetha, Tirupathi, Andhra Pradesh, India

Corresponding Author: Dr. V.Kalyana Chakravarthy

**Abstract:** This paper aims to look at the concept of optimization at capsule level in a supply chain to avoid bottlenecks that exists in manufacturing facility. Usually a supply chain consists of raw material supplier, manufacturer, wholesaler, retailer and customer. This paper concentrates on specific capsule called process operations in which the inspection line process operations in an Liquefied Petroleum Gas (LPG) cylinder manufacturing unit is optimized.

Task Bottleneck Generalized Assignment Problem (TBGAP) aims at assigning the suitable quality supervisors to each process inspection stage and at the same time avoiding the defects in all these stages. Here optimizing the response time of the system and the effect of quality personnel in these stages treated as one of the capsule level optimization in the outsourcing of supply chain.

We present the response time of the process operations using Lexi Search methodology. The outcome would be treated as the optimal solution in terms of response time reduction, which shows effect on the manufacturing cost, and time bound assignment.

**Key Words:** Task Bottleneck, Generalized assignment problem, capsule, supply chain, response time.

<sup>\*</sup>Associate Professor, SGI-SET, Villupuram, TN <sup>\*\*</sup> Sr.Technical Director, National Informatics Centre, Vijayawada, A.P <sup>\*\*\*</sup> Registrar, Rashtriya Sanskrit Vidyapeetha, Tirupathi A.P India

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### I. Introduction

Bottlenecks occurs in manufacturing or fabrication process in Liquefied Petroleum Gas (LPG) cylinder manufacturing very frequently because of its more number of process inspection stages, though there are many process inspection stages available to avoid the bottlenecks because of the continuous working of machinery and quality supervisors by mistake or by accident. In LPG gas cylinder manufacturing unit there are 21 stages of process inspection in major are mentioned here as strip circumferential welding stage, foot ring welding stage, blanking operation stage, hydraulic press of providing non defective circular halves inspection stage, Physical material inspection of the incoming material stages, testing the sample materials for shear strength and compressive strength stage, quality control stage, coat thickness inspection stage, torque test, pneumatic inspection stage. Assigning the appropriate supervisor to a specific inspection stage plays a major role because of manufacturing cost and delivery time saving of the product. Here a study has conducted and tested numerically in the assignment of supervisors to appropriate process inspection stages based on the importance of process operations.

The Generalized Assignment Problem (GAP) [8] is comprised of a set of agents and with a set of tasks. The costs of each agent to perform each of the jobs, the amount of resources consumed by each agent performing them and the resource capacity available with each of the agents are also known. An assignment of jobs to agents is to be made in such a way that all the jobs are to be performed and the total resource needs on any agent does not exceed its capacity; such that the sum of the costs corresponding to the assignments is minimum.

VenkataRamana [33] discussed in his doctoral thesis about the various combinatorial programming problems in detail. The GAP with a mini-max (bottleneck) objective function instead of the usual minimum sum objective is studied. Many times, the appropriate objective is not the sum of the costs of the active assignments, but the maximum of the costs of these assignments. Problems of this type are called as the **Task Bottleneck Generalized Assignment Problems** (TBGAP) [1]. It can be defined as follows: there are a number of agents and number of tasks. Any agent can be assigned to perform any task, incurring some cost that may vary

depending on the agent-task assignment. It is required to perform all tasks by assigning exactly one agent to each task in such a way that the maximum cost among the individual assignments is minimized. The term "bottleneck" is explained by a common type of application of the problem, where the cost is the duration of the task performed by an agent. In this setting the "maximum cost" is "maximum duration", which is the bottleneck to schedule the overall job, to be minimized. The objective of TBGAP is to maximize the rate of production of the entire assembly line.

## II. Supply Chain Management

In the present global market scenario, the manufacturer is facing many challenges and one such is meeting the demand that is uncertain and even after meeting this requirement, subsequent supply of spare parts is once again an uncertain commodity. To overcome this, the concept called Supply Chain Management (SCM) was evolved to the rescue of many manufacturers with its outstanding solutions at every link level in the supply chain. The Supply Chain Management can be viewed in four different modules vis., 1. Supplier Management, which includes various aspects like supplier relation, supplier selection, and material selection at different stages etc., 2. The producer module, which discusses about the order, inventory and manufacturing managements (including the assembly line balancing),

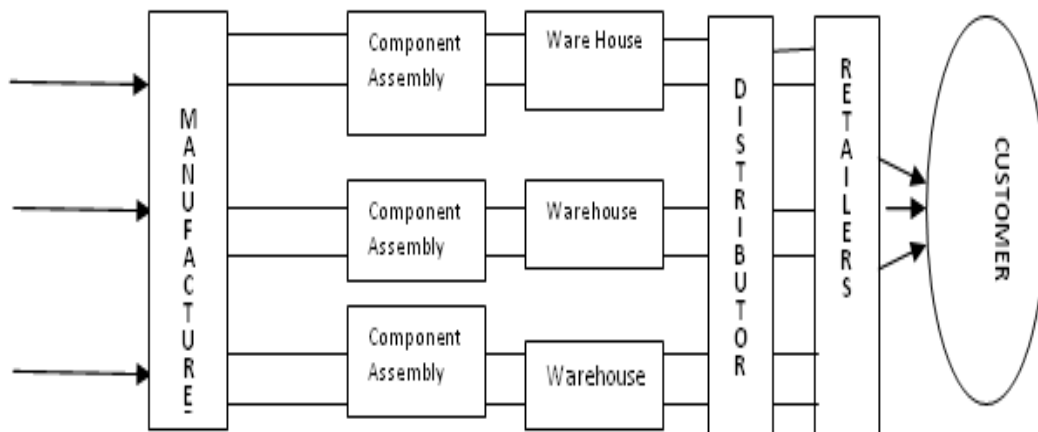


Figure: A Simple Supply Chain

3. The wholesaler module, which takes care of the logistics, distribution of various products at multi product manufacturer level and the last module, 4. The customer module, which reflects the performance of the above modules in chain like fashion which should result in the customer delight.

Douglas J. Thomas [11] defines the concept of Supply Chain Management (SCM) as the management of material and information flows both in and between facilities, such as vendors, manufacturing and assembly plants and distribution centers (DC). SCM is an area that has recently received a great deal of attention in the business community that is shown in figure. The very essence right from the manufacturer to customer delight every stage of the defined supply chain is to be optimized to its corresponding constraints so as to establish a précised processes and which leads to the Supply Chain Optimization concept which was discussed by Romero Morales [28].

In Supply Chain Management, utmost care is to be exercised between the suppliers – manufacturer tie up with regard to meeting the required specifications. It is at this stage assigning the supplier to a manufacturer based on the requirement is the management problem in the sense to assign the best of the supplier to a manufacturer(s) which gives rise to an exciting Assignment Problem through Supply Chain, visualising this concept the Generalized Assignment criteria has been evolved to Supply Chain Management, which leads the analyst to suggest novel procedures in optimizing the supplier assignments to different manufacturers is the concept of application of assignment problems in Supply Chain Management.

Generally, the initial consideration of Supply Chain Management is to choose the best of the available suppliers considering its corresponding cost relate to the manufacturer, lead time reduction, shipping, delivery date commitment, customer satisfaction, supply in right time, right quality with right quantity at the right location. This necessitated the researcher to study and establish different modeling aspects of Supply Chain Management in choosing the best of the supplier pave way for different types of Assignment Problems like Classical, Continuous, Quadratic and Discrete, which can be used in optimizing the supplier – producer assignment.

The information on the application of algorithmic aspects revealed fact that initially the Lexi Search, Branch and Bound, Simulated Annealing, Heuristics etc., are applied and later by the Genetic Algorithms. Compared to the different algorithmic aspects of application, for certain reasons the Genetic Algorithmic approach is proved to be an efficient method, for which reason the author is fascinated to study, probe and to report the Supply Chain Optimization problem solving methods through the application of several categories of GAP with Genetic Algorithmic approach.

**2.1 TBGAP in relation to SCM:** The Task Bottleneck Assignment Problem and Constrained Assignment Problem are the Combinatorial Optimization Problems in the branch of optimization or Operations Research in Mathematics. Maximum cost" is "maximum duration", which is the bottleneck to schedule the overall job, to be minimized. When there are a number of suppliers and a number of orders, any supplier can be assigned to perform any order in the supplier manufacturer module of SCM, incurring some cost that may vary depending on the supplier order assignment. It is required to perform all orders by assigning exactly one supplier to each order in such a way that the total cost of the assignment is minimum which very fact is the objective of the TBGAP. To minimize the total cost with minimum assignment, TBGAP problem is considered to study at supplier – manufacturer capsule of SCM.

### III. Problem Description

Let  $I=\{1,2,\dots,m\}$  and  $J=\{1,2,\dots,n\}$  be the index sets of QC Supervisors and Inspection stages respectively. For  $i \in I, j \in J$ , let  $C_{ij}$  be the cost of agent  $i$  to perform job  $j$ ,  $R_{ij}$  be the corresponding inspection stage requirement and let  $B_i$  be the resource capacity of QC Supervisor  $i$ . The TBGAP is to assign each inspection stage to one QC supervisor so that total resource requirement for any inspection stage does not exceed its availability and the maximum cost incurred is minimized. Thus, we have, mathematically

$$\begin{aligned} & \text{Minimize } (\text{Max } \{C_{ij} X_{ij}\}) & (1) \\ & \text{Subject to } \sum_{i=1}^n R_{ij} X_{ij} \leq B_i, i \in I & (2) \\ & \sum_{i=1}^n X_{ij} = 1, j \in J & (3) \\ & X_{ij} = 1, \text{ if job } j \text{ is assigned to agent } i & (4) \\ & = 0, \text{ otherwise.} \end{aligned}$$

It is to be noted that equations (2) - (4) define the usual constraint set of the general minimum GAP, whose objective function is  $\text{Min } \sum \sum C_{ij} X_{ij}$  (5). It is well known that GAP is NP-Hard in the strong sense, since even its feasibility question is so [41]. Hence, the same results apply to TBGAP.

### IV. Lexi - Search Algorithm Method:

The name **Lexicographic-Search** or **Lexi-Search** method implies that the search is made for an optimal solution in a systematic way, just as one searches for meaning of a word in a **dictionary**. When the process of feasibility checking of a partial word becomes difficult, though lower bound computation is easy, Pattern Recognition Technique can be used. Lexi-Search algorithms, in general, require less memory, due to the existence of a **Lexicographic** order of partial words. If **Pattern Recognition Technique** is used, the dimension requirement of the problem can be reduced, since it reduces the two-dimensional cost array into a linear array and the problem can be reduced to a linear form of finding an optimal word of length  $n$  [32,33], and hence reduces computational work in getting an optimal solution.

Ravi Kumar has considered the maximization version of the GAP as it is considered by Martello & Toth [15] and has presented a Lexi Search Algorithm using Data Guidedness. He has also considered the following aspects, to speed up the infeasibility checks as a Pre-Processing aid so that if a problem is found infeasible, the search can be terminated faster than any other algorithm.

- (i). All  $C_{ij}, W_{ij}, B_i$ , are positive integers ,
- (ii).  $W_{ij} < B_i$  and
- (iii).  $B_i \geq \text{Min } \{W_{ij}\} \forall i \in I$ .

If assumption (i) is violated, then fractions can be handled by multiplying through a proper factor; Knapsacks with  $B_i \leq 0$  can be eliminated for each item  $j$  having  $\min(C_{ij}) \leq 0 \quad \forall i \in I$ , one can add  $\lfloor \text{Min}(C_{ij}) \rfloor + 1, \forall i \in I$  to  $C_{ij}$  and subtract the same from the resulting objective function value.

If assumption (ii) is violated, then the item cannot be assigned to any Knapsack and hence GAP is infeasible.

If assumption (iii) is violated, then those Knapsacks can be eliminated from the instance

The degree of difficulty of answering the question of feasibility increases with the increase in the number of jobs. It is also observed that when a problem of  $2 \times 20$ , is infeasible even Martello & Toth's [14, 15] algorithm took inordinately more time to confirm the infeasibility. Hence Ravi Kumar developed a Lexi Search algorithm with Data Guidedness, tested it and found that it quickly confirms the infeasibility else gives optional solution. He has also used the Knapsack Problem terminology in his work. He developed two algorithms: one based on the ordering of the costs [LVGAP] and another based on the requirements [LVGAP] and found to be very efficient in different classes of problem data sets. Particularly when a problem is infeasible, the infeasibility is confirmed by LWGAP within **no time**. It is clearly shown that Lexi Search algorithm is very efficient and faster than the Martello & Toth algorithm.

Lexi search (pattern recognition) algorithm:

Ramana & Umashankar [31] have developed Lexi Search algorithm using Pattern Recognition Technique and found that this is also efficient and faster than Martello & Toth's algorithm in most of the cases. Observing the importance and advantage of the assumptions made by Martello and Toth, the above three assumptions are incorporated into the Lexi Search algorithm of Ramana and Umashankar [32]. A novel condition that all the  $m$  agents are to be assigned is also considered here and the computational results are reported.

#### 4.1 LEXI SEARCH ALGORITHM FOR THE TBGAP:

Step 0: Read the Cost, Requirement and the Resource Limit data. Arrange the alphabet Table, by arranging the costs column wise, in the increasing order of various agents for each job. Store the corresponding row indices. BTCA gives the bottleneck cost of assignment at any point of time. BND is the bound value on the cost; VT is the trial value for bound (999999) at the start. Array LW will contain the optimal assignment.  $J=0, K=1$ .

Step 1: Given cost matrix is converted into reduced form.

Step 2: Sorting the values of the matrix with index position in terms of rows of the given cost matrix.

Step 3: Rearrange the values of each column in the ascending order of the modified cost matrix

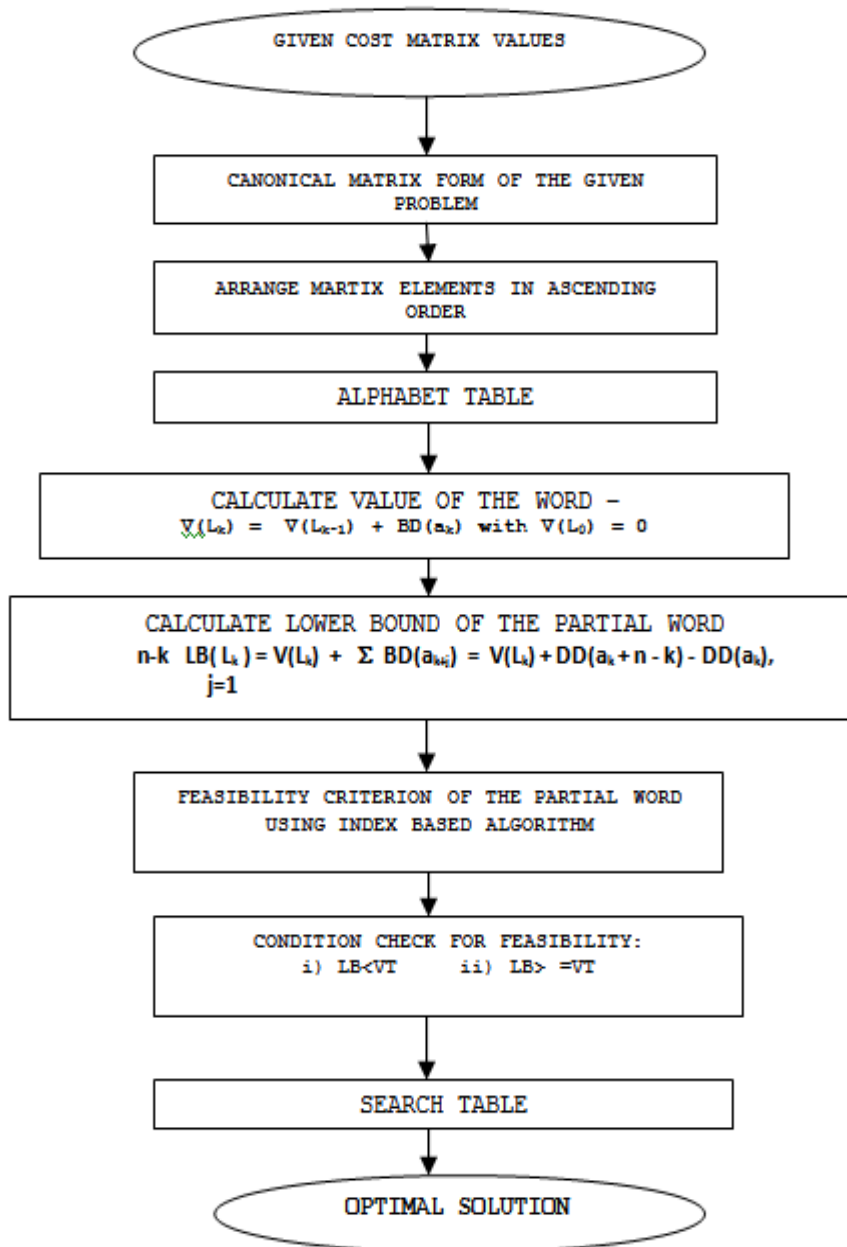
Step 4: Lexi search calculations to carry out for bound calculations, value of the word (VLK) and also the length of the word (LK) to be checked for sensitivity

Step 5:  $LB < VT$  go to feasible checking otherwise comeback

Step 6: As a part of feasible check, column repletion to be checked and the requirement matrix should not violate the maximum capacity.

At the end, if  $VT = 999999$  there is no feasible solution, otherwise solution with least bottleneck value is optimal.

4.2 LSA FLOW CHART:



4.3 CASE STUDY:

Consider the following problem of 3 Quality Control personnel and 4 process inspection stages, as given in the below Table. The Alphabet Table of the search is as given in the Table, where in each cell; the first letter denotes the actual row identity while the next entry is the corresponding cost, after arranging the original cost matrix column wise in ascending order. Thus we can see the striking difference in the Alphabet Table preparation.

In this case, one should not subtract the column minima from the respective columns, as the problem is to find out the bottleneck objective value. The Search Table for the same is given in the below Table.

QUALITY SUPERVISORS	COST MATRIX C				INSPECTION STAGE R				BUDGET LIMIT
	40	51	43	18	30	12	26	18	90
	20	42	34	55	15	29	22	28	100
	49	22	35	49	12	23	23	16	80

**ALPHABET TABLE**

**Modified Cost Matrix C'**

<b>2-20</b>	<b>3-22</b>	<b>2-34</b>	<b>1-18</b>
<b>1-40</b>	<b>2-42</b>	<b>3-35</b>	<b>3-49</b>
<b>3-49</b>	<b>1-51</b>	<b>1-43</b>	<b>2-55</b>

**SEARCH TABLE**

Sno	1	2	3	4	Value	Remarks
1.	2-20				20	Accept
2.		3-22			22	Accept
3.			2-34		34	Accept
4.				1-18	18	Accept, VT=18
5.				3-49	49	Reject, > VT
6.			3-35		35	Reject, > VT
7.		2-42			42	Reject, > VT
8.	1-40				40	Reject, > VT

A complete solution word for the above problem is thus (2, 3, 2, 1) implying the assignment that jobs 1,2,3,4 are respectively to be assigned to the agents 2,3,2,1 and the optimal bottleneck objective value is 18.

**V. Conclusion**

From the above it is observed that the Quality Control personnel obtain the optimal cost with minimum response time in the system at each process inspection stage. The optimal manufacturing costs obtained from the below table used in capsules of the supply chain in a time bound assignment that optimize the process operations module of the supply chain.

Time taken by the TBGAP algorithm - LSA includes optimal cost  
 COMPUTATIONAL RESULTS ON INTEL CORE DUO E7500 System\* 3GHZ PROCESSOR / 3GB RAM

6. COMPUTATIONAL EXPERIENCE:

S.No	1	2	3	4	5	6	7	8	9	10
<b>PROBLEM DESCRIPTION-MATRIX SIZE</b>	4 x 6	5 x 5	5 x 10	5 x 15	5 x 20	10 x 15	10 x 25	15 x 20	15 x 25	25 x 50
<b>COMPUTATIONAL TIME - LEXISEARCH METHOD</b>	0.0275	0.1375	0.165	0.188	0.38	0.55	0.825	0.1325	0.597	1.19
<b>OPTIMAL COST</b>	133	98	279	325	355	211	301	181	173	166

\* TIME IN SECONDS

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