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On Soft b-open sets In Soft Tritopological Space

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Abstract: *In this paper the soft bitopological space has been extended to Soft Tritopological Space and* $(1,2,3)^*$ *soft b open sets and* $(1,2,3)^*$ *soft b closed sets in soft Tritopological Space has been defined .Some properties on* $(1,2,3)^*$ *soft b open sets has been proved. Also the relation between* $(1,2,3)^*$ *soft regular open,* $(1,2,3)^*$ *soft preopen,* $(1,2,3)^*$ *soft semi open,* $(1,2,3)^*$ *soft* α *open,* $(1,2,3)^*$ *soft* β *open in soft tritopological space has been discussed.*

Keywords: $(1,2,3)^*$*soft b open set,*$(1,2,3)^*$ *soft b closed set,*$(1,2,3)^*$*soft regular open,*$(1,2,3)^*$*soft pre open,*$(1,2,3)^*$*soft semi open,*$(1,2,3)^*$*soft* α *open,*$(1,2,3)^*$*soft* β*open.*

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I. Introduction

In the year 1999,Moldtsov D[7]proposed the theory of soft sets with new dimension to explain the practical difficulties in engineering physics,computerscience,economics,social science and medical science.Muhammad ShabirandMunazzaNaz[6] setforth soft topological spaces which are defined over an initial universe with a fixed set of criteria.Hazra H,Majumdar P,Samanta S K[4] described the topic topology on soft subsets and soft topology. Basavaraj,Ittanagi M[3] explained soft bitopological spaces in subtle manner.RevathiN, BageerathiK [8]initiated soft b open sets and soft b closed sets in soft bitopologicalspace.Barathi B,Sathiya S,Ramesh Kumar T[2] introduced a new class of soft sgb closed sets in soft bitopological spaces.AsmhanFlienHassan[1] introduced soft Tritopological spaces.Indhu S,MathiSujitha T,Ramesh Kumar T [5] given Soft Gsr-Closed Sets In soft Bitopological space and some of its characteristics are investigated.

II. Preliminaries

Definition 2.1 [5]
A pair (F,A) is called a soft set over X, Where F is a mapping given by F:A → P(X).In other words ,a soft set over X is a parameterized family of subset of the universe X. For e∈ $A, F(e)$ may be considered as the set e-approximate elements of the soft set (F,A).

Definition 2.2[5]
For two soft sets (F,A) and (G,B) over a common universe X, we say that (F,A) is a soft subset of (G,B) if
i) A ⊆ B
ii) ∀ e ∈ A,F(e) ⊆ G(e).
we write (F,A) ⊆ (G,B).(F,A) is said to be a soft super set of (G,B).if (G,B) is a soft subset of (F,A) and is denoted by (F,A) ⊇ (G,B).

Definition 2.3[5]
For two soft sets (F,A) and (G,B) over a common universe X, union of two soft sets of (F,A) and (G,B) is the soft (H,C),where C=A∪B and ∀ e ∈ C

$$H(e)=\begin{cases} F(e) & if\ e \in A-B \\ G(e) & if\ e \in B-A \\ F(e)\cup G(e) & if\ e \in A\cap B \end{cases}$$

We write (F,A) ∪(G,B)=(H,C)

Definition 2.4[5]
The intersection (H,C) of two soft sets (F,A) and (G,B) over a common universe X denoted by (F,A) ∩(G,B) is defined as C=A∩B and H(e)=F(e)∩G(e) ∀ e ∈ C

Definition 2.5 [5]
Let $\bar{X} \in S(X)$ power set of $\bar{X}$ is defined by $P(\bar{X}) = \{\bar{X_i} \subseteq \bar{X},\ i \in I\}$and its cardinality is defined by $|P(\bar{X})| = 2^{\sum x \in E|F(x)|}$, where $|F(X)|$ is cardinality of F(x).

Example 2.6
Let $X = \{l, m\}$,$E = \{e_1, e_2\}$ and$\bar{X} = \{(e_1, \{l, m\}), (e_2, \{l, m\})\}$ .Then

$A_1 = \{(e_1, \{l\}), (e_2, \{l\})\}$ $A_9 = \{(e_1, \{X\}), (e_2, \{l\})\}$
$A_2 = \{(e_1, \{l\}), (e_2, \{m\})\}$ $A_{10} = \{(e_1, \{X\}), (e_2, \{m\})\}$
$A_3 = \{(e_1, \{l\}), (e_2, \{X\})\}$ $A_{11} = \{(e_1, \{X\}), (e_2, \{X\})\}$
$A_4 = \{(e_1, \{l\}), (e_2, \{\emptyset\})\}$ $A_{12} = \{(e_1, \{X\}), (e_2, \{\emptyset\})\}$
$A_5 = \{(e_1, \{m\}), (e_2, \{l\})\}$ $A_{13} = \{(e_1, \{\emptyset\}), (e_2, \{l\})\}$
$A_6 = \{(e_1, \{m\}), (e_2, \{m\})\}$ $A_{14} = \{(e_1, \{\emptyset\}), (e_2, \{m\})\}$
$A_7 = \{(e_1, \{m\}), (e_2, \{X\})\}$$A_{15} = \{(e_1, \{\emptyset\}), (e_2, \{X\})\}$
$A_8 = \{(e_1, \{m\}), (e_2, \{\emptyset\})\}$ $A_{16} = \{(e_1, \{\emptyset\}), (e_2, \{\emptyset\})\}$

A_1, A_2……………….$A_{16}$are all soft subsets of $\bar{X}$ .So $|P(\bar{X})| = 2^4$=16

Definition 2.7 [5]
Let τ be the collection of soft sets over X ,then τ is called a soft topology on X if τ satisfies the following axioms
i) $\emptyset, \bar{X}$belongs to τ
ii) The union of any number of soft sets in τ belongs to τ
iii)The intersection of any two soft sets in τ belongs to τ
The triplet (X, τ, E) is called a soft topological space over X.

Example 2.8
Let us consider the soft subsets of X that are given in example 2.6 then
$\tau_1 = \{\varphi, \bar{X}, A_1, A_{13}\}$,$\tau_2 = \{\varphi, \bar{X}\}$are soft topologies on X.

Definition 2.9 [5]
Aset X together with two different soft topologies is called soft bitopologicalspace.
It is denoted by (X, τ_1, τ_2,).

Example 2.10
Let us consider the soft subsets of X that are given in example 2.6 then
$\tau_1 = \{\varphi, \bar{X}, A_5, A_{13}\}$, $\tau_2 = \{\varphi, \bar{X}, A_1, A_4, A_9\}$
$\tau_{1,2} = \{\varphi, \bar{X}, A_1, A_4, A_5, A_9, A_{13}\}$are called soft open set
$\tau_{1,2} = \{\varphi, \bar{X}, A_2, A_6, A_7, A_{10}, A_{14}\}$are called soft closed set

Definition 2.11
A set X together with three different soft topologies is called soft tritopological space.
It is denoted by (X, τ_1, τ_2, τ_3)

Example 2.12
Let $X = \{l, m\}$,$E = \{e_1, e_2\}$and consider the soft sets over X in Example 2.6 where
$\tau_1 = \{\varphi, \bar{X}, A_5, A_{13}\}$, $\tau_2 = \{\varphi, \bar{X}, A_1, A_4, A_9\}$,$\tau_3 = \{\varphi, \bar{X}, A_1, A_{13}\}$
The soft open sets are $\tau_{1,2,3} = \{\varphi, \bar{X}, A_1, A_4, A_5, A_9, A_{13}\}$
The soft closed sets are $\tau_{1,2,3} = \{\varphi, \bar{X}, A_2, A_6, A_7, A_{10}, A_{14}\}$

Definition 2.13
A soft set (A,E) in a soft tritopological space $\bar{X}$ is called
i) (1,2,3)*soft regular open set if $(A, E) = \tau_{1,2,3} - int(\tau_{1,2,3} - cl(A, E))$ and (1,2,3)* soft regular closed set if $(A, E) = \tau_{1,2,3} - cl(\tau_{1,2,3} - int(A, E))$
ii)(1,2,3)*softα open set if $(A, E) \subseteq \tau_{1,2,3} - int(\tau_{1,2,3} - cl(\tau_{1,2,3} - int(A, E)))$and (1,2,3)* soft α closed set if $\tau_{1,2,3} - cl(\tau_{1,2,3} - int(\tau_{1,2,3} - cl(A, E))) \subseteq (A, E)$
iii) (1,2,3)*soft preopen if $(A, E) \subseteq \tau_{1,2,3} - int(\tau_{1,2,3} - cl(A, E))$ and (1,2,3)* softpre closed if $\tau_{1,2,3} - cl(\tau_{1,2,3} - int((A, E)) \subseteq (A, E)$
iv) (1,2,3)* soft semi open if $(A, E) \subseteq \tau_{1,2,3} - cl(\tau_{1,2,3} - int(A, E))$ and (1,2,3)* soft semi closed if $\tau_{1,2,3} - int(\tau_{1,2,3} - cl((A, E)) \subseteq (A, E)$
v) (1,2,3)*softβ open if$(A, E) \subseteq \tau_{1,2,3} - cl(\tau_{1,2,3} - int(\tau_{1,2,3} - cl(A, E)))$ and (1,2,3)* soft β closed if$\tau_{1,2,3} - int(\tau_{1,2,3} - cl(\tau_{1,2,3} - int(A, E))) \subseteq (A, E)$

Lemma 2.14
In (X, τ_1, τ_2, τ_3)be a soft tritopological space .we have the following results.
i)Every$(1,2,3)^*$ soft regular open set is $(1,2,3)^*$soft open
ii) Every$(1,2,3)^*$soft open set is $(1,2,3)^*$ soft α open
iii)Every$(1,2,3)^*$ softα open set is $(1,2,3)^*$ soft semi open
iv)Every$(1,2,3)^*$ soft preopen set is $(1,2,3)^*$ soft βopen
v)Every$(1,2,3)^*$ soft semi open set is $(1,2,3)^*$ soft β open
Proof:
Let (X, τ_1, τ_2, τ_3) be a soft tritopological space and $(A, E) \in \bar{X}$.Suppose (A,E) be a $(1,2,3)^*$ soft regular open set.Then $(A, E) = \tau_{1,2,3} - int(\tau_{1,2,3} - cl(A, E))$ since $\tau_{1,2,3} - cl(A, E)$ is a closed set in soft tritopological space and interior of any set is open. Hence the lemma (i) is proved
Let (A,E) be a $(1,2,3)^*$ soft open set .This implies $(A, E) = \tau_{1,2,3} - int(\tau_{1,2,3} - cl(A, E))$ since $(A, E) \subseteq \tau_{1,2,3} - cl(A, E) = \tau_{1,2,3} - cl(\tau_{1,2,3} - int(\tau_{1,2,3} - cl(A, E)))$.Hence the lemma (ii) is proved.
Let (A,E) be a $(1,2,3)^*$ softα open set .This implies $(A, E) \subseteq \tau_{1,2,3} - int(\tau_{1,2,3} - cl(\tau_{1,2,3} - int(A, E))) \subseteq \tau_{1,2,3} - cl(\tau_{1,2,3} - int((A, E))$ .Hence the lemma (iii) is proved.
Let (A,E) be a $(1,2,3)^*$ soft pre open set .This implies $(A, E) \subseteq \tau_{1,2,3} - int(\tau_{1,2,3} - cl(A, E)) \subseteq \tau_{1,2,3} - cl(\tau_{1,2,3} - int(\tau_{1,2,3} - cl(A, E)))$. Hence the lemma (iv) is proved
Let (A,E) be a $(1,2,3)^*$ soft semi open set .This implies$(A, E) \subseteq \tau_{1,2,3} - cl(\tau_{1,2,3} - int(A, E)) \subseteq \tau_{1,2,3} - cl(\tau_{1,2,3} - int(\tau_{1,2,3} - cl(A, E)))$. Hence the lemma (v) is proved

Remark 2.15
The converse of the above is need not be true as seen in the following examples.

Example 2.16
Let $X = \{l, m\}$,$E = \{e_1, e_2\}$and consider the soft sets over X in Example 2.6 where
$\tau_1 = \{\varphi, \bar{X}, A_5, A_{13}\}$, $\tau_2 = \{\varphi, \bar{X}, A_1, A_4, A_9\}$,$\tau_3 = \{\varphi, \bar{X}, A_1, A_{13}\}$
The soft open sets are $\tau_{1,2,3} = \{\varphi, \bar{X}, A_1, A_4, A_5, A_9, A_{13}\}$
The soft closed sets are $\tau_{1,2,3} = \{\varphi, \bar{X}, A_2, A_6, A_7, A_{10}, A_{14}\}$
i) $A_1$is a $(1,2,3)^*$soft open but not $(1,2,3)^*$soft regular open
ii) $A_3$is a $(1,2,3)^*$soft α open but not $(1,2,3)^*$ soft open
iii)$A_{15}$is a $(1,2,3)^*$soft semi open but not $(1,2,3)^*$ softα open
iv)$A_{15}$is a $(1,2,3)^*$soft β open set but not $(1,2,3)^*$soft pre open

Example 2.17
Let us consider the soft subsets of Xthat are given in Example 2.6.Let(X, τ_1, τ_2, τ_3) be a soft tritopologicalspace,where
$\tau_1 = \{\varphi, \bar{X}, A_4\}$, $\tau_2 = \{\varphi, \bar{X}, A_8\}$,$\tau_3 = \{\varphi, \bar{X}, A_{12}\}$
The soft open sets are $\tau_{1,2,3} = \{\varphi, \bar{X}, A_4, A_8, A_{12}\}$
The soft closed sets are $\tau_{1,2,3} = \{\varphi, \bar{X}, A_3, A_7, A_{15}\}$
v) The soft setA_{13} is $(1,2,3)^*$ soft β open set but not $(1,2,3)^*$ soft semi open set

III. $(1, 2, 3)^*$Soft b -Open Sets

In this section we introduce $(1,2,3)^*$soft b-open sets in soft tritopological spaces and study some of their properties.

Definition 3.1
Let (X, τ_1, τ_2, τ_3) be a soft tritopological space and $(A, E) \subseteq \bar{X}$.Then (A,E) is called $(1,2,3)^*$ soft b open set if $(A, E) \subseteq \tau_{1,2,3} - int(\tau_{1,2,3} - cl(A, E)) \cup \tau_{1,2,3} - cl(\tau_{1,2,3} - int(A, E))$

Example 3.2
In example 2.12 the$(1,2,3)^*$soft b-open sets are $\{\varphi, \bar{X}, A_1, A_2, A_3, A_4, A_5, A_7, A_9, A_{13}, A_{15}\}$

Theorem 3.3
Let (X, τ_1, τ_2, τ_3) be a soft tritopological space. Then
i) Every $(1,2,3)^*$ soft preopen open set is $(1,2,3)^*$ soft b-open set
ii)Every$(1,2,3)^*$ soft b open open set is $(1,2,3)^*$ soft β-open set
iii)Every$(1,2,3)^*$ soft semi openopen set is $(1,2,3)^*$ soft b-open set
Proof

Let (X, τ_1,τ_2,τ_3) be a soft tritopological space and $(A,E) \subseteq \bar{X}$.Let (A,E) be a $(1{,}2{,}3)^*$ soft preopen set. Then
$(A,E) \subseteq \tau_{1,2,3} - int(\tau_{1,2,3} - cl(A,E)$
$$\subseteq \tau_{1,2,3} - int(\tau_{1,2,3} - cl(A,E)) \cup \tau_{1,2,3} - int(A,E)$$
$$\subseteq \tau_{1,2,3} - int(\tau_{1,2,3} - cl(A,E)) \cup \tau_{1,2,3} - cl(\tau_{1,2,3} - int(A,E))$$
Thus (i) is proved
Let (A,E) be a $(1{,}2{,}3)^*$ soft b open set. Then
$(A,E) \subseteq \tau_{1,2,3} - cl(\tau_{1,2,3} - int(A,E)) \cup \tau_{1,2,3} - int(\tau_{1,2,3} - cl(A,E))$
$$\subseteq \tau_{1,2,3} - cl(\tau_{1,2,3} - int(\tau_{1,2,3} - cl(A,E))) \cup \tau_{1,2,3} - int(\tau_{1,2,3} - cl(A,E))$$
$$\subseteq \tau_{1,2,3} - cl(\tau_{1,2,3} - int(\tau_{1,2,3} - cl(A,E)))$$
Thus (ii) is proved.
Let (A,E) be a $(1{,}2{,}3)^*$ soft semi open set. This implies
$(A,E) \subseteq \tau_{1,2,3} - cl(\tau_{1,2,3} - int(A,E))$
$$\subseteq \tau_{1,2,3} - cl(\tau_{1,2,3} - int(A,E)) \cup \tau_{1,2,3} - int(A,E))$$
$\subseteq \tau_{1,2,3} - cl(\tau_{1,2,3} - int(A,E))) \cup \tau_{1,2,3} - int(\tau_{1,2,3} - cl(A,E))$
Thus (iii) is proved

Remark 3.4

The converse of the above lemma is need not be true as seen in the following example.

Example 3.5

Let us consider the soft subsets of X that are given in Example 2.6.Let (X, τ_1,τ_2,τ_3) be a soft tritopologicalspace,where
$\tau_1 = \{\varphi, \bar{X}, A_4\}$, $\tau_2 = \{\varphi, \bar{X}, A_8\}$,$\tau_3 = \{\varphi, \bar{X}, A_{12}\}$
The soft open sets are $\tau_{1,2,3} = \{\varphi, \bar{X}, A_4, A_8, A_{12}\}$
The soft closed sets are $\tau_{1,2,3} = \{\varphi, \bar{X}, A_3, A_7, A_{15}\}$
i) The soft set A_1 is $(1{,}2{,}3)^*$soft b-open set but not $(1{,}2{,}3)^*$soft pre-open set
ii)The soft set$A_{14}$is $(1{,}2{,}3)^*$soft β open set but not $(1{,}2{,}3)^*$soft b-open set
iii) The soft set $A_{13}$is$(1{,}2{,}3)^*$soft b open set but not $(1{,}2{,}3)^*$soft semi open set

Remark 3.6

The above discussions are summarized in the following diagrams

$(1{,}2{,}3)^*$Soft regular open
↓
$(1{,}2{,}3)^*$Soft open
↓
$(1{,}2{,}3)^*$Softαopen
↓
$(1{,}2{,}3)^*$Soft semi open
↓
$(1{,}2{,}3)^*$Soft b open
↓
$(1{,}2{,}3)^*$Softβ open

IV. $(1,2,3)^*$Soft b-Closed Sets

In this section we introduce $(1{,}2{,}3)^*$soft b-closed sets in soft tritopological spaces and study some of their properties.

Definition 4.1

Let (X, τ_1,τ_2,τ_3) be a soft tritopological space and $(A,E) \subseteq \bar{X}$ .Then (A,E) is called $(1{,}2{,}3)^*$ soft b closed set if $\tau_{1,2,3} - int(\tau_{1,2,3} - cl(A,E)) \cup \tau_{1,2,3} - cl(\tau_{1,2,3} - int(A,E)) \subseteq (A,E)$

Example 4.2

In example 2.12 the$(1{,}2{,}3)^*$soft b-closed sets are $\{\varphi, \bar{X}, A_2, A_4, A_5, A_6, A_7, A_8, A_{10}, A_{12}, A_{14}\}$

Theorem 4.3

Let (A,E) be a $(1{,}2{,}3)^*$ soft b closed set in soft tritopological space
i) If $(A,E)$is a $(1{,}2{,}3)^*$ soft regular closed set then $(A,E)$is a$(1{,}2{,}3)^*$ soft semi closed set.
ii) If (A,E) is a $(1{,}2{,}3)^*$ soft regular open set then (A,E) is a $(1{,}2{,}3)^*$ soft pre closed set.

Proof

Since (A,E) be a $(1,2,3)^*$ soft b closed set, $\tau_{1,2,3}-cl(\tau_{1,2,3}-int(A,E)) \cap \tau_{1,2,3}-int(\tau_{1,2,3}-cl(A,E)) \subseteq (A,E)$ .Since (A,E) is a $(1,2,3)^*$ soft regular closed set, $(A,E)=\tau_{1,2,3}-cl(\tau_{1,2,3}-int(A,E))$ .Therefore $(A,E) \cap \tau_{1,2,3}-int\left(\tau_{1,2,3}-cl(A,E)\right) \subseteq (A,E)$ .Thus $\tau_{1,2,3}-int\left(\tau_{1,2,3}-cl(A,E)\right) \subseteq (A,E)$ .Hence (i) is proved.

Since (A,E) be a $(1,2,3)^*$ soft regular open set ,$(A,E)=\tau_{1,2,3}-int\left(\tau_{1,2,3}-cl(A,E)\right)$.

Therefore, $(A,E) \cap \tau_{1,2,3}-cl\left(\tau_{1,2,3}-int(A,E)\right) \subseteq (A,E)$ .This implies, $\tau_{1,2,3}-cl\left(\tau_{1,2,3}-int(A,E)\right) \subseteq (A,E)$. Hence (ii) is proved.

V. Conclusion

This paper deals with the concept of $(1,2,3)^*$soft b open sets and $(1,2,3)^*$ soft b closed sets in soft tritopological space .Some properties on $(1,2,3)^*$ soft b open and closed sets has been proved. This paper will be supportive to solve various type of open sets on soft tritopological space.

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