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Shape Preserving Curves and Surfaces Interpolation: State of the Art

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Summary: *Computer aided geometric design (CAGD) is concerned with the approximation and representation of curves and surfaces when they are subjected to computer processing. CAGD is a relatively new field. The idea in CAGD is to find representations of curves and surfaces which are easy to treat on a computer, and to render on a graphical device such as a computer screen. The work in this field was started in the mid-1960s. Barnhill and Riesenfeld established the field of CAGD in 1974 when they organized a conference on the topic at the University of Utah in the United States of America (USA).A significant element is embodied by the design of curves and surfaces within the building of various items like vehicle bodies, wings and fuselages for aircraft, ship hulls,in additiontothe definitionofphysical, geologicaland medical occurrences. Innovative aspects of CAGD use encompass computer vision and scrutinyof produced items, the film industry, and image evaluation for medical research.From this discussion paper, positivity,monotonicityand convexity are important shapes. These are independent shapes which are found inherited in data. Previous studies had discussed these shapes independently using different mathematical models and methodologies.*

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Computer aided geometric design (CAGD) is concerned with the approximation and representation of curves and surfaces when they are subjected to computer processing. CAGD is a relatively new field. The work in this field was started in the mid-1960s. Barnhill and Riesenfeld established the field of CAGD in 1974 when they organized a conference on the topic at the University of Utah in the United States of America (USA). The conference is considered as the founding event of the field. The first textbook of CAGD was "Computational Geometry for Design and Manufacture" byFaux and Pratt appeared in 1979. The first journal of "Computer Aided Geometric Design" was founded in 1984 by Barnhill and Boehm. Another early conference was heldinParisin1971, whichfocused on automotivedesign.The conferencewas organizedbyBézier. Therewasalsoaseriesofworkshops startedin1982attheMathematics Research Institute at Oberwolfach, which were organizedby Barnhill(Farin, 2002).

Asignificant element is embodied by the design of curves and surfaces within the building of various items like vehicle bodies, wings and fuselages for aircraft, ship hulls, in additiontothe definitionofphysical, geological and medical occurrences. Innovative aspects of CAGD use encompass computer vision and scrutiny of produced items, the film industry, and image evaluation for medical research. Spline functions in their simplest and most useful form are nothing more than pieces of polynomials joined together smoothly at certain knots, were first introduced into CAGD by Ferguson (1964) from Boeingin 1963. At about the same time deBoor and Gordon studied these curves at General Motors (Böhm et al., 1984).

In the efforts of engineers and scientists, spline functions comprise an essential and common element. They comprise thekeyinstrumentinCAGD, which is essential to build curves and surfaces comprising particular shape features for most of CAGD applications. Splines remain very important toolsinamultitudeof applicationsinvolving curve fitting and design. The main reason for this is their excellent approximation properties. Also they are easy to manipulate, store, and evaluate on a computer. Polynomial splines do not retain the shape properties of the data. This problem is known as the problem of shape preserving. During the last two decades, different authors have developed various algorithms of spline approximation with both local and global shape control. Based on spline functions, such methods are usually called methods of shape preserving spline approximation.

One of the main applications of shape preserving spline approximation is CAGD. The idea in CAGD is to find representations of curves and surfaces which are easy to treat on a computer, and to render on a graphical device such as a computer screen. To be of most use, these representations should have convenient handles consisting of a set of parameters which can be varied by the user to make well-defined changes in the curve or surface. Hence the main challenge is to develop algorithms that select these parameters automatically. Very strong requirements must be met in industrial design. Usually, a designer provides the envelopes of a car body, ship hull, airplane fuselage, engine details of complex shape as a discrete set of points. Hence to produce the body we need to describe these points as lying on some curves or some surfaces. Anydiscontinuitiesofthe firstandeven second derivative may leadtoflow separation that is to an increase in friction. By this reason, the designer is often interested in a very smooth approximation which preserves the shape of the data(Kvasov, 2000).

Preservation of shape features, inherent in the data, by an interpolant is one of important research subjects in CAGD. In particularly, when data from some scientific observation are considered, a user may be interested to visualize it graphically. There aresplineswhichcanproducesmoothcurvesbutunabletopreservethe inherentshape of given data. In many interpolation problems, the solution that preserves some shape properties, such as positivity, monotonicity, and convexity is important. Numerous physical cases have entitiesthatget meaningonlywhentheirvaluesappearinapositive,a monotone, or a convex shape. Therefore, discussing shape preserving interpolation problems is important to provide a visually pleasing and computationally economical solution to various scientific events.

Positivity is an important property of the data that occurs in visualizing a physical quantity that cannot have a negative value, which may arise if the data is taken from some scientific, social or business environments, furthermore stability of radioactive substance and chemical reactions, solvability of solute in solvent, population statistics ,observationofgas discharge when certain chemicalexperimentisin process(Hussain and Sarfraz,2008), depreciationofthepriceof computersinthemarket(Abbasetal., 2014), monthly rainfall amounts resistance offered by an electric circuit, probability distribution,volumeand density(Hussainand Hussain, 2006b), dissemination rate of drugsintheblood,and half-lifeofa radioactive substance(Tahatetal.,2015a),are few examples of entities which are always positive. Therefor the negative graphical display of these physical quantities is meaningless.

Monotonicity is another important shape property that is applied in manyscientific applications such as physical situations and engineering problems, where entities only have a meaning when their values are monotone. Data generated from stress of a material, uric acid level in patients suffering from gout(Tahat et al., 2015b), erythrocyte sedimentation rate (E.S.R.) in cancer patients, rate of dissemination of drug in blood (Hussain and Hussain, 2007), dose-response curves and surfaces in biochemistry and pharmacology, design of aggregation operators in multi criteria decision making and fuzzy logic, approximation of copulas and quasi-copulas in statistics, empirical option pricing models infinance, approximation of potential functionsinphysicaland chemical systems(Beliakov,2005)arefewexamplesof entities which arealways monotone. Convexity is another important shape property and plays a major role in various applications including telecommunication systems designing, nonlinear programming, engineering, optimal control, optimization, parameter estimation, approximation theory and others (Sarfraz et al., 2012).

Many researchers have addressed the problem of data visualization. Schumaker (1983)used piecewise quadratic polynomial to preserve the shape of monotone data by introducing an additional knot in each subinterval where the shape of the data is not preserved. Brodlie and Butt (1991)developed a C1 piecewise cubic interpolation to preserve the shape of convex data. They divided the interval where convexity was lost into twosubintervals byinserting extra knotsinthat interval. Buttand Brodlie(1993) used the same technique to develop a C1 positivity preserving scheme for 2D data. Goodman et al.(1991)developed two interpolating methods to maintain the shape of constrained data utilizing a rational cubic interpolant. Firstly they preserved the shape of the databy scaling the weightsbysome scale factor. Secondly theyintroducedanew data point to retain the shape of the data. Unlike(Brodlie and Butt, 1991),(Butt and Brodlie, 1993), and(Goodman et al., 1991), the data visualization scheme for shape preserving curvedeveloped in this thesis neither require the specification of the interval in which the shape of data is lost nor scaling of weights. The schemes developed in this thesis, assure an automated selection of parameters in each subinterval.

Hussainetal.(2010)introduced rational interpolant (cubic/linear)withoneshape parameterto visualizetheshapeofpositive, constrained and monotonedatabyimposing data dependent constraints on the shape parameter. Sarfraz et al. (2000, 2001), Sarfraz (2002) and Sarfraz and Hussain (2006)presenteda C1 rational cubic function with two shape parameters to maintain the shape properties of the shaped data. Data dependent conditions were derived on shape parameters to preserve the positivity of positive data(Sarfraz et al., 2000), positivity and monotonicity of shaped data (Sarfraz et al., 2001), positivity and convexity of 2D data (Sarfraz, 2002) and positivity, monotonicity and convexity (Sarfraz and Hussain, 2006). The rational functions used in (Sarfraz, 2003) and (Hussain and Hussain, 2007) have also two parameters which are constrained to visualize the shape of data. However, no flexibility is provided for the user to refine the curves further if needed, whereas the schemes developed in this thesis has four parameters, where two of the parameters are constrained to visualize the data and the other two parameters provide the user with a degree of freedom to adjust the shape of the generated curve which is more suitable for interactive curve design. Hussain and Sarfraz(2008, 2009)used a rational cubic function in its most generalized form (four shape parameters) to preserve the shape of positive and monotone data in(Hussain and Sarfraz, 2008)and(Hussain and Sarfraz, 2009)respectively. Sarfrazetal. (2012)proposed piecewise rational functionina cubic/cubicform, which involves four shape parameters in each interval in its construction.Twoof these shape parameters are constrained to preserve the shape of convex, monotone, and positive data while the other two parameters are used to modify positive, monotone and convex curves to obtain a visually pleasing curve.

A rational cubic Ball interpolantwasdevelopedby Piahand Unsworth (2013)with two shape parameters that can be used to generate the desired monotone curves from monotone data. However, no flexibility is provided for the user to refine the curves further if needed, so it is unsuitable for interactive curve design. Shape preserving interpolation problem for visualization of 3D data is one of the basic problem in computer graphics, CAGD, data visualization and engineering. It also arises frequently in manyfields including military, education, art, medicine, advertising, and transport military, art and many otherfields. Data are noticed from mathematical description, scientific phenomenon and real sciences, and one of the main interests for the designer in data visualization environment is to convert this data into anygraphical representation that makes the content easier to understand and provides an insight into the noticed data. These data may havesome special shape properties such as positivity, monotonicity and convexity.

Inmanyshapepreserving interpolationproblems,itisrequiredthatthefunctionexhibits the shape features ingrained in the data and the problem become critical when it fails to retain this shape property. Furthermore, smoothness is also required to demonstrate the data in a visual pleasant display. Ordinary spline methods usually ignore these characteristics thus exhibiting undesirable inflections or oscillations in resulting curvesandsurfaces.Duetothis reasonagood amountofworkhasbeen published that focuses on surfaces shape preserving. Piahetal. (2005)have discussedthe problemof positivity preservingfor scattered data interpolation. Sufficient conditions are derived on the ordinates of the Bézier control points in each triangle to preserve the positivity of data. Hussain and Sarfraz (2008)utilized a C1 rational cubic function to preserve the shape of positive data, then they extended it to an interpolating rational bi-cubic form, involving eight shape parameters. Constraints were derived on four shape parameters in the description of the rational bi-cubic function to visualize the shape of positive data in the view of positive surfaces and the remaining four shape parameters were left to the user to refine the shape of the surfaces.

The problem of visualization of constrained data which is a generalized case of problemof positive data visualization consideredbyafew authors. This problem usually arises in the comparative study of data(Hussain et al., 2008). Brodlie et al.(2005) proposed the method of visualizing constrained data. They modified the quadratic Shepard method, which interpolates scattered data of anydimensionality to preserve positivity. Brodlie et al.(1995) discussed the problem of surface data interpolation subject to simple linear constraints. Theydeveloped a piecewise bi-cubic function from data on a rectangular grid. The problem of positivity was generalized to the case of linearly constrained interpolation, where it was required that the function lie between bounds which were linear functions.

ChanandOng (2001)constructedarange restricted C1interpolant to scattered data, sufficient non-negativity conditions derived on the Bézier ordinates to ensure the non-negativity of a cubic Bézier triangular patch. Constraints are derived on derivatives and the gradients modified at the data points if needed to guarantee the achievement of non-negativity conditions. Carlsonand Fritsch (1985)developedabi-cubic polynomial interpolation scheme to preserve the shape of monotone data. Necessary and sufficient conditions are derived on derivatives, such that the resulting bi-cubic polynomial is monotone. Beatson and Ziegler (1985) presented a visualization of monotone data arranged over a rectangular grid by C1 monotone quadratic spline. In(Brodlie et al., 1995), (Chan and Ong, 2001), (Carlson and Fritsch, 1985), and (Beatson and Ziegler, 1985) the necessary and sufficient conditionswere derived on derivatives values at grid points to preserve the shape of the 3D data. Thus the derivative values at the data sites were fixed and the proposed schemes were not applicable to data with derivatives at the data points. Hussain and Hussain(2006b)developed a rational bi-cubic interpolant to preserve the shape of positive surface data and the surface data that lies abovea plane. Simple data dependent conditions were derived on shape parametersto conservetheshapeofsurfacedata. Hussainand Hussain(2006a) preserved the shape of monotonic surface databy utilizinga rational bi-cubic function with four shape parameters in it is description. Simple constraints are derived on shape parameters to preserve the shape of data. Asmooth surface interpolation scheme for positive and convex data has been developed in(Hussain et al., 20111). The scheme has been extended from the rational quadratic spline function of Sarfraz to a rational bi-quadratic spline function. Simple data dependent constraints are derived on the shape parameters in the description of rational bi-quadratic spline function to preserve the shape of 3D positive and convex data.

Hussain and Hussain(2006b) extended the rational cubic functiondevelopedby HussainandAli(2006)to rational bi-cubic partially blended function(Coons patches). Simple constraints are developed on the shape parameters in the description of rational bi-cubic function to visualize positive data and data that lies above the plane. Sarfraz et al.(2010)developeda C1 piecewise rational cubic interpolant, with two shape parameters. Data dependent shape conditions are imposed on the shape parameters to preserve the shape of data. The rational cubic spline has been extended to a rational bi-cubic partially blended surface (Coons-patches) and derived constraints on parameters to visualizetheshapeofpositivesurfacedata. Shaikhetal.(2011) extendthe rational cubic functiondevelopedby Hussainetal.(2011)toa rational bi-cubicpartially blended function. Data dependent constraints are derived on shape parameters to visualize surface lie above the plane. Hussain and Hussain(2007)used piecewise rational cubic function to visualize monotone data in the view of monotone curves by making constraints on shape parameters in the description of rational cubic function. The rational cubic function is extended to rational bi-cubic partially blended function, simple constraints were derived on the parameters in the description of rational bi-cubic partially blended patches to visualize the monotone data in the view of monotonesurfaces. Hussainetal.(2010-)extendedthepiecewise rationalcubic functionfor monotone curve designdevelopedby Hussainand Sarfraz(2009)to rational bi-cubic partially blended function to preserve the shape of 3D monotone data. The rational cubic function presentedin Sarfrazand Hussain(2006)has beenextendedto rational bi-cubic partially blended function to visualize the shape of 3D positive data by Hussain et al.(2011). Hussain et al.(2012)utilized the same rational bi-cubic function to preserve the shape of monotone and convex data. Simple data dependent constraints are developed on shape parameters in each rectangular patch to assure the preservation of the shape of data.

Hussain and Bashir(2011)presented surface data visualization scheme for the visualization of positive, constrained and monotone data using rational bi-cubic functions with linear denominator. To visualize surface data arranged over a rectangular mesh, a rational bi-cubic function has been developed which is an extension of the rational cubic function in Hussain et al.(2010). Data dependent conditions have been derived onshape parametersto preservetheshapeof data. Hussainetal.(2015) extendeda piecewise rational cubic function presentedin(Sarfrazetal., 2012)toa bi-cubicpartially blended rational function with eight shape parameters to preserve the inherent shape features of the shaped data. Data dependent sufficient constraints are developed on four of the parameters to preserve the shape of data while remaining are free to refine the shape of data at user choice.

The C1 rational cubic spline interpolantof Karim andKong(2014)has beenextended to a partially blended bi-cubic rational spline with 12 shape parameters in the descriptionsbyKarimetal. (2015).Sufficient conditionsarederivedonfourshapeparameters and the remaining8of them were free parameters which are used to change the shape of the final surfaces of the positive data. From the previous discussion, positivity,monotonicity,and convexity are important shapes. These are independent shapes which are found inherited in data. Previous studies had discussed these shapes independently using different mathematical models and methodologies.

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