IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 15, Issue 6 Ser. V (Nov – Dec 2019), PP 77-81
www.iosrjournals.org

Permutation Graphs with Inversion on Γ_1-non deranged Permutations

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Abstract: *In this paper, we define permutation graphs on Γ_1-non deranged permutations using the set of inversion as edge set, and the values of permutation as the set of vertices. From the graphs, we observe that radius of the graph of any ω_1 is zero, the graph of any $\omega_1 \in G_p^{\Gamma_1}$ is null, and by restricting 1, the graph of ω_{p-1} is complete, other properties of the graphs were also observed.*

Keywords: *inversion numbers, co-inversion, permutation graph, Γ_1 -non deranged permutations.*

Date of Submission: 27-11-2019 Date of Acceptance: 12-12-2019

I. Introduction

A Permutation f of the Γ_1-non deranged permutations presents as inversion, is a pair (i, j) such that $i < j$ and $f(i) > f(j)$, Permutation statistics were first introduced by [10] and then extensively studied by [3].in the last decades much progress has made, both in the discovery and the study of new statistics, and in extending these to other type of permutations such as words and restricted permutation. The concept of derangements in permutation groups (that is permutations without a fix element) has proportion in the underlying symmetric group S_n. [4] used concept to develop a scheme for prime numbers $P \le 5$ and $\Omega \subseteq N$ which generate the cycles of permutations (derangements) using $\omega_i = ((1)(1+i)_{mp}(1+2i)_{mp}...(1+(p-1)i)_{mp})$ to determine the arrangements. It is difficult for a set of derangements to be a permutation group because of the absence of the natural identity element (a non derangement),The construction of the generated set of permutations from the work of [4] as a permutation group was done by [11] .They achieved this by embedding an identity element into the generated set of permutation(strictly derangements) with the natural permutation composition as the binary operation (the group was denoted as G_p) With no doubt, patterns in permutations have been well studied for over a century. As seem to be the case, these patterns were studied on permutations arbitrary. The symmetric group S_n is the set of all permutations of a set Γ of cardinality n. There are several types of other smaller permutation groups (subgroup of S_n) of set Γ,a notable one among them is the alternating group A_n .,Afterwards, [6] studied the representation of Γ_1-non deranged permutation group $G_P^{\Gamma_1}$ via group character, hence established that the character of every $\omega_i \in G_p^{\Gamma_1}$ is never zero. Also the non standard Young tableaux of Γ_1-non deranged permutation group $G_P^{\Gamma_1}$ has been studied by [5], they established that the Young tableaux of this permutation group is non standard. [1] studied pattern popularity in Γ_1-non deranged permutations they establish algebraically that pattern τ_1 is the most popular and pattern τ_3, τ_4 *and* τ_5 are equipopular in $G_P^{\Gamma_1}$ they further provided efficient algorithms and some results on popularity of patterns of length-3 in $G_P^{\Gamma_1}$.[2] studied Fuzzy on Γ_1-non deranged permutation group $G_P^{\Gamma_1}$ and discover that it is a one sided fuzzy ideal (only right fuzzy but not left) also the $\alpha-$level cut of f coincides with $G_P^{\Gamma_1}$ if $\alpha=\frac{1}{p}$. [7] studied ascent on Γ_1-non deranged permutation group $G_P^{\Gamma_1}$ and discover that the union of ascent of all Γ_1-non derangement is equal to identity also observed that the difference between $Asc(\omega_i)$ and $Asc(\omega_{p-1})$ is one. [8] provide very useful theoretical properties of Γ_1-non deranged permutation s in relation to excedance and shown that the excedance set of all ω_i in $G_P^{\Gamma_1}$ such that $\omega_i \neq e$ is $\frac{1}{2}(p-1)$. More recently [9] established that the intersection of descent set of all Γ_1-non derangement is empty, also observed that the descent number is strictly less than ascent number by $p-1$. Hence we will in this paper show that radius of the graph of any ω_1 is zero, the graph of any $\omega_1 \in G_p^{\Gamma_1}$ is null, and by restricting 1, the graph of ω_{p-1} is complete, other properties of the graphs were also observed.

II. Preliminaries

Definition 2.1 [6]

$\Gamma-$ non deranged permutation group $G_P^{\Gamma_1}$ is a permutation group with a fixed element on the first column from the left.

Definition 2.2

$\omega_i \in G_p^{\Gamma_1}$ [6]: let Ω be a non empty ordered set such that $\Omega \subset N$. Let $G_P^{\Gamma_1}=\omega_i \quad 1\le i \le (p-1)$ be a subgroup of symmetry group S_p such that every ω_i is generated by arbitrary set Ω for any prime $p \ge 5$ using the following

$$\omega_i=\begin{pmatrix} 1 & 2 & 3 & \cdot & \cdot & \cdot & p \\ 1 & (1+i)_{mp} & (1+2i)_{mp} & \cdot & \cdot & \cdot & (1+(p+1)i)_{mp} \end{pmatrix} \quad (1)$$

Example 2.2.1

For $p=5$ equation (1) will generate permutation group $G_P^{\Gamma_1}$

$\omega_1=\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$={e} (the identity permutation)

$\omega_2=\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & 2 & 4 \end{pmatrix}$,

$\omega_3=\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 2 & 5 & 3 \end{pmatrix}$,

$\omega_4=\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 5 & 4 & 3 & 2 \end{pmatrix}$.

Definition 2.3

An inversion of permutation $f=\begin{pmatrix} 1 & 2 & 3 & . & . & . & n \\ f(1) & f(2) & f(3) & . & & . & f(n) \end{pmatrix}$ is a pair (i,j) such that $i<j$ and $f(i)>f(j)$. The inversion set of f, denoted as $Inv(f)$, is given by $Inv(f)=\{(i,j):1\le i<j\le n \text{ and } f(i)>f(j)\}$, the inversion number of f, denoted by $inv(f)=|Inv(f)|$.

Definition 2.4

A co- inversion of permutation $f = \begin{pmatrix} 1 & 2 & 3 & . & . & . & n \\ f(1) & f(2) & f(3) & . & . & . & f(n) \end{pmatrix}$ is a pair (i, j) such that $i < j$ and $f(i) < f(j)$.The co- inversion set of f, denoted as $Coinv(f)$, is given by $Coinv(f) = \{(i, j): 1 \le i < j \le n \text{ and } f(i) < f(j)\}$, the number of co-inversion f, denoted by $coinv(f) = |Coinv(f)|$.

III. Main Results

In this section, we present some Permutation graphs with inversion number results of subgroup $G_P^{\Gamma_1}$ of S_p (Symmetry group of prime order with $p \ge 5$).

Proposition 3.1

Let $G_P^{\Gamma_1}$ be a Γ_1-non derangement permutations, Then the graph of $\omega_i\left(G_{\omega i}\right)$ is simple.

Proof.

We define permutation graphs by using the set of inversion as the edge set, and for any edge set no element is repeated, and there is no any edge $e = (i, j)$ such that $i = j$. Hence there is no multiple edge and loop. Therefore, the graph is simple

□

Proposition 3.2

Suppose that $G_P^{\Gamma_1}$ is Γ_1-non derangement permutations, Then for any $\omega_{p-1} \in G_p^{\Gamma_1}$.

The graph $G_{\omega p-1} - \{1\}$ is complete.

Proof.

For any $G_{\omega p-1}$ all the vertices are adjacent to each other except vertex (1) Hence by restricting vertex (1).The graph is complete.

□

Corollary 3.3

The graph $G_{\omega p-1} - \{1\}$ is regular.

Proof.

By proposition 3.2 $G_{\omega p-1} - \{1\}$ is complete. Hence the prove since every complete graph is regular.

□

Proposition 3.4

For any $\omega_1 \in G_p^{\Gamma_1}$. The graph $G_{\omega 1}$ is null /empty.

Proof.

ω_1 is the identity permutation. Therefore, its inversion is empty. Hence no vertices are adjacent. Thus the graph is null

□

Corollary 3.5

For any $\omega_i \in G_p^{\Gamma_1}$.The graph $G_{\omega 1}$ is regular.

Proof.

This follows proposition 3.4

□

Proposition 3.6

For any $\omega_{p-1} \in G_p^{\Gamma_1}$. The

$$diam\left(G_{\omega p-1}\right) = 1$$

Proof.

By proposition 3.2 $G_{\omega p-1} - \{1\}$ is complete. Hence the distance between any two vertices is 1. Therefore, the maximum eccentricity is 1. Thus

$$diam\left(G_{\omega p-1}\right) = 1$$

□

Proposition 3.7

Let $G_P^{\Gamma_1}$ be a Γ_1-non derangement permutations, Then the

$$Rad\left(\omega_i\right) = 0$$

Proof.

Since for any $\omega_i \in G_p^{\Gamma_1}$, the $ecc(1)$ is zero then the

$$\min_{v \in V(G)} \{ecc(v)\} = 0$$

Hence $Rad\left(\omega_i\right) = 0$

□

Lemma 3.8

Suppose that $G_P^{\Gamma_1}$ is Γ_1-non derangement permutations, Then the

$$G_{\omega_i} \cup G_{\omega p-i} = G_{\omega_1}$$

Proof.

Given $\omega_i = a_1 a_2 a_3 \ldots a_{p-1} a_p$, then $\omega_{p-i} = a_1 a_p a_{p-1} \ldots a_3 a_2$. By restricting a_1 because it has no effect on the inversion since it is the least of all values and it is at the first position in ω_i and ω_{p-i}, we have,

$$Inv\left(\omega_i\right) = Coinv\left(\omega_{p-i}\right) \qquad (1)$$

and

$$Coinv\left(\omega_i\right) = Inv\left(\omega_{p-i}\right). \qquad (2)$$

It is obvious form the definition 2.3 and definition 2.4 that

$$Inv\left(\omega_i\right) \cap Coinv\left(\omega_i\right) = \phi \qquad (3)$$

Substituting (2) into (3) we have

$$Inv\left(\omega_i\right) \cap Inv\left(\omega_{p-i}\right) = \phi$$

Hence the graph $G\left(\omega_i\right) \cap G\left(\omega_{p-i}\right) = G_{\omega i}$ □

Corollary 3.9

Let $\omega_i \in G_p^{\Gamma_1}$. Then the graph $G_{\omega i} \cap G_{\omega p-i}$ is empty/ null.

Proof.

This follows since by proposition 3.8 the graph $G_{\omega i} \cap G_{\omega p-i}$ is empty.

□

Proposition 3.10

Let $G_P^{\Gamma_1}$ be a Γ_1-non derangement permutations, Then the

$$G_{\omega i} \cup G_{\omega p-i} = G_{\omega p-1}$$

Proof.
Suppose $\omega_i = a_1 a_2 a_3 \dots a_{p-1} a_p$, then $\omega_{p-i} = a_1 a_p a_{p-1} \dots a_3 a_2$, and $\omega_{p-1} = 1 - p(p-1)\dots 2$. By restricting a_1 because it has no effect on the inversion since it is the least of all values and it is the first position in ω_i and ω_{p-i}. Since ω_{p-1} is a strictly decreasing sequence when a_i is restricted.

$$Inv(\omega_{p-1}) = \{(j,k): j < k and a_j > a_k\} \cup \{(j,k): j < k and a_j < a_k\}$$
$$= Inv(\omega_i) \cup Coinv(\omega_i)$$
$$= Inv(\omega_i) \cup Inv(\omega_{p-i}), \text{ by (2)}$$

By this we can see that

$$Inv(\omega_i) \cup Inv(\omega_{p-i}) = Inv(\omega_{p-1})$$

Hence, $G_{\omega i} \cup Gi = G_{\omega p-1}$ □

Corollary 3.11 Let $\omega_i \in G_p^{\Gamma_1}$. Then the $G_{\omega i} \cup G_{\omega p-i}$ is complete.

Proof.
This follows by proposition 3.10 the graph $G_{\omega i} \cup G_{\omega p-i} - \{1\}$ is complete.
□

IV. Conclusion

This paper has provided very useful theoretical properties of this scheme called Γ_1-non deranged permutations in relation to the inversion We have shown that radius of the graph of any ω_1 is zero, the graph of any $\omega_1 \in G_p^{\Gamma_1}$ is null, and by restricting 1, the graph of ω_{p-1} is complete, other properties of the graphs were also observed.

Acknowledgment
The Authors are very grateful to the referees for their valuable suggestion and opinions.

References

[1] Aremu K.O., Ibrahim A.H., Buoro S. and Akinola F.A.Pattern Popularity in Γ_1-non deranged permutations: An Algebraic and Algorithmic Approach.*Annals. Computer Science Series* (2017);**15**(2): 115-122.

[2] Aremu K.O.,Ejima O. and Abdullahi M.S. On the Fuzzy Γ_1-non deranged permutation group $G_P^{\Gamma_1}$.*Asian Journal of Mathematics and Computer Research* (2017);18(4):152-157.

[3] Euler L. Institutiones Calculi differntialis in "*opera omnia series prime* ,(1913);" Volx, Teubner,Leipzig.

[4] Garba A.I. and Ibrahim A.A. A New Method of Constructing a Variety of Finite Group Based on Some Succession Scheme. *International Journal of Physical Sciences*(2010); **2**(3): 23-26.

[5] Garba A.I.,Ejima O.,Aremu K.O. and Hamisu U. Non standard Young tableaux of Γ_1-non deranged permutation group $G_P^{\Gamma_1}$.*Global Journal of Mathematical Analysis* (2017);**5**(1):21-23.

[6] Ibrahim A.A,Ejima O. and Aremu K.O. On the Representation of Γ_1-non deranged permutation group $G_P^{\Gamma_1}$ *Advance in Pure Mathematics*, (2016); **6:**608-614.

[7] Ibrahim M., Ibrahim A.A, Garba A.I and Aremu K.O. Ascent on Γ_1-non deranged permutation group $G_P^{\Gamma_1}$ *International journal of science for global sustainability*, (2017); 4(2):27-32.

[8] Ibrahim M. and Garba A.I Exedance on Γ_1-non deranged permutations *proceedings of Annual National Conference of Mathematical Association of Nigeria (MAN)* , (2018);197-201.

[9] Ibrahim M. and Garba A.I Descent on Γ_1-non deranged permutation group *Journal of Mathematical Association of Nigeria ABACUS* , (2019);46(1):12-18.

[10] MacMahon P.A. *Combinatory Analysis* Vol. 1 and 2 Cambridge University Press (1915);(reprinted by Chesea,New York,1955)