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# Model with Periodic Coefficients in the Pollution Elimination Function, Almost Normal Form

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***Abstract:*** *In this article, we have analyzed international concerns about the environment, in particular current United Nations policy, indicating the goal is for the world to be committed to the elimination of contaminants in order to achieve a healthy, non-polluting environment and achieving that the results reach an environment with acceptable pollutant concentrations. Here we present a model through a system of periodic differential equations in the elimination function, which simulates the elimination of pollution by oxidation ponds. The system is reduced to the normal combined form, a qualitative study is made, and necessary and sufficient conditions are given on the future behavior of the trajectories.*

***Keywords:*** *Environment, pollution, mathematical model.*

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## I. Introduction

Following the United Nations Conference on the Human Environment in June 1972 in Stockholm, the world focused on preserving the environment. Words like depollution, imbalance, favoritism, conservation have been heard all over the planet. Several sectors, companies, governments, students, organized civil society, among others, discussed in depth the issue that created a sense or unison of discomfort, that is, has been the insomnia of many people around the planet and not for less, once the effects of imbalance emerge everywhere imaginable, frightening and bringing hopelessness.

The environmental situation is becoming a more current problem [6]. The Secretary-General Antonio Guterres has urged statesmen not to step on the podium without "concrete and transformative plans to stop global warming, achieve carbon neutrality and reduce carbon emissions by 45%. It's not a summit. to talk, we talk a lot It's not a negotiating summit because it's not negotiated with nature It's a summit of action Governments have come to show how committed they are, who are the leaders to invest in a green future. they are demanding that they act urgently and be right", said the Secretary-General, according to AFP.

The 16-year-old Greta Thunberg was strict: "They stole my dreams, my hope, with their empty words. The only thing they talk about is money and they tell us stories about perpetual economic growth. How dare they? I want them to panic, to feel the fear I feel every day and after they act! Let them act like the house is on fire". But politicians do not see the fire on the roof, so many, even presidents, act irresponsibly and do not worry about the future of the planet.

Points to the moment in which we live in a chaotic and alarming situation, driven by scientific and technological advances, which enabled and stimulated new consumption classes and new markets. Most of these occurred without the mechanisms for the preservation or restoration of the environment having to be structured or contemplated, not only to improve the quality of life of present generations, but mainly to enable future generations to exist [7].

Pollution control is related to the ability of a group of people to stay in aenvironment without harming them and is committed to preserving the natural environment by ensuring the future of the environment and the human species. Thinking about the best solutions for the preservation of humanity in an intelligent and committed way, the discussion comes to Brazil through Rio 92 and, almost 25 years later, the debates permeate various sectors, such as universities.

To clear the waves for the development of environmental awareness, [12] points to the need for information sharing in society, according to the author the information will bring awareness that will point to the need to preserve and conserve ecosystems and biodiversity. It also indicates that urban and rural social empowerment, together with the strengthening of environmental awareness, is of utmost importance for the relationship between human beings and the environment. What is behind this thinking is the possibility of transforming the inhabitants of certain areas, such as the Amazon, into guardians of this immense heritage, agents of preservation and conservation of natural resources and sustainable use.

National Curriculum Parameters / Presentation of Cross-cutting Themes, Secretariat for Fundamental Education - Ministry of Education [11], makes the following contribution: "Life has grown and developed on Earth as a plot, a large network of interconnected beings. This network intertwines intensely and involves a set of living beings and physical elements.For every living being that inhabits the planet there is a space around it all other living elements and beings that interact with it through exchange relations. of energy: this set of elements, beings and relationships constitute its environment, and it may seem that when it comes to the environment, it is only about biological aspects and on the contrary, the human being is part of the environment and the relationships that are established social, economic and cultural relations - they are also part of this environment and, therefore, are objects of the environmental area ".

It is exactly the medium of energy exchange relations and no set of elements that records the involvement of the masses and scholars who seek to understand and convey an idea of depollution; Thus, to involve the theme that will foster participation and co-responsibility for the collective and solidary life, based on the guarantee of quality of life and environmental sentiment. [8] in the paper entitled Ecological Literacy: A Discussion on the Philosophical and Sociological Aspects of Environmental Education as Considerations: "A human civilization and its consumer culture, driven in recent years by the advent of technology, have led to a devastating process of its fundamental ecosystems and, consequently, a crisis in a society, as economic, social, educational and why not philosophical ". We draw attention to the terms "devastation" and "crisis" as they complement each other. There is no devastation without crisis and vice-versa, but the world has only come to understand this very recently. The picture gets worse when the focus is on consumer culture driven by scientific and technological advances. In fact, depollution must focus on environmental re-education, or the human SOS, because when we look at parts of the American Indians' letter to "white" chiefs, we come to understand a little of the prevailing destruction and heralds a tragic event. ; not so distant, imminent, if not opposed to the agents of pollution, in the face of the growing wave of imbalance that is already operating in our midst.

Every process of pollution and depollution is directly related to information held by society, whatever it may be. [5] warns against understanding what one sees, in fact, the author expresses the concern of being able to contribute positively: which we are confronted to better understand that information is only a necessary condition of knowledge. Perhaps most perverse is that knowledge construction is as easy as current access to information at the touch of a key. "

In fact, the speed of information forces the change of social thinking, requires effort, perseverance, commitment. Understanding these mechanisms involves breaking common sense, breaking down barriers, analyzing and even breaking down paradigms.

Focuses on environmental balance and its relation to the issue of justice, defines an exhausted environment as a stage to meet our current needs without compromising the ability of future generations to meet their needs. In fact, it explains in its emergence, as a notoriously vague expression, a consensual term that made it possible to accommodate the various positions and expectations of different countries and the multiple intellectual currents [10]. The author also refers to the 1972 Stockholm Conference, as one whose debates followed the Malthusian line, for the sake of clarity, pointed to catastrophic consequences for population growth and the increase of natural resources due to economic growth.

Points out the structural differences between developed and underdeveloped countries regarding the impact and differentials brought about by the new form of exploitation of the environment combined with the new initiative that focuses on industrialization and development policy, economic and social structures. , because inequalities in resource appropriation between countries and between groups within countries are conflicting [8].

Given this rhetoric, the present work also proposes a model of depollution of any place, space, zone, etc. From this it is possible to establish ordinary differential equations, which allow us to assign variables in decontamination processes. After tabulation, information processing, testing, collection and other procedures, it is possible to establish a mathematical model that allows the measurement of pollution and depollution levels efficiently and effectively in the various values shown and the verification of a model that involves a system of equations that will work in theory and practice, allowing to evaluate the levels and control of such widespread pollution.

When you want to eliminate pollution from certain results, which could be in a liquid, solid or gaseous form, you should think about how to achieve our goals rather than the environment, as you should not eliminate a form. contaminant and introduce another that may be even more aggressive. In general, these processes of depollution are presented in combination, since a certain matter is contaminated and it is desired to send to nature with the least possible affectation to the environment; These results can be generated by sewage or the operation of an industry, among others.

In this sense, there are experiences such as those practiced in the depollution of the Tietê de São Paulo, where through certain plants pollutants can be eliminated in a high percentage of the initial concentration. On the other hand, it is common to work using oxidation ponds to eliminate contaminants from the liquid endings left by the population, where by using chemicals and sometimes natural products, it is possible to bulk clean up the liquid part, and with the removal of solids. , these goals are usually achieved.

The treatment we will do in this case corresponds to other models presented in the research of other diseases, especially the case of sicklemia, well treated and with many models already developed, will only mention some of these works. In [13] and [14] applies the qualitative study of differential equations for different models in an autonomous and non-autonomous form corresponding to the formation of polymers.

Authors such as [15] treat dynamic insulin glucose, in which two critical cases are treated to reach conclusions, applying the Qualitative Theory of Differential Equations. In [16] applied the Qualitative Theory of Differential Equations to study a combined critical case; here the Qualitative Theory of Differential Equations will be applied to the critical case when a null value appears.

In [1] and [2] was simulated the process of elimination of pollution by means of a system of differential equations with constant coefficients, but in general the additions of pollution occur periodically, so it is more accurate a non-autonomous model. and especially periodic with respect to time.

## II. Development

We present the case of two oxidation lagoons, where a decontamination procedure is applied to each one of them. Here is considered as the compartment a first pond; compartment two is the second tank and compartment three is given by the material leaving the environment. This procedure is being used in the city of Leticia, capital of the Colombian Amazon state. Initially, we will give some basic principles that we will take into account in the writing of the model; let's denote by: $\bar{x}_1, \bar{x}_2$ and $\bar{x}_3$, the allowable concentration values of pollutants in compartments one, two and three; Let's indicate the following other variables to consider:

- $\tilde{x}_1$ the concentration of pollutant in compartment one at time t.
- $\tilde{x}_2$ the concentration of pollutant in compartment two at time t.
- $\tilde{x}_3$ the concentration of pollutant in compartment three at time t.

In the system, let's consider the variables $x_1, x_2$ and $x_3$, defined as follows, $x_1 = \tilde{x}_1 - \bar{x}_1$, $x_2 = \tilde{x}_2 - \bar{x}_2$, and $x_3 = \tilde{x}_3 - \bar{x}_3$, when $(x_1, x_2, x_3) \to (0,0,0)$, então $\tilde{x}_1 \to \bar{x}_1$, $\tilde{x}_2 \to \bar{x}_2$ e $\tilde{x}_3 \to \bar{x}_3$.

It is good to realize that in our model will appear the pollutants that will be placed in both ponds, moreover, if it is considered that there is no other supply of contaminated material from that initial moment, then we have that a possible model would have the next path

$$\begin{cases} x_1' = -a_{12}(t)x_1 + X_1(t, x_1, x_2, x_3) \\ x_2' = a_{12(t)}x_1 - a_{23}(t)x_3 + X_2(t, x_1, x_2, x_3) \\ x_3' = a_{23}(t)x_3 + X_3(t, x_1, x_2, x_3) \end{cases} \quad (1)$$

On here $X_1, X_2$ and $X_3$ represent depollution functions, that is, the action of chemicals or natural products supplied for the elimination of pollution, it is assumed that these functions contain only nonlinear terms and their powers will depend on the speed with which pollution is eliminated; $a_{ij} x_i(t)$ represents the coefficient with the transfer variable from compartment i to compartment j. Thus, there is the Cauchy problem, given by system (1) with the initial conditions,

$x_1(0) = N, x_2(0) = 0$ and $x_3(0) = 0$; In this case it is considered that at the initial moment all the contamination is concentrated in the first lagoon, that is, if the process of elimination of the pollution is starting.

In [3] Floket's theory was applied to transform a System with periodic coefficients in general into a System where the matrix of the linear part of the system has constant coefficients and where the nonlinear part has periodic coefficients, which case will be studied at that time.

This is the autonomous case, the non-autonomous case can be treated as it does in Sánchez, S., Fernández, G. A. A., Ruiz. A. I., & Carvalho, E. F (2015), where the case in which time dependence is periodic is specifically treated. We will consider the combined critical case, when the eigenvalues of the matrix of the linear part of the system are $\lambda_1 = 0$, $\lambda_2 = \sigma i$ and $\lambda_3 = -\sigma i$. By means of a non-degenerate linear transformation $X = QY$, o system (2) can be reduced to the shape,

$$\begin{cases} y_1' = Y_1(y_1, y_2, y_3) \\ y_2' = \sigma i y_2 + Y_2(y_1, y_2, y_3) \\ y_3' = -\sigma i y_2 + Y_3(y_1, y_2, y_3) \end{cases} \quad (3)$$

**Theorem 1:** The exchange of variables,

$$\begin{cases} y_1 = z_1 + h_1(t, z_1) + h^0(t, z_1, z_2, z_3) \\ y_2 = z_2 + h_2(t, z_1) \\ y_3 = z_3 + h_3(t, z_1) \end{cases} \tag{4}$$

transforms the system (3) into the system,

$$\begin{cases} z_1' = Z_1(z_1) \\ z_2' = i\sigma z_2 + Z_2(t, z_1, z_2, z_3) \\ z_3' = -i\sigma z_3 + Z_3(t, z_1, z_2, z_3) \end{cases} \tag{5}$$

at where $z_3 = \overline{z}_2$, $P$ and $\overline{P}$ are conjugated, in addition $Z_2$, $Z_3$ and $h^0$ annul for $z_2 = z_3 = 0$.

**Demonstration:** By deriving the transformation (4) along the trajectories of systems (3) and (5) we obtain the system of equations,

$$\begin{cases} h_1' + h^{0'} + (p_2 - p_3)\sigma i h^0 + Z_1(z_1) = Y_1(z_1 + h_1 + h^0, z_2 + h_2, z_3 + h_3) \\ \qquad - \frac{dh_1}{dz_1} Z_1(z_1) - \frac{\partial h^0}{\partial z_1} Z_1 - \sum_{i=2}^{3} \frac{\partial h^0}{\partial z_i} Z_i \\ h_2' - \sigma i h_2 + Z_2 = Y_2(z_1 + h_1 + h^0, z_2 + h_2, z_3 + h_3) - \frac{dh_2}{dz_1} Z_1 \\ h_3' + \sigma i h_3 + Z_3 = Y_3(z_1 + h_1 + h^0, z_2 + h_2, z_3 + h_3) - \frac{dh_3}{dz_1} Z_1 \end{cases} \tag{6}$$

To determine the series that intervene in the systems and the transformation, we will separate the coefficients of the power of degree $p = (p_1, p_2, p_3)$ in the following two cases:

**Case I)** Making in the system (6) $z_2 = z_3 = 0$, is to say to the vector $p = (p_1, 0, 0)$ the system

,

$$\begin{cases} h_1' + Z_1 = Y_1(t, z_1 + h_1, h_2, h_3) - \frac{dh_1}{dz_1} Z_1 \\ h_2' - i\sigma h_2 = Y_2(t, z_1 + h_1, h_2, h_3) - \frac{dh_2}{dz_1} Z_1 \\ h_3' + i\sigma h_3 = Y_3(t, z_1 + h_1, h_2, h_3) - \frac{dh_3}{dz_1} Z_1 \end{cases} \tag{7}$$

The system (7) allows to determine the coefficients of the series, $Z_1$, $h_1$, $h_2$ and $h_3$, where for being the resonant case, and the remaining series are determined in a unique way. For the first equation of (7) to have a solution $2\pi$-periodic it is necessary and enough that, $2\pi -$

$$Z_1 = \frac{1}{2\pi} \int_0^{2\pi} F(t)dt$$

And the coefficients of the series can be determined in the form,

$$h_1(t) = \frac{1}{2\pi} \int_0^{t} F(\tau)d\tau - tZ_1$$

And in the second equation the coefficients are determined by the expression,

$$h_2(t) = (e^{2\pi\sigma i} - 1)^{-1} \int_t^{t+2\pi} e^{\sigma i(t-\tau)} G(\tau)d\tau$$

Similarly, to the third.

**Case II)** This is the case when $z_2 \neq 0$ and $z_3 \neq 0$ of the system (6),

$$\begin{cases} h^{0'} + (p_1 - p_2)h^0 = Y_1(t, z_1 + h_1 + h^0, z_2 + h_2, z_3 + h_3) \\ \qquad - \frac{\partial h^0}{\partial z_1} Z_1 - \sum_{i=2}^{3} \frac{\partial h^0}{\partial z_i} Z_i \\ Z_2 = Y_2(t, z_1 + h_1 + h^0, z_2 + h_2, z_3 + h_3) \\ Z_3 = Y_3(t, z_1 + h_1 + h^0, z_2 + h_2, z_3 + h_3) \end{cases} \tag{8}$$

In the first equation of (8) the resonant case must be presented when $p_2 = p_3$, and in this case the coefficients of the series $h^0(t) = 0$. In the non-resonant case these coefficients are determined by the expression,

$$h_2(t) = \left(e^{2\pi(p_2-p_3)i} - 1\right)^{-1} \int_t^{t+2\pi} e^{(p_2-p_3)i(t-\tau)} G(\tau)d\tau$$

where G (t) is the second member of the first equation of the system (8).

Because the series of the system (5) are known expressions, the system (8) allows to calculate the series $h^0$, $Z_2$ and $Z_3$. This proves the existence of the exchange of variables.

**Theorem 2:** The transformation of coordinates,

$$\begin{cases} z_1 = u_1 \\ z_2 = u_2 + h_2(t, u_2, u_3) \\ z_3 = u_3 + h_3(t, u_2, u_3) \end{cases} \quad (9)$$

reduces the system (5) to the combined normal form,

$$\begin{cases} u_1' = U_1(u_1) \\ u_2' = i\sigma u_2 + u_2 P_2(u_2 u_3) \\ u_3' = -i\sigma u_3 + u_3 P_3(u_2 u_3) \end{cases} \quad (10)$$

**Demonstration:** Deriving the transformation (9) to the logo of the trajectories of systems (5) and (10) we obtain the system of equations,

$$\begin{cases} U_1(u_1) = Z_1(z_1) \\ h_2' + (p_1 - p_2 - 1)i\sigma h_2 + u_2 P_2 = Z_2 - \sum_{r=1}^{2} \frac{\partial h_2}{\partial u_r} u_r P_r \\ h_3' + (p_1 - p_2 + 1)i\sigma h_3 + u_3 P_3 = Z_2 - \sum_{r=1}^{2} \frac{\partial h_3}{\partial u_r} u_r P_r \end{cases} \quad (11)$$

The system (11) allows the series $\bar{h}_2$, $\bar{h}_3$, $P$ and $\bar{P}$ thus the theorem being proved. Because $P$ and $\bar{P}$ are different from zero in the resonant case, ie when satisfying equations $p_1 - p_2 - 1 = 0$and $p_1 - p_2 + 1 = 0$This ensures the shape previously indicated for these series; However $\bar{h}_2$ and $\bar{h}_3$ being non-resonant are uniquely determined.

In the system (10) the functions $P$ and $U_1$ admit the following development in series of powers:

$$P(u_2 u_3) = \sum_{n=k}^{\infty} a_n (u_2 u_3)^n + i \sum_{n=l}^{\infty} b_n (u_2 u_3)$$

and

$$U_1(u_1) = \alpha u_1^s + \ldots$$

**Theorem 3:** If $\alpha < 0$, $s$ odd and $a_k < 0$, then the trajectories of the system (10) are asymptotically stable, otherwise they are unstable.

**Demonstration:** Consider the positive defined Lyapunov function,

$$V(u_1, u_2, u_3) = \frac{u_1^2}{2} + u_2 u_3$$

The function $V$ is such that its derivative along the trajectories of the system (10) has the following expression,

$$V'(u_1, u_2, u_3) = \alpha u_1^{s+1} + a_k (u_2 u_3)^{k+1} + R(u_1, u_2, u_3)$$

This function is defined as negative because in $R$ potencies of degrees higher than those indicated in the initial part of the expression of the derivative of $V$ , this allows us to state that the equilibrium position is asymptotically stable; in this case, it follows that$\tilde{x}_1(t) \to \overline{x_1}$, $\tilde{x}_2(t) \to \overline{x_2}$ and $\tilde{x}_3(t) \to \overline{x_3}$ .

## III. Conclusions

1. It is interesting to study modeling using non-autonomous systems and even more with periodic coefficients, because the procedures are applied periodically, making the simulation more real.
2. Theorem 1 and 2 provide the methodology for reducing the system to the combined normal form, which allows for a more simplified qualitative study.
3. Theorem 3 allows, by means of qualitative differential equations, to draw conclusions about the behavior of the trajectories of the original system.
4. If $\alpha < 0$, $s$ odd, and $a_k < 0$theconcentrationofpollution converges totheoptimum, otherwisetheprocesshasto continue toavoiddamagetotheenvironment.

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