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Solution Of One-Dimensional Radial Fin By Frobenius Method Versus Improved Classical Radial Fin With Application On Finned Electric Motor

Élcio Nogueira[1]
[1]*Department of Mechanics and Energy –DME/FAT / UERJ - Resende - RJ – Brazil*

ABSTRACT
An industrial application of extended surfaces occurs in electric motors, which are of vital importance in the industry, as they are used in machines of all types. In this work, one-dimensional models are used to define physical and thermal characteristics to be applied to extended surfaces. Results for temperature, heat transfer rate, efficacy, and efficiency are obtained to compare one-dimensional radial fin model, "Frobenius Method," with the mathematical model already established and validated, called "Improved One-dimensional Classic Radial Fin" in the literature. The results obtained by the one-dimensional models, for effectiveness and efficiency, were compared numerically with the results of the two-dimensional model for the radial fin. The finned electric motor, with an aspect ratio of 5.82, is used to generate the numerical and graphical results. The equivalence of one-dimensional models is evident, considering the Reynolds number range and the aspect ratio value.
KEYWORDS:*Radial Fin; Frobenius Method; Improved Classical Method; Finned Electric Motor.*

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I. Introduction

Recently, a one-dimensional model to predict the heat transfer rate from continuous fin-and-tube heat exchangers was presented. It has been shown that the one-dimensional model presented makes it possible to estimate the fin efficiency with a higher level of precision than any other known alternative[1].

R.J. Moitsheki, M.M. Rashidi, A. Basiriparsa, andA. Mortezaei[2] consider a model describing the temperature profile in a longitudinal fin with rectangular, concave, triangular, and convex parabolic profiles. An optimal homotopy analysis method (OHAM) is employed to analyze the problem. The solutions are obtained, and the validity of obtained solutions is verified by the Runge–Kutta fourth- order method and numerical simulation. Analytical and numerical results are in excellent agreement.

A. Aziz, Mohsen Torabi, and KailiZhang [3] study a radial fin of uniform thickness with convective heating at the base and convective–radiative cooling at the tip. The fin is assumed to experience uniform internal heat generation. The differential transformation method is used to generate results, andthe effects on the dimensionless parameters on the thermal performance of the fins are illustrated.

Marcus V. F Soares & Élcio Nogueira [4,5] presents developed analytical solutions for determining the temperature variations in the fins, performance, and electric motor efficiency, considering heat flow constant at the base of the fins and the possible differences intemperature of surround media.The results obtained characterize a range of values for the internal and external coefficients of heat convection.

The two-dimensional straight radial fin was used as a reference for comparison with the generalized one-dimensional radial fin, where the expansion in modified power series, the Frobenius Method, is applied. The authorsconclude the smaller the value of the aspect ratio, the higher the range of Biot number in which the one-dimensional model works correctly. The one-dimensional model is suitable for compact fins systems, where the aspect ratio is relatively low ($K \leq 6$) [6].

Élcio Nogueira [7] presents an exact analytical solution of two-dimensional, steady-state heat conduction in an extending rectangular surface and the improved classic one-dimensional model for radial fins. He concludes, enters others that the effect of cross-heat exchange can be better observed for higher aspect ratio values, and low aspect ratio value, the results obtained for the models almost coincide. There is also a qualitative difference between the results presented in the highest aspect ratio. The one-dimensional model responds late to the elevation of the fin height, and the two-dimensional model effects of cross-heat exchange, for relatively low Biot numbers, areconsiderable.

II. Objective

Validate the one-dimensional radial fin model, "Frobenius Method," using an already established and validated mathematical model, called "Improved classic one-dimensional radial fin" in the literature.

To present and compare results for heat transfer rate, efficiency, and effectiveness in an electric motor with fins.

III. Methodology

Frobenius Method

Consider steady-state, one-dimensional heat conduction through a radial fin, with constant conductivity, k, and subjected an ambient temperature, T_∞.

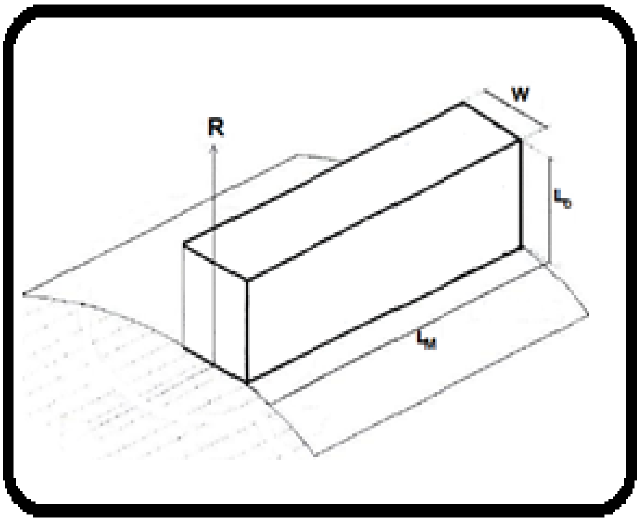

Figure 01 – Geometric Representation of the consider one-dimensional Radial Fin

The differential Equation 01 represents the Classical One-dimensional Radial Fin problem:

$$\frac{1}{R}\frac{d}{dR}\left[R\frac{d\theta(R)}{dR}\right] - B_{i2}K^2\theta(R) = 0\,;\;\; 0 \le \mathrm{R} \le 1 \qquad 01$$

And the boundary conditions for prescribed temperature on the base and convection at the tip of the fin are:

$$\theta(0) = 1 \qquad 01.1$$

$$\frac{d\theta(1)}{dR} + B_{i2}K\theta(1) = 0 \qquad 01.2$$

Then, we have the following altered equation:

$$\mathbb{P}(R)\frac{d^2\theta(R)}{dR^2} + \mathbb{Q}(R)\frac{d\theta(R)}{dR} - \mathbb{W}(R)\theta(R) = 0 \qquad 02$$

At where

$$\mathbb{P}(R) = P_1\,R^2;\; \mathbb{Q}(R) = Q_1R;\; \mathbb{W}(R) = W_1R^2 \qquad 03$$

and

$$P_1 = 1.0;\; Q_1 = 1.0;\; W_1 = B_{i2}K^2 \qquad 04$$

with dimensionless groups defined as:

$$R = \frac{r}{r_o - r_i};\ K = \frac{2L_o}{w};L_o = r_o - r_i;\ B_{i2} = \frac{h_2 \mathrm{w}}{2k};\quad \theta = \frac{T(r) - T_\infty}{T(r_i) - T_\infty} \qquad 05$$

In this work, the equation 02 is more convenient because the interest is in obtaining the solution by the expansion in the modified series of power, called the "Frobenius Method." Then, we have

$$R^2 \frac{d^2\theta(R)}{dR^2} + R\frac{d\theta(R)}{dR} - W_1 R^2 \theta(R) = 0 \qquad 06$$

For convenience, was defined

$$\beta^2 = W_1 \ \ and \ \ R^{'} = \beta R \qquad 07$$

In this case,

$$R^{'2} \frac{d^2\theta(R)}{dR^2} + R^{'}\frac{d\theta(R)}{dR} - R^{'2}\theta(R) = 0 \qquad 08$$

The equation 08 has a singular regular point in $R^{'}$=0 and By Georg Frobenius (1849-1917), Boyce & Diprima [8, pag.243], Erwin Kreyzig [9, pag.190], Arpaci [10, pag.231], Hildebrand [11, pag.143], Schneider [12, pag.46-59], Carslaw and Jaeger[13, pag.374-376]:

$$\theta(R^{'}) = \sum_{n=0}^{\infty} a_n R^{'n+s} \qquad 09$$

$$\theta^{'}(R^{'}) = \frac{d\theta(R^{'})}{dR^{'}} = \sum_{n=1}^{\infty} a_{n-1}(n+s-1)R^{'n+s} \qquad 10$$

$$\theta^{''}(R^{'}) = \frac{d^2\theta(R^{'})}{dR^{'2}} = \sum_{n=2}^{\infty} a_{n-2}(n+s-2)(n+s-3)R^{'n+s} \qquad 11$$

Then

$$R^{'2}\sum_{n=0}^{\infty} a_n(n+s)(n+s-1)R^{'n+s-2} + R^{'}\sum_{n=0}^{\infty} a_n(n+s)R^{n+s-1} - R^{'2}\sum_{n=0}^{\infty} a_n R^{n+s} \ = 0 \qquad 12$$

and the following indicial equation was obtained:

$$a_0[(s^2 - s) + s]R^{'s} = 0 \qquad with \qquad a_0 \neq 0 \ \ and \ s = 0 \qquad 13$$

The recurrence rule is given by

$$a_n = \frac{a_{n-2}}{n^2} \qquad 14$$

or

$$a_2 = \frac{a_0}{2^2};\ a_4 = \frac{a_0}{2^2 4^2}\ ;\ a_6 = \frac{a_0}{2^2 4^2 6^2} \dots \qquad 15$$

For the situation in analysis, two equal roots, there are two linearly independent solutions, which constitute a fundamental system of solution Kreyszig [9]. The first one is:

$$\theta_1(R) = 1 + \sum_{m=1,2,3..}^{\infty} a_{2m}(\beta R)^{2m};\quad a_{2m} = \frac{1}{2^{2m}(m!)^2} \qquad 16$$

The second linearly independent solution contains a logarithmic term and has a form:

$$\theta_2(R) = [\ln(\beta R)]\theta_1(R) + \sum_{m=1,2,3\ldots}^{\infty} A_m(\beta R)^m \tag{17}$$

By Carslaw and Jaeger[13], and Boyce & Diprima [8] the more convenient expression is

$$\theta_2(R) = -\left[\ln\left(\frac{\beta R}{2}\right) + \gamma\right]\theta_1(R) + \sum_{m=1,2,3\ldots}^{\infty} a_{2m}H_m(\beta R)^{2m} \tag{18}$$

At where

$$H_m = \frac{1}{m} + \frac{1}{m-1} + \cdots + \frac{1}{2} + 1 \quad and\ \gamma \cong 0.5772 \tag{19}$$

γ is known as the Euler-Mascheroni Boyce & Diprima [8, pag.247] constant.

Then

$$\theta(R) = a_0\theta_1(R) + a_1\theta_2(R) \tag{20}$$

$$\theta(R) = a_0[1 + \sum_{m=1,2,3..}^{\infty} a_{2m}(\beta R)^{2m}] + a_1[-\left[\ln\left(\frac{\beta R}{2}\right) + \gamma\right]\theta_1(R) + \sum_{m=1,2,3\ldots}^{\infty} a_{2m}H_m(\beta R)^{2m}\ ] \tag{21}$$

or

$$\theta(R) = a_0[1 + \sum_{m=1,2,3..}^{\infty} a_{2m}(\beta R)^{2m}] - a_1\left\{\left[\ln\left(\frac{\beta R}{2}\right) + \gamma\right]\left[1 + \sum_{m=1,2,3..}^{\infty} a_{2m}(\beta R)^{2m}\right] - \sum_{m=1,2,3\ldots}^{\infty} a_{2m}H_m(\beta R)^{2m}\right\} \tag{23}$$

The first boundary condition is defined by Cotta and Mikhailov [14]:

$$\theta(0) = 1 \quad \rightarrow \quad a_0 = 1 + a_1\left[\ln\left(\frac{\beta R_b}{2}\right) + \gamma\right] \tag{24}$$

Finally,

$$\theta(R) = \theta_1(R) + a_1\left\{\left[\ln\left(\frac{\beta R_b}{2}\right) + \gamma\right]\theta_1(R) + \theta_2(R)\right\} \tag{25}$$

$$\theta'(R) = \theta_1{}'(R) + a_1\left\{\left[\ln\left(\frac{\beta R_b}{2}\right) + \gamma\right]\theta_1{}'(R) + \theta_2{}'(R)\right\} \tag{26}$$

where (Figure 01)

$$R_b = R \rightarrow 0 \tag{27}$$

For the second boundary conditions:

$$\theta'(1) = -B_{i2}K\theta(1) \tag{28}$$

Then,

$$-[\theta_1(1) + B_{i2}K\theta_1'(1)] = a_1\left[\ln\left(\frac{\beta R_b}{2}\right) + \gamma\right][\theta_1(1) + B_{i2}K\theta_1'(1)] + [\theta_2(1) + B_{i2}K\theta_2'(1)] \tag{29}$$

In this case,

$$a_1 = \frac{-[\theta_1(1) + B_{i2}K\theta_1'(1)]}{\left[\ln\left(\frac{\beta R_b}{2}\right) + \gamma\right][\theta_1(1) + B_{i2}K\theta_1'(1)] + [\theta_2(1) + B_{i2}K\theta_2'(1)]} \qquad 30$$

The total exchange heat transfer is given by

$$\dot{q} = \frac{-kA_b(T_b - T_\infty)\theta'(0)}{L_0} \qquad 31$$

The dimensionless heat transfer rate is, by definition

$$Q_b = \frac{\dot{q}}{h_2 A_b(T_b - T_\infty)} \quad \rightarrow \quad Q_b = \frac{-1}{B_{i2}K}\left(\frac{d\theta}{dR}\right)_{R=0} \qquad 32$$

A_b *and* T_b *are the base area and the base temperature respectively*

Efficiency is given by

$$\eta = \frac{-1}{B_{i2}K(1+K)}\left(\frac{d\theta}{dR}\right)_{R=0} \qquad 33$$

Improved Classical Radial Fin

The classical solution of the one-dimensional radial fin problem is represented by the equationCotta and Mikhailov [14]:

$$\theta(R) = C_1\mathbb{I}_\nu(R) + C_2\mathbb{K}_\nu(R) \qquad 34$$

Where $\mathbb{I}_\nu$ and $\mathbb{K}_\nu$ are modified Bessel functions of order ν of the first and second kind, respectively. The functions $\mathbb{I}_\nu(R)$ and $\mathbb{K}_\nu$ are two linearly independent solutions of Equation 01 and are valid for all values of ν. Figure 02 shows a plot of zero endsfirst-order modified Bessel functions. It is be noted that $\mathbb{K}_\nu$ functions become infinite as R goes to zero, whereas $\mathbb{I}_\nu(R)$ functions become infinite as R goes to infinity.

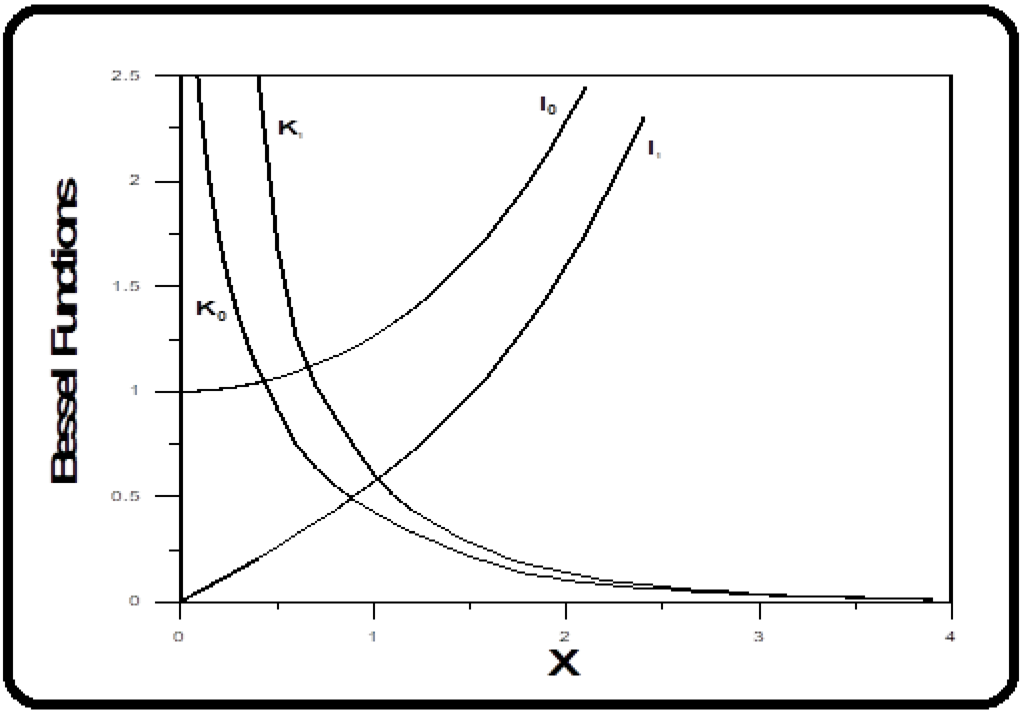

Figure 02 – Zero and first-order modified Bessel functions

Can be proven, Boyce & Diprima [8], Hildebrand [10], the equation 01 is a case of the modified Bessel equation, whose solution is given by Butkov [15],M. N. Özisik [16], M. D. Mikhailov and M. N. Özisik [17]:

$$\theta_{Av}(R) = \frac{\{\mathbb{K}_0(\sqrt{B_{i2}^+}KR)[B_{i2}^+\mathbb{I}_0(\sqrt{B_{i2}^+}K)+\sqrt{B_{i2}^+}\mathbb{I}_1(\sqrt{B_{i2}^+}K)]-\mathbb{I}_0(\sqrt{B_{i2}^+}KR)[[B_{i2}^+\mathbb{K}_0(\sqrt{B_{i2}^+}K)-\sqrt{B_{i2}^+}\mathbb{K}_1(\sqrt{B_{i2}^+}K)]\}}{\{\mathbb{K}_0(\sqrt{B_{i2}^+}KR_b)[B_{i2}^+\mathbb{I}_0(\sqrt{B_{i2}^+}K)+\sqrt{B_{i2}^+}\mathbb{I}_1(\sqrt{B_{i2}^+}K)]-\mathbb{I}_0(\sqrt{B_{i2}^+}KR_b)[[B_{i2}^+\mathbb{K}_0(\sqrt{B_{i2}^+}K)-\sqrt{B_{i2}^+}\mathbb{K}_1(\sqrt{B_{i2}^+}K)]\}} \quad 35$$

and

$$Q_b = \frac{K}{4}\sqrt{B_{i2}^+}\frac{\{\mathbb{K}_1(\sqrt{B_{i2}^+}KR_b)[B_{i2}^+\mathbb{I}_0(\sqrt{B_{i2}^+}K)+\sqrt{B_{i2}^+}\mathbb{I}_1(\sqrt{B_{i2}^+}K)]+\mathbb{I}_1(\sqrt{B_{i2}^+}KR_b)[[B_{i2}^+\mathbb{K}_0(\sqrt{B_{i2}^+}K)-\sqrt{B_{i2}^+}\mathbb{K}_1(\sqrt{B_{i2}^+}K)]\}}{\{\mathbb{K}_0(\sqrt{B_{i2}^+}KR_b)[B_{i2}^+\mathbb{I}_0(\sqrt{B_{i2}^+}K)+\sqrt{B_{i2}^+}\mathbb{I}_1(\sqrt{B_{i2}^+}K)]-\mathbb{I}_0(\sqrt{B_{i2}^+}KR_b)[[B_{i2}^+\mathbb{K}_0(\sqrt{B_{i2}^+}K)-\sqrt{B_{i2}^+}\mathbb{K}_1(\sqrt{B_{i2}^+}K)]\}} \quad 36$$

where the improved solution, extending the range for the Biot number, is obtained with the following defined parameter Aparecido and Cotta [18;19]:

$$B_{i2}^+ = \frac{B_{i2}}{1.0+\frac{B_{i2}}{4}} \quad 37$$

$$\eta = \frac{Q_b}{B_{i2}K^2} \quad 38$$

$\mathbb{K}_0$, $\mathbb{K}_1$, $\mathbb{I}_0$, $\mathbb{I}_1 are$ modified Bessel functions of the first and second kind:

where,

$$\mathbb{I}_\mu(x) = \sum_{k=0}^{\infty}\frac{1}{k!\,\Gamma(\mu+k+1)2^{\mu+2k}}x^{\mu+2k} \quad 39$$

And

$$\mathbb{K}_m(x) = \frac{(-1)^m}{2}\left[\frac{\partial\mathbb{I}_{-\mu}(x)}{\partial\mu}-\frac{\partial\mathbb{I}_\mu(x)}{\partial\mu}\right]_{\mu=m} \quad 40$$

For small R [Özisik; 9]:

$$\mathbb{I}_n(R) \cong \frac{1.0}{2^n n!}R^n \quad 41$$

$$\mathbb{K}_n(R) \cong -l_n R forn = 0 \ and \mathbb{K}_n(R) \cong \frac{2^{n-1}(n-1)!}{R^n} forn \neq 0 \quad 42$$

For $R \geq 10$ [$\ddot{O}zisik$; 9]:

$$\mathbb{I}_0(R) \cong \frac{0.3989e^R}{R^{\frac{1}{2}}}\left\{1+\frac{1}{8R}+\frac{9}{128R^2}+\frac{75}{1024R^3}\right\} \quad 43$$

$$\mathbb{I}_1(R) \cong \frac{0.3989e^{-R}}{R^{\frac{1}{2}}}\left\{1+\frac{3}{8R}-\frac{15}{128R^2}+\frac{105}{1024R^3}\right\} \quad 44$$

$$\mathbb{K}_0(R) \cong \frac{1.2533e^{-R}}{R^{\frac{1}{2}}}\left\{1-\frac{1}{8R}+\frac{9}{128R^2}-\frac{75}{1024R^3}\right\} \quad 45$$

$$\mathbb{K}_1(R) \cong \frac{1.2533e^{-R}}{R^{\frac{1}{2}}}\left\{1+\frac{3}{8R}-\frac{15}{128R^2}+\frac{105}{1024R^3}\right\} \quad 46$$

For large R (Özisik):

$$\mathbb{I}_n(R) \cong \frac{e^R}{\sqrt{2\pi R}} \quad 47$$

$$\mathbb{K}_n(R) \cong \sqrt{\frac{\pi}{2R}} e^{-R} \quad 48$$

Despite the relative simplicity of the equations approximated above, it is difficult to define the limits of the application of each of them. A solution found to solve the problem of application ranges, and facilitate the use of modified Bessel Functions, was to implement polynomial interpolations that satisfy the equations within specific ranges of the variable Élcio Nogueira [6;7].

IV. Results And Discussion

The R_b value, defined by equations (24 - 27), has a fundamental importance in the solution by the Frobenius method presented in this work. R_b depends heavily on the value of the aspect ratio and the Biot number. In Figure 03, we show the variation of R_b as a function of the number of Biot for the aspect ratio analyzed in this work (Figure 08, Table 01 and Equation 49), that is, for K = 5.82. A more in-depth analysis of the dependence of R_b onBiot number and aspect ratio should be the object of deeper mathematical analysis and is beyond the scope of this work.

Figure 04 shows the dimensionless temperature versus the radial position of the fin, with the Biot number as a parameter, for the two models analyzed in this work. For a wide range of Biot numbers, a satisfactory equivalence between models can be observed. The most unfavorable result occurs for a Biot value equal to 0.05.

The results for the dimensionless heat transfer rate are shown in Figure 05, for an aspect ratio of 5.82. Equivalently to that observed for the values of radial temperature, Figure 04, the models present close results, within the range of Biot number analyzed.

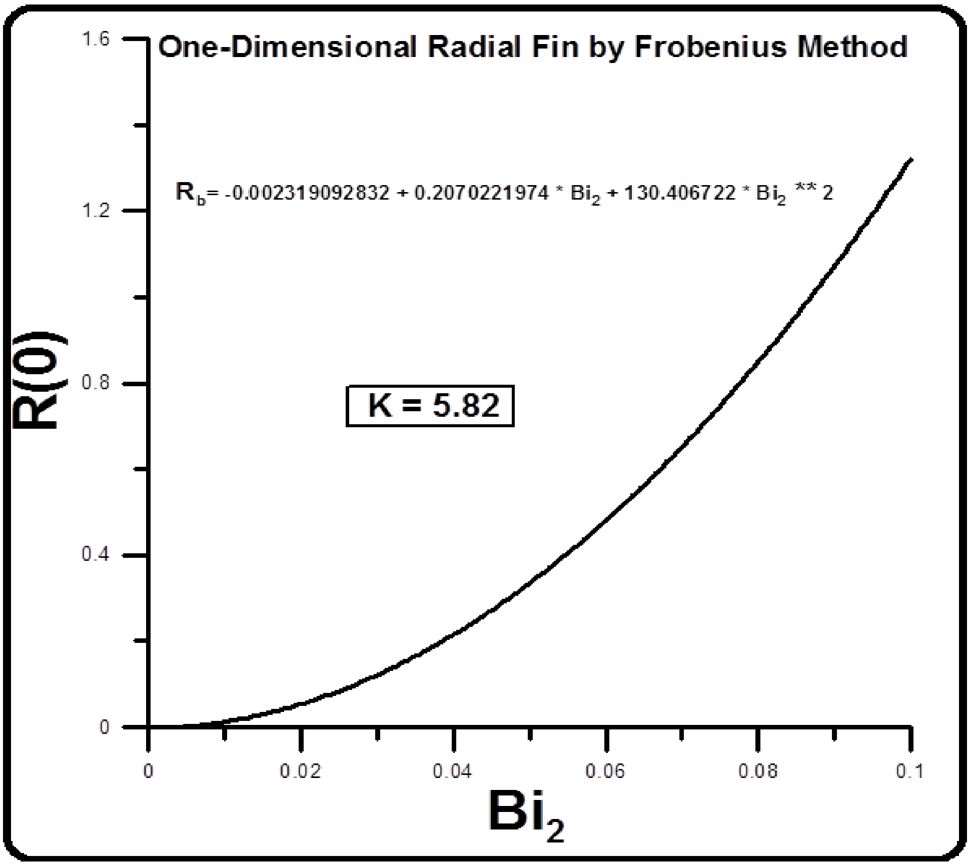

Figure 03 - R_b versus Biot number for aspect relation fin K=5.82

The results for efficiency are shown in Figure 06. Equivalently to that observed for the values of radial temperature, in Figure 04, the models present close results, within the range of the analyzed Biot number.

The results for the efficacy are shown in Figure 07. The models present close results, within the range of Biot number analyzed.

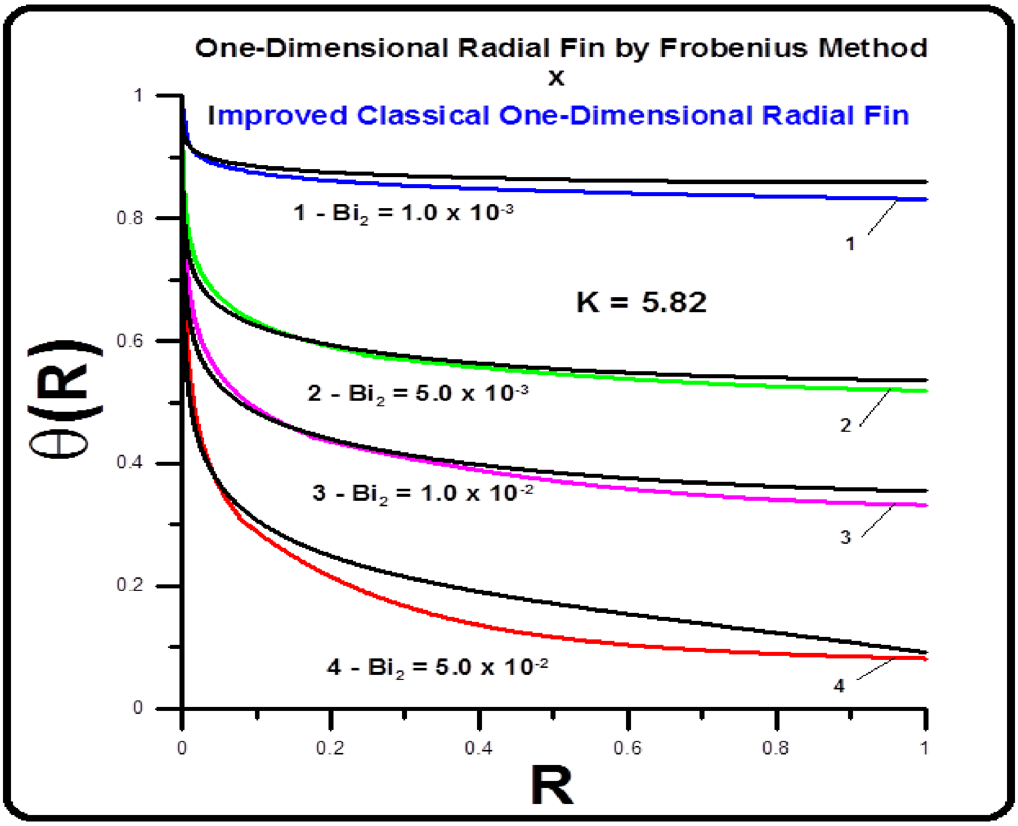

Figure 04–Comparison of dimensionless temperature versus radial positionfor the two considered models

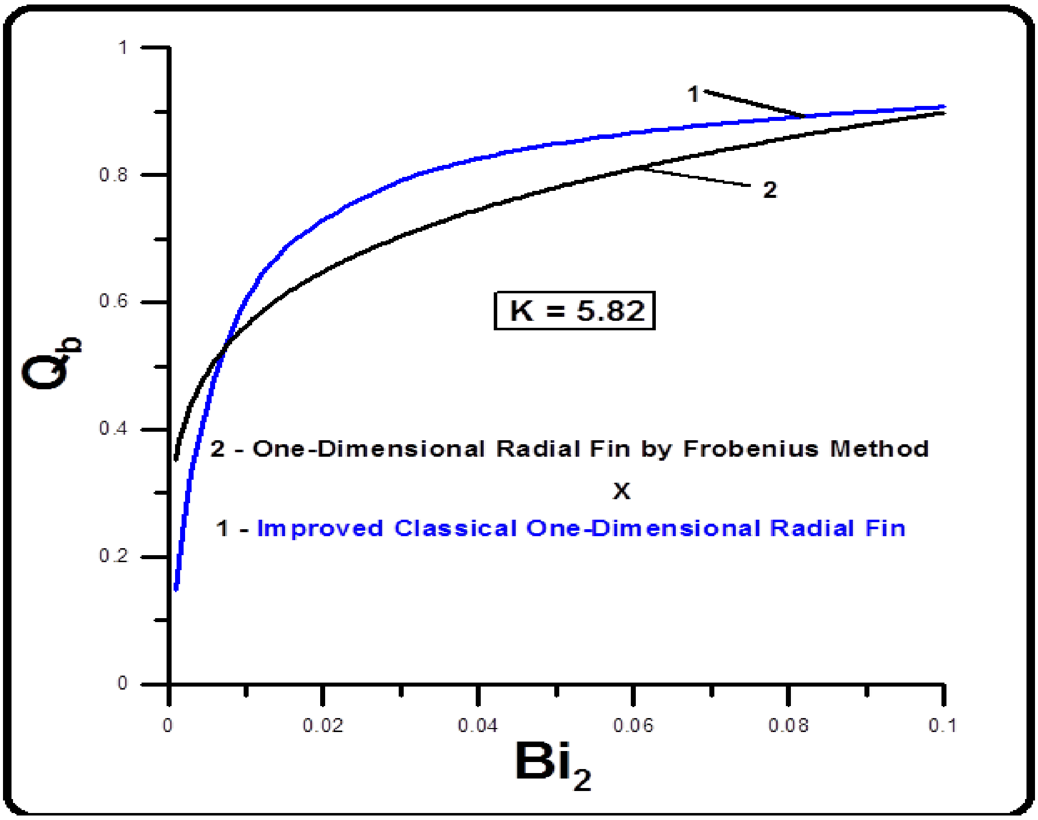

Figure 05–Comparisonofdimensionlessheat transfer rate versus Biot number for the two considered models

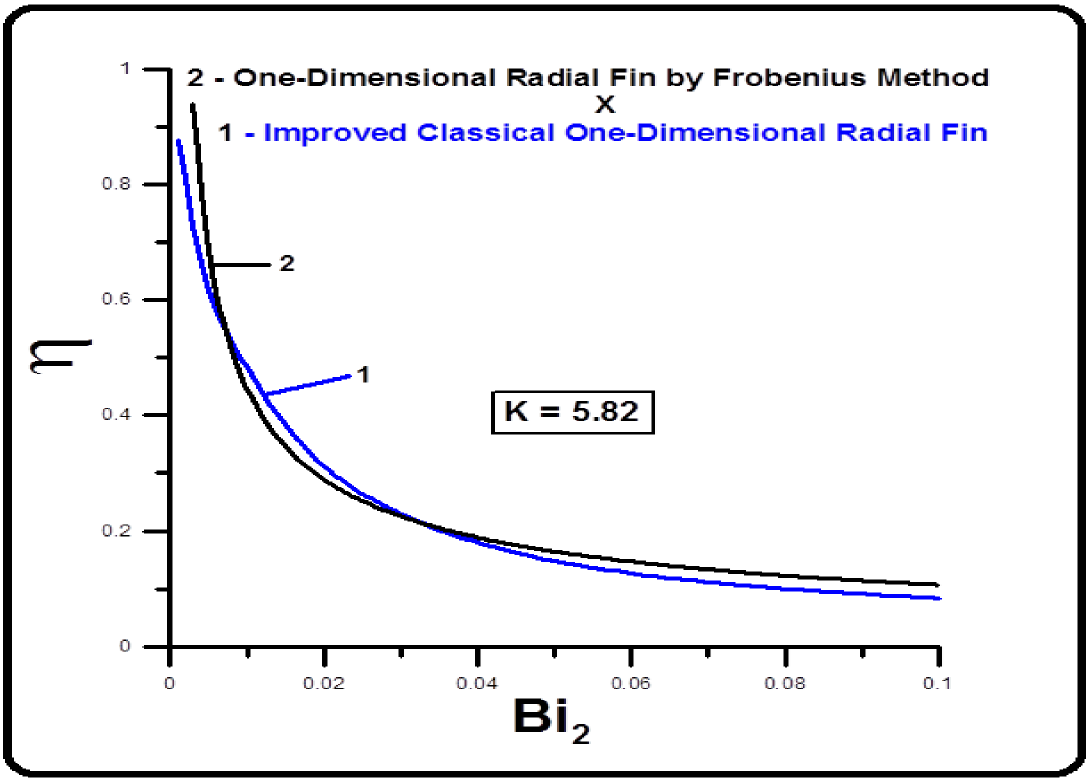

Figure 06–Comparison of efficiency versus Biot number for the two considered models

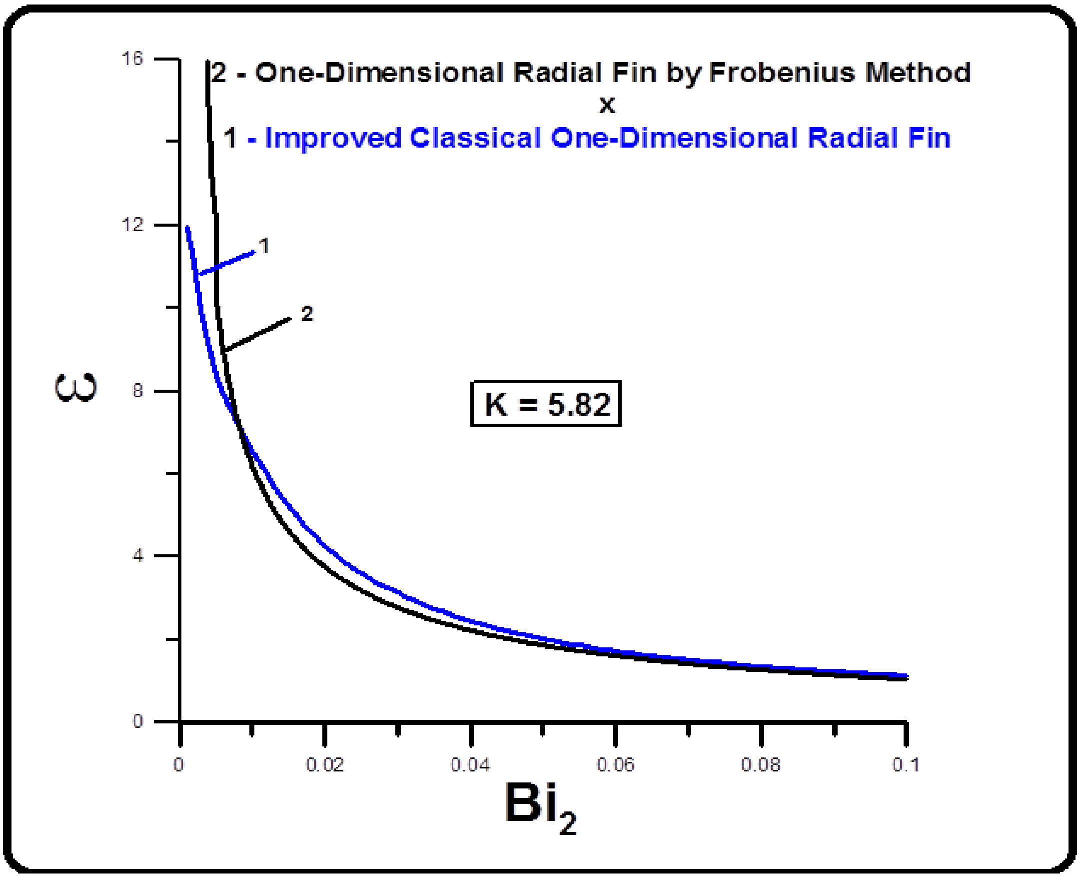

Figure 07 – Comparison of efficacy versus Biot number for the two considered models

Application

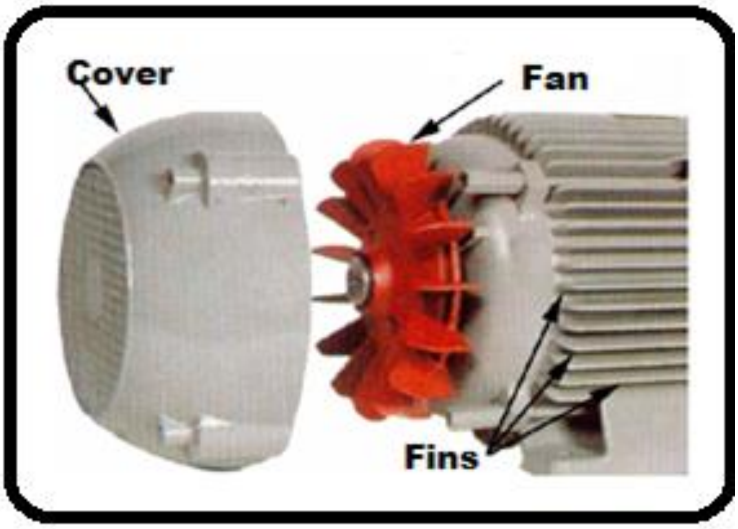

Figure 08 – Representation of an Electric Motor with Fins

A well-dimensioned fin and ventilation system can contribute to significant energy savings in Electric Motor. The data below, Table 01, refer to the quantities associated with the fins and to some operational conditions used in this work.

Table 01 – Data for finned Electric Motor DC

Essential information for the fin system	
Engine width - L_M	**130,13 mm**
Width of fin base - w	**5,84 mm**
Fin height - L_o	**17,00 mm**
Conductivity - k	**80W/(m.K)**
Maximum base Temperature - T_b	**98ºC**
Maximum external Temperature - T_∞	**40ºC**

In this case, we have for the aspect relation of the fin:

$$K = \frac{L_o}{w/2} = \frac{34.00}{5.84} \longrightarrow K = 5.82 \quad (49)$$

Table 02 – Comparison of efficiency and the effectiveness with bidimensional and improved classical solutions

Bi2	h2 W/(m^{2}K)	η % 2D	ε 2D	η%/Error% Frobenius	ε/Error% Frobenius	η%/Error% Classical	ε/Error% Classical
10^{-3}	27.4	97.35	13.63	96.71/0.7	15.31/12	87.69/10	11.96/12
10^{-2}	274	77.17	10.91	50.22/35	6.07/44	48.11/38	6.56/40
10^{-1}	27400	18.80	2.63	10.76/43	1.06/60	8.45/55	1.14/57

Table 02 presents the results of efficiency and effectiveness for the fin system in Figure 08, considering the variation in the number of Biot and the aspect ratio equal to 5.82.

For relatively low values of the Biot number, less than 0.05 (see Figure 04), the efficiency and effectiveness justify the placement of the fins and use of one of the two one-dimensional models. The results obtained by the one-dimensional models presented in this work were compared with the results of the two-dimensional model in Table 02. The two-dimensional model was implemented by Élcio Nogueira [7].

V. Conclusions

The comparison between the one-dimensional radial fin model, “Frobenius Method,” with the mathematical model already established and validated, called “Improved Classical Radial Fin” in the literature, was presented.

The physical quantities used to compare the models are the dimensionless temperature, heat transfer rate, the efficiency, and the effectiveness of an electric motor with fins and an aspect ratio of 5.82.

The results obtained for efficiency and the effectiveness of the one-dimensional models presented in this work were compared with the two-dimensional model, and for a wide range of Biot numbers, a satisfactory equivalence between one-dimensional models can be observed.

The equivalence of the one-dimensional models presented in this work is evident, considering the Reynolds number range and the value of the aspect ratio considered for analysis.In fact, for relatively low values of the Biot number, less than 0.05, the efficiency and effectiveness justify the placement of the fins and use of one of the two one-dimensional models.

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