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Proof of Goldbach's Conjecture

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Abstract: *The mathematical proof of Goldbach's conjecture in number theory is drawn in this paper by applying a specific bounding condition from Bertrand's postulate or Chebyshev's theorem.*
Keywords: *Bertrand's postulate & Chebyshev's theorem, Goldbach's conjecture, prime number, even & odd number, natural numbers series.*

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I. Introduction

It is already known that Goldbach's conjecture in number theory is: Every even integer greater than 2 can be expressed as the sum of two primes. If n be an integer, where n>1; then 2n is an even integer, where 2n>2. Thus the mathematical formulation of above conjecture is $2n=p_1+p_2$; where p_1 & p_2 are two prime numbers. Again from the other way the conjecture states that: Every even integer greater than 4 can be expressed as the sum of two odd primes. These even numbers (>4) are called Goldbach's numbers.

II. Notes of Proof

Bertrand's postulate (Chebyshev's theorem) states that:

(i) There exists at least a prime number (p) between n and 2n for any integer n>1. Such that $n<p<2n$. Let it be considered that n_1 and n_2 are two integers; where n_1 & n_2 both are greater than 1. Now $2n_1$ & $2n_2$ are the twice of n_1 & n_2 respectively. Suppose p_1 be at least a prime in between n_1 & $2n_1$ and p_2 be at least a prime in between n_2 & $2n_2$. Hence from the above postulate it is written that $n_1<p_1<2n_1$ and $n_2<p_2<2n_2$. So from these relations it can be determined that $n_1+n_2<p_1+p_2<2n_1+2n_2$ or $n_1+n_2<p_1+p_2<2(n_1+n_2)$. As $n_1>1$ & $n_2>1$, so if $n_1=u$=constant i.e. any fixed value of $n_1=2, 3, 4, \ldots$ (any integer greater than 1) & $n_2=m$, where m=2, 3, 4, … (any integer greater than 1); then $u+m< p_1+p_2<2(u+m)$ or $m+u< p_1+p_2<2(m+u)$. After addition of -u, it is obtained that $m+u-u< p_1+p_2-u<2(m+u)-u$ or $m< p_1+p_2-u<2m+u$. Now the above relation shows that $p_1+p_2-u<2m+u$, so there is at least the possibility either $p_1+p_2-u+r=2m+u$ or $p_1+p_2-u=2m+u-r$; where r be an integer>0. Hence $p_1+p_2=2(m+u)-r$. As $p_1+p_2-u<2m+u$, so r=u+x; where x=0, 1, 2, 3, … (any integer). Again every even number (2n) is the twice of a natural number (n). Thus 2(m+u) is even for any value of m and u. Now to consider Goldbach's number for even numbers except 4, p_1 & p_2 both are always odd (because of all primes are odd in natural numbers series except 2), as a result p_1+p_2 is always even as (odd+odd)=even. That means r is always even as (even-even)=even. Hence r is even when x=0, 2, 4, 6, … (any even integer) if u is an even & x=1, 3, 5, 7, … (any odd integer) if u is an odd because of (even+even)=even & (odd+odd)=even. Suppose u=2, x=0 & m=2, 3, 4, …; then $p_1+p_2=6, 8, 10, \ldots$ etc (all even integers>4). In this way by choosing the proper values of m, u & r from the above bounding condition it can be determined that every even integer greater than 4 can be expressed as the sum of at least two primes. This is nothing but a specific situation of Goldbach's conjecture.

However the above proof shows that $p_1+p_2\geq 6$ (according to consideration the lowest values of m, u & x are 2, 2 & 0 respectively). Thus $2(m+u)-r\geq 6$. Hence $2(m+u)-(u+x)\geq 6$ or $2m+u-x\geq 6$ or $2m+u-6\geq x$. i.e. $x\leq(2m+u)-6$.

(ii) There exists at least one prime number (p) for integer n>3 with $n<p<2n-2$. Let it be considered that n_1 and n_2 are two integers; where n_1 & n_2 both are greater than 3 and p_1 & p_2 are the at least prime numbers with $n_1<p_1<2n_1-2$ and $n_2<p_2<2n_2-2$ respectively . In the above way it can be drawn that $n_1+n_2<p_1+p_2<2(n_1+n_2)-4$. Here as $n_1>3$ & $n_2>3$, so if $n_1=u$=constant i.e. any fixed value of $n_1=4, 5, 6, \ldots$ (any integer greater than 3) & $n_2=m$, where m= 4, 5, 6, …(any integer greater than 3); then $u+m< p_1+p_2<2(u+m)-4$ or $m+u< p_1+p_2<2(m+u)-4$. After addition of -u, it is obtained that $m< p_1+p_2-u<2m+u-4$. Now the above relation shows that $p_1+p_2-u<2m+u-4$, so there is at least the possibility either $p_1+p_2-u+r=2m+u-4$ or $p_1+p_2-u=2m+u-4-r$; where r be an integer>0. Hence $p_1+p_2=2(m+u)-4-r$. As $p_1+p_2-u<2m+u-4$, so r=u+x; where x=0, 1, 2, 3, … (any integer). Again every even number (2n) is the twice of a natural number (n). Thus 2(m+u) is even for any value of m and u. Now to consider Goldbach's number for even numbers except 4, p_1 & p_2 both are always odd (because of all primes are odd in natural numbers series except 2), as a result p_1+p_2 is always even as (odd+odd)=even. That means r is always even as (even-even)=even and 4 is even number. Hence r is even when x=0, 2, 4, 6, … (any even integer) if u is an even & x=1, 3, 5, 7, … (any odd integer) if u is an odd because of(even+even)=even &

(odd+odd)=even. Suppose u=4, x=0 & m=4, 5, 6, …; then p_1+p_2=8, 10, 12, … etc (all even integers>6). In this way by choosing the proper values of m, u & r from the above bounding condition it can be determined that every even integer greater than 6 can be expressed as the sum of at least two primes. Here it is also nothing but a specific situation of Goldbach's conjecture.
However the above proof shows that $p_1+p_2\geq 8$ (according to consideration the lowest values of m, u & x are 4, 4 & 0 respectively). Thus $2(m+u)-4-r\geq 8$. Hence $2(m+u)-4-(u+x)\geq 8$ or $2m+u-4-x\geq 8$ or $2m+u-12\geq x$. i.e. $x\leq(2m+u)-12$.

III. Conclusion

Thus Goldbach's conjecture can be proved from Bertrand's postulate or Chebyshev's theorem with applying a special bounding condition for even integers n>4 (Goldbach's numbers). However the proof cannot be applicable for even number 4. Because 4=2+2; where 2 is only the even prime.

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