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A Mathematical Model for Pressure Distribution in a Bounded Oil Reservoir Subject to Single-Edged and Bottom Constant Pressure

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Original Research Article

Abstract

Well test analysis of a horizontal well is complex and difficult to interpret. Most horizontal well mathematical models assume that horizontal wells are perfectly horizontal and are parallel to the top and bottom boundaries of the reservoir. As part of effort towards correct horizontal well test analysis, the purpose of this study is to develop a mathematical model using source and Green's functions for a horizontal well completed in an oil reservoir at late time flow period, where the reservoir is bounded by an edge and bottom constant pressure boundaries.

The purpose of the derivation is to understand the effects of well completion, well design and reservoir parameters on pressure and pressure derivative behavior of the well at late flow time, when all these external boundaries are presumed to have been felt. If the model is applied for well test analysis therefore information like reservoir natural permeability distribution, actual external boundary types and even the well completion performance will be decidable easily. Dimensionless variables were used to derive throughout the derivations.

Results of the derivation show that the dimensionless pressure and dimensionless pressure derivatives increase with increase in dimensionless well length. This means that higher well productivity is achievable with extended well length when the reservoir is surrounded partially by constant pressure boundaries. Furthermore, the models show that higher directional permeabilities would also encourage higher well productivity at late flow time. The dimensionless pressure derivative will, as a result of a constant dimensionless pressure, potentially collapse gradually to zero at the moment the dimensionless pressure begins to exhibit a constant trend. Finally, the dimensionless pressure and dimensionless pressure derivatives vary inversely with the reservoir dimensionless width at late flow time.

Keywords: *Oil Reservoir, Horizontal Well, Two Constant Pressure Boundaries, Late Flow Period.*

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1. Introduction

One of the primary objectives of petroleum engineers is concerned with the optimization of ultimate recovery from oil and gas reservoirs. Advances in drilling and completion technologies have placed horizontal wells among the techniques used to improve production performance. In the case of bottom water drive, horizontal wells prevent coning without introducing the flow restriction seen in partial penetration wells. Horizontal drilling is also efficient to increase the well surface area for fluid withdrawal, thus improving the productivity. In order to develop oil reservoirs and forecast their future reservoir performance, it is important to attain accurate reservoir descriptions. Horizontal wells can greatly increase the contact area of the wellbore so they are commonly applied in oil reservoirs to enhance the production and ultimate recovery of oil especially in low permeability formations. Pressure behavior of horizontal wells considering various reservoir conditions have been analyzed in various previous studies.

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The tables of instantaneous Green's and source functions that can be used with the Newman's product method to generate solutions for a wide variety of reservoir flow problems gives new solutions for infinite conductivity sources. This provides the point source solution as part of a more general theory of Green's functions[1]. Mathematical model to study the transient pressure behavior in a well with an infinite-conductivity horizontal drainhole in an infinite slab reservoir was developed by [2], where the physical model included a fluid of small and constant compressibility flowing through an infinitely large anisotropic reservoir with upper and lower impermeable boundaries, analytical solution was obtained using concepts of instantaneous sources and Green's functions. The performance of a horizontal well subject to bottom water drive delineates conditions under which completion mode is more appropriate. The productivities of horizontal wells operating under bottom water drive depend on displacement efficiencies[3].

Mathematical model of horizontal well pressure drawdown and buildup in a single porosity reservoir and a double porosity reservoir by developing the necessary mathematical analysis,[4] applied quadrature theorem to derive pressure drawdown and buildup formulae of the uniform line source. Their formulae, which did not include the sum of infinite series, was more reasonable and are easily used in well testing analysis.

Technique for the interpretation of transient pressure based on dimensionless pressure and pressure derivative was developed by [5] wherepressure behavior of the wells for different conditions was analyzed. The effect of the outer boundaries of the reservoir on the pressure behavior of the horizontal wells was investigated for different configurations and rectangular shaped reservoir with different dimensions was used to study the pressure response in the well. [6] derived theoretical expressions for predicting dimensionless breakthrough times of horizontal wells in a two layered reservoir of architecture like letter 'B', experiencing bottom water drive mechanism of different patterns, with or without a top gas. The theoretical breakthrough times was based on dimensionless pressure and dimensionless pressure derivative distributions of each identified model. [7] deduced that, through a study of instantaneous source functions for a horizontal and vertical well completed in the layers of the reservoir, a reservoir is layered and the layering precipitates an architecture in the form of letter 'H', then modeling pressure distribution or understanding the character of the influencing external boundaries can be achieved. He considered both crossflow and no-crossflow layers and the external boundaries were varied as sealed and constant-pressured. the study showed that a two-layered reservoir of latter 'H' architecture has no inverted form in terms of external boundary variation, no matter the axis of symmetry considered for inversion.

Comparative analysis between the pressure behavior of a horizontal well and a vertical well both subject to edge water drive, Source and Green's functions were used to evaluate the performance of horizontal well and vertical well, [8] compared dimensionless pressures and pressure derivatives computed by varying the reservoir geometry. Results presented show that the rate of decline of the pressure derivative curve is sharper and more sensitive in vertical well than horizontal well of the same geometry indicating a shorter period of clean oil production in vertical well than the horizontal well. Further [9] derived ten models for pressure distribution for horizontal well under different boundary variation. They resolved that; three instantaneous source functions are involved in the pressure distribution expression. Wells in a bounded reservoir with constant pressure at the Top and Bottom for the interpretation of pressure responses in the reservoir based on dimensionless pressure and pressure derivative. Results showed that dimensionless lateral extent does not directly affect the dimensionless pressure and dimensionless pressure derivative for very short well lengths. Further dimensionless pressure increased with reservoir pay thickness and delayed the time for steady state conditions. [10] Carried out a study using Source functions and Newman's product rule to develop model predicting the performance and behavior of a horizontal well which was subjected by double edge water drive. Numerical methods were used to perform the computation and Sensitivity analysis of the parameters used.

The purpose of this work is to derive a model that can be used to determine pressure distribution and well test procedure for ultimate enhanced oil recovery in a horizontal well.

2. Reservoir and Mathematical Model Description

When the reservoir is bounded by single edge and bottom water drives, pressure distribution information is fundamental in determining wellbore pressure flow regimes for efficient oil production. Figure 1 shows anisotropic rectangular shaped reservoir model under study. It consists of a horizontal well where the vertical permeability is greater than the horizontal permeability, $k_v > k_h$. The horizontal well is designed such that $d_x \ll x_e - D_x$, $d_y = y_e - D_y$ and $d_z \gg z_e - D_z$ to avoid early encroachments of the bottom water drives.
From Figure 1, the Instantaneous source functions for the reservoir are now selected as described below.

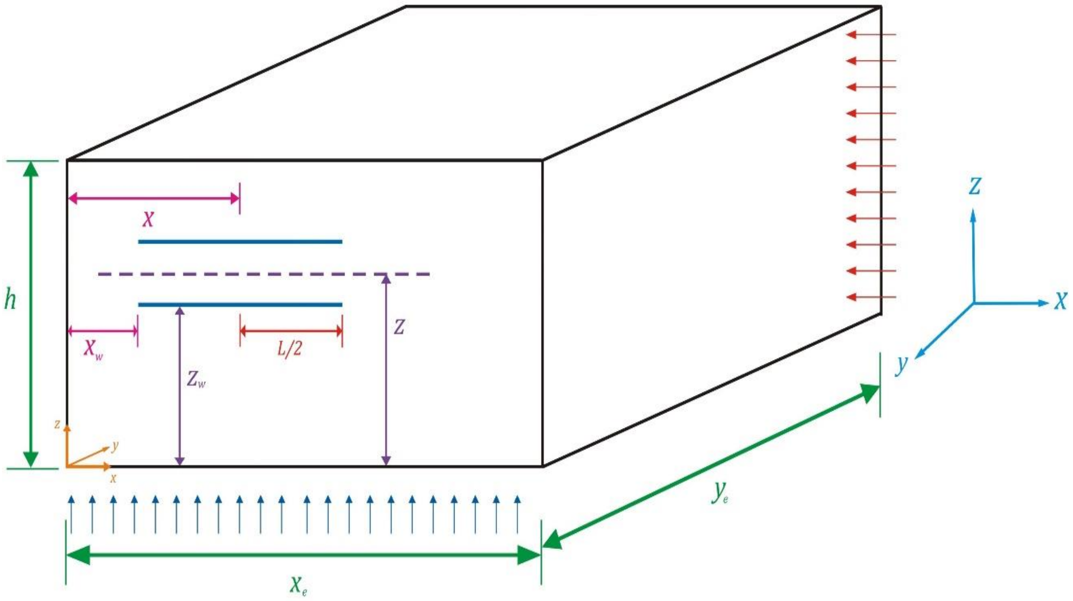

Figure 1: Horizontal well acting in an infinite reservoir

x – $Axis$ Physical and Mathematical Description

The reservoir is bounded both at the bottom and at the top by sealing fault as shown in Figure 2.

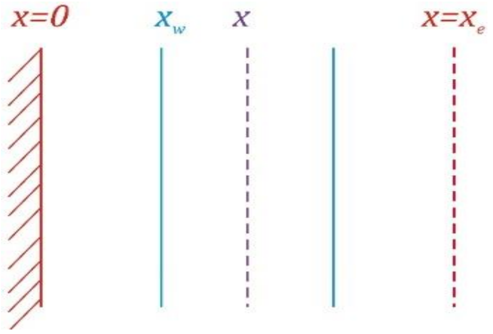

Figure 2 (a): Physical Description of the Reservoir along the $x - Axis$

Since the well is aligned with this axis, the source function responsible for flow is an infinite plane source in an infinite slab reservoir with bottom end sealed and the exposed to a constant pressure boundary. Hence, according to Gringarten *et. al* and Adewole *et. al* the appropriate instantaneous source function is given as: -

$$s(x,t) = \frac{8}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n+1}exp\left(-\frac{(2n+1)^2\pi^2\eta_x t}{4x_e^2}\right)\sin\frac{(2n+1)\pi x_w}{2x_e}\cos\frac{(2n+1)\pi x}{2x_e}\cos\frac{(2n+1)\pi x}{2x_e} \quad (1)$$

$y - Axis$ Physical and Mathematical Description

The reservoir is assumed to be sealed at both ends as shown in Figure 2b.

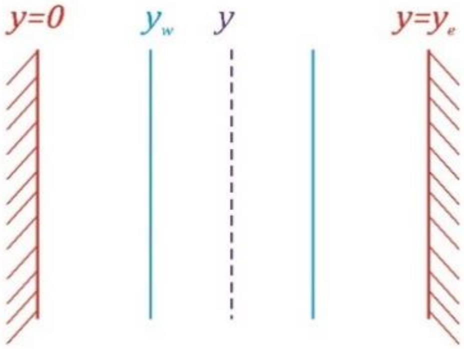

Figure 2 (b): Physical Description of the Reservoir along the $y - Axis$

Therefore, the source function at late flow is an infinite plane source in an infinite slab reservoir. Mathematically, this corresponds to the source function below: -

$$s(y,t) = \frac{1}{y_e}\left[1 + 2\sum_{n=1}^{\infty} exp\left(-\frac{n^2\pi^2\eta_y t}{y_e^2}\right)\cos\left(\frac{n\pi y_w}{y_e}\right)\cos\left(\frac{n\pi y}{y_e}\right)\right] \tag{2}$$

$z - Axis$ Physical and Mathematical Description

Along the z axis, the reservoir has a constant pressure boundary at the bottom and sealed at the top as shown in Figure 2c.

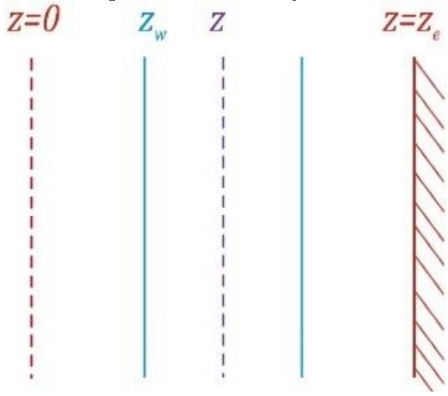

Figure 2 (c): Physical Description of the Reservoir along the $z - Axis$

Hence the appropriate source function responsible for fluid flow is given as shown below: -

$$s(z,t) = \frac{1}{z_e}\sum_{n=1}^{\infty} exp\left(-\frac{(2n-1)^2\pi^2\eta_z t}{4z_e^2}\right)\sin\left(\frac{(2n-1)\pi z_w}{2z_e}\right)\sin\left(\frac{(2n-1)\pi z}{2z_e}\right) \tag{3}$$

3. Mathematical model description

Using dimensionless parameters in equations (1) to (3), and considering a point source as the intersection of three perpendicular infinite plane surfaces normal to the principal axes of permeability, the instantaneous point source function is obtained by Newman's product method as: -

$$P_D = 2\pi h_D \int_0^{\tau_D} s(x_D,t_D).s(y_D,t_D).s(z_D,t_D)d\tau \tag{4}$$

Where: -

$$s(x_D,t_D) = \frac{8}{\pi}\sum_{n=1}^{\infty}\frac{1}{n+1}exp\left(-\frac{(2n+1)^2\pi^2 t_D}{4x_{eD}^2}\right)\sin\left(\frac{(2n+1)\pi x_{wD}}{2x_{eD}}\right)\cos\left(\frac{(2n+1)\pi x_D}{2x_{eD}}\right)\cos\left(\frac{(2n+1)\pi x_D}{2x_{eD}}\right) \quad (5)$$

$$s(y_D,t_D) = \frac{1}{y_{eD}}\left(1+2\sum_{n=1}^{\infty}exp\left(-\frac{n^2\pi^2 t_D}{y_{eD}^2}\right)\cos\left(\frac{n\pi y_{wD}}{y_{eD}}\right)\cos\left(\frac{n\pi y_D}{y_{eD}}\right)\right) \quad (6)$$

and

$$s(z_D,t_D) = \frac{1}{h_D}\sum_{n=1}^{\infty}exp\left(-\frac{(2n-1)^2\pi^2 t_D}{4h_D^2}\right)\sin\frac{(2n-1)\pi z_{wD}}{2h_D}\sin\left(\frac{(2n-1)\pi z_D}{2h_D}\right) \quad (7)$$

Substituting equation (5), (6) and (7) into equation (4) we have: -

$$\begin{aligned} P_D &= \frac{16}{y_{eD}}\int_0^{\tau_D}\left[\sum_{n=1}^{\infty}\frac{1}{n+1}exp\left(-\frac{(2n+1)^2\pi^2\tau_{Di}}{4x_{eD}^2}\right)\sin\left(\frac{(2n+1)\pi x_{wD}}{2x_{eD}}\right)\cos\left(\frac{(2n+1)\pi x_D}{2x_{eD}}\right)\cos\left(\frac{(2n+1)\pi x_D}{2x_{eD}}\right)\right. \\ &* \left(1+2\sum_{n=1}^{\infty}exp\left(-\frac{n^2\pi^2\tau_{Di}}{y_{eD}^2}\right)\cos\left(\frac{n\pi y_{wD}}{y_{eD}}\right)\cos\left(\frac{n\pi y_D}{y_{eD}}\right)\right) \\ &* \left.\sum_{n=1}^{\infty}exp\left(-\frac{(2n-1)^2\pi^2\tau_{Di}}{4h_D^2}\right)\sin\left(\frac{(2n-1)\pi z_{wD}}{2h_D}\right)\sin\left(\frac{(2n-1)\pi z_D}{2h_D}\right)\right]d\tau_D \end{aligned} \quad (8)$$

The dimensionless pressure derivative at steady-state is given as: -

$$P_D' = t_D\frac{\partial P_D}{\partial t_D} \quad (9)$$

Substituting equation (8) into equation (9), we have: -

$$\begin{aligned} P_D' = \frac{16t_D}{y_{eD}}&\left[\sum_{n=1}^{\infty}\frac{1}{n+1}exp\left(-\frac{(2n+1)^2\pi^2 t_{Di}}{4x_{eD}^2}\right)\sin\left(\frac{(2n+1)\pi x_{wD}}{2x_{eD}}\right)\cos\left(\frac{(2n+1)\pi x_D}{2x_{eD}}\right)\cos\left(\frac{(2n+1)\pi x_D}{2x_{eD}}\right)\right. \\ &* \left(1+2\sum_{n=1}^{\infty}exp\left(-\frac{n^2\pi^2 t_{Di}}{y_{eD}^2}\right)\cos\left(\frac{n\pi y_{wD}}{y_{eD}}\right)\cos\left(\frac{n\pi y_D}{y_{eD}}\right)\right) \\ &* \left.\sum_{n=1}^{\infty}exp\left(-\frac{(2n-1)^2\pi^2 t_{Di}}{4h_D^2}\right)\sin\left(\frac{(2n-1)\pi z_{wD}}{2h_D}\right)\sin\left(\frac{(2n-1)\pi z_D}{2h_D}\right)\right] \end{aligned} \quad (10)$$

4. Conclusions

Equation (8) is the dimensionless pressure distribution of a reservoir having an edge constant pressure boundary at the upper end and a constant pressure boundary at the bottom. Equation (10) is the corresponding dimensionless pressure derivative of the reservoir. The edge constant pressure is similar to edge water drive and the bottom constant pressure boundary is similar to bottom water drive frequently encountered in reservoirs. The two models show clearly that: -

i. Both dimensionless pressure and its derivatives are affected by reservoir dimensionless length, thickness, fluid properties and well design.
ii. The dimensionless pressure and derivatives are particularly inversely affected by reservoir width.
iii. The dimensionless pressure will exhibit a constant trend with dimensionless time.
iv. The dimensionless pressure derivative will, as a result of a constant dimensionless pressure, potentially collapse gradually to zero at the moment the dimensionless pressure begins to exhibit a constant trend.
v. The models are capable of predicting actual constant pressure influences at late time.

Competing interests

Authors have declared that no competing interests exists.

Dimensionless Parameters

$$P_D = \frac{kh\Delta p}{141.2q\mu B}$$

$$t_D = \frac{4kt_i}{\phi\mu c_t L^2}$$

$$\eta_i = \frac{k_i}{\phi\mu c_t}$$

$$i_D = \frac{2i}{L}\sqrt{\frac{k}{k_i}}$$

$$i_{wD} = \frac{2i_w}{L}\sqrt{\frac{k}{k_i}}$$

$$i_{eD} = \frac{2i_e}{L}\sqrt{\frac{k}{k_i}}$$

$$h_D = \frac{2h}{L}\sqrt{\frac{k}{k_z}}$$

$$L_D = \frac{L}{2h}\sqrt{\frac{k}{k_x}}$$

$$h_D = \frac{1}{L_D}\sqrt{\frac{k^2}{k_x k_z}}$$

$$r_{wD} = z_D - z_{wD}$$

$$i = x, y, z$$

$$\eta = \frac{k_i}{\mu\phi c_t}$$

Nomenclatures

B *oil volumetric factor, rbbl/STB*
ct *total compressibility, 1/psi*
i *axial flow directions x, y, or z*
h *formation thickness, ft*
k *average geometric permeability, md*
k_i *formation permeability in the i^{th} direction, md*
L *total length of horizontal well, ft.*
L_D *dimensionless well length*
Δp *pressure drop, psi*
P_D *dimensionless pressure*
P_D' *dimensionless pressure derivative*
x_e *half the distance to the boundary in the $x-direction$*
y_e *half the distance to the boundary in the $y-direction$*
x_w *the X coordinate of the production point*
y_w *the Y coordinate of the production point*
z_w *the Z coordinate of the production point*
z_D *the dimensionless distance from the bottom of the reservoir to the centre of the wellbore*
z_{wD} *the dimensionless distance from the bottom of the reservoir to the bottom of the wellbore*
d_x *the shortest distance between the well and the $x-$boundary, ft*
d_y *the shortest distance between the well and the $y-$ boundary, ft*
d_z *the shortest distance between the well and the $z-$ boundary, ft*
D_x *the longest distance between the well and the $x-$ boundary, ft*
D_y *the longest distance between the well and the $y-$boundary, ft*
D_z *the longest distance between the well and the $z-$ boundary, ft*
q *flow rate, bbl/day*
s *source*
t *time, hours*
D *dimensionless*
e *external*
w *wellbore*

Greek symbols

μ *reservoir fluid viscosity, cp*
ϕ *porosity, fraction*
τ *dummy variable of time*
η *Diffusivity constant, md-psi/cp*

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