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Role of Cut Points and Puncture Points in Space -Time Measure Manifold to Prevent the Spared Of Corona Virus in Real World Manifold

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Abstract: - *We conduct a study on Topological Manifold and some of its properties Theorems, structures on topological manifold and its structural characteristics. The main properties in Topological Manifold is connectedness, path connectedness, compactness, boundedness etc, which are the module in the real time manifold. These modules are use in day to day life to prevents spared of corona virus in the world Manifold. Actually Real World is space- time Manifold. Cut points are Used on this manifold as ,To maintain the social distance, to follow the rules of lock down , Also these topological properties are studied the path connected ,locally connected ,locally path connected with cut point, punctured point and punctured space on real World Manifold. Cut point and punctured points are additional properties of manifold helps for quarantine, curfew, social distance, isolation and lock down in society. The punctured space is quarantine space which prevents spread of corona virus in world Manifold.*

Key-word: *Connectedness, locally connected, path connected, Locally path connected, cut point, punctured points, corona virus, quarantine, lockdown curfew, isolation. Boundedness.*

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I. Introduction

We are using basic concepts like, connectedness, subspace of Topological manifold, Topological space like(sub space) product space, quotient space, Equivalence Space .The Connectivity of Topological Manifold Plays an important role .In connectivity of Topological manifold William Basener [12] introduced the basic concepts of all types of connectivity in Topological Manifold. Also properties on measure manifold [10] the concept of path connectivity is discussed by Lawrence Conlon [8].Real world Manifold is Space-Time Measure manifold which was discussed in[11]

Devender Kumar Kamboj introduced the concept of cut points which plays a very important role in Topological space. I[2][5][6]developed the cut points and punctured points on topological manifold, space time measure manifold and measure manifold To study cut point, punctured points and punctured space on space-time measure manifold [6][11] as well as on topological space , topological manifold are assumed to be connected. The idea of cut points in a topological manifold, space-time manifold comes dates back to 2013's. After 2013 by Dr. Haloli H G[5][6] uplifted the concepts of cut points and punctured points . These cut points, punctured points plays an important role to prevents social distance during the lock down period, Whole world is suffering from corona virus spared in the society, to control the rate of spared of such corona virus in space – time real world manifold we have to maintain the following things

1) **Lockdown** : compactness and bounded
2)**Curfews**: boundedness
3) **Seals**: boundedness and compactness
4) **Quarantine:** Home, institute and village :- Disconnected space-time Measure manifold, punctured space(Dirichlets condition)
5) **Area restricted**:- path disconnectedness
6) **Isolations: bounded**

The topological subspace is also important part of the work.

An injective continuous map that is a homeomorphism onto its image in the subspace topology is called topological embedding. If f: A → X is such a map we can think of the image set f(A) as a homeomorphic copy of A embedded in X. The main role of path connectedness in a connected space gives the non cut point connected so called strongly connected

We are imbibing these following basic definitions in our discussion
1) Topological Manifolds :- Space –time Measure Manifold
2)Connectedness :
3) Path connected spaces
4) Locally connected
5) Locally path connected
Of all the spaces which one studies in topology the Euclidean space and their subspace are the most important also the metric spaces R^n serve as a topological model for Euclidean space E^n for finite dimensional vector space.

II. Some Basic Defination

DEFINATION 2.1

A Topological space **M** is a Manifold of dimensions n (an - n- manifold) if
i) **M** is locally Euclidean and dim of **M** =n
ii) **M** is 2^{nd} countable
Iii) **M** is Hausdorff
Here **Hausdorff** means that any two distinct points lie in disjoint open sets but in the same space. **Second countable** means that there is a countable family of open subsets is the union of a subfamily. **Locally homeomorphic** or local Euclidean means that every point has an open neighborhood homeomorphic with an open subset of R^n

EXAMPLE 2.2

Let **M** be an open subset of R^n with the subspace topology then **M** is an n-manifold

Definition 2.3 Connectedness [12]

A topological space **M** is said to be connected if the only subset of **M** that are both open and closed are itself and the empty set.

Example 2.4

R is connected and any interval in R is connected.

DEFINATION 2.5

A subset A of a topological space **N** is called a component of q if A is connected (in the sub-space topology) and if there is no connected subset of q that properly contains A.
i.e. A topological space is connected if it has only one piece. The connected pieces are called components.
Real world is topological space which is connected.

DEFINATION 2.6 [12] [7]

A Topological Space **M** is said to disconnected if there exists two non-empty subset A, B of **M** satisfying
(i) $A \cup B =$ **M**
(ii) $A \cap B = \emptyset$
(iii) A and B are both open and closed.

DEFINATION2.7. Path connected [8] [12]

A Topological Space **M** is path connected if given any two – points p, q $\in$ **M** ,there exists a map γ: [0,1]
M such that $\gamma(0)=p$ and $\gamma(1)=q$. The map γ is called path in **M** from p to q
Particularly a surface is a path connected if given any two points p,q $\in$ S there is a map $\gamma[0,1] \rightarrow S$ such that $\gamma(0) = p$ and $\gamma(1) = q$ .The map γ is called a path in S from p to q.

Definition 2.10 [12]

A Topological space **M** is locally connected if every p $\in$ **M** has a connected neighborhood. A Space **M** is locally path connected if every p $\in$ **M** has a path connected neighborhood.

Example 2.11

Any open subset $O \subseteq R^n$ is locally path connected for each p $\in$ O. There is an open ball
$B_r(p) \subseteq O$ and this ball is a path connected neighborhood of p.

Theorem 2.12 [7]
Every manifold is locally path connected.

III. Topological Properties are model on Real World Manifold M.

As per government of all countries(India) taken decision for the lock down, particularly curfew, social distance, road blocks ,quarantine , isolation etc. to control the spread of corona virus . World is a space –time manifold, so that decision taken by govt. time to time. The spread of corona virus is spreading vigorously in world. Those countries are followed rules and model strictly then rate of spread of such virus is in the control. According to the mathematical model and topological properties. In the previous paper[2] [4][6] [6] I had covered all basics of this paper. On the basis of all topological properties I had correlated the very important topological properties to control the spread of corona virus in the real world space-time Manifold. They are mathematical model.
In this paper I had use basic property as connectedness, compactness, cut point and punctured points 0n the real world space –time manifold.

3.1 Connectedness
The real world manifold **M is** topological manifold.[6]

Theorem 3.1.
World is a real space- time manifold M ,which is locally path connected.

Theorem3.2.
Let **M** be an open subset of R^n .If **M** is connected then **M** path connected. **Theorem 3.3**
Every connected space need not be locally connected
Proof
We have every path connected space is connected but converse is need not be true for path connectivity that space is must be open subset of R^n , then it is possible also every path connected implies locally connected.
If p is connected but not an open sub-set of R^n then p is not locally connected because there does not exist connected neighborhood which contains $q \neq p$ for all $p \in$ **M** which is open.
$p \neq q$ does not exist connected neighborhood. This shows that **M** fails locally connected condition. Therefore every connected space need not be locally connected
Every path connected space is connected converse need not be true.

Theorem 3.4
Every locally path connected space **M** is locally connected (converse not true)
Proof
By William Basener, every path connected space is connected implies local property i.e by definition of locally connected space
Every locally path connected space is locally connected
i:e for any point $p \in$ **M** in locally path connected space exists a neighborhood of p which is connected neighborhood for **M**,which is locally connected, hence the Theorem .

Corollary 3.5:-
Every connected manifold is locally connected.
Remark:- 3.6
1) Real- world Manifold M is locally connected.
2) Real- world Manifold M is locally path connected.
3) Real- world Manifold M is connected.
On real world Manifold M all above statements are equivalent .
Statement 2 implies statement 1 implies Statement 3 implies statement 2

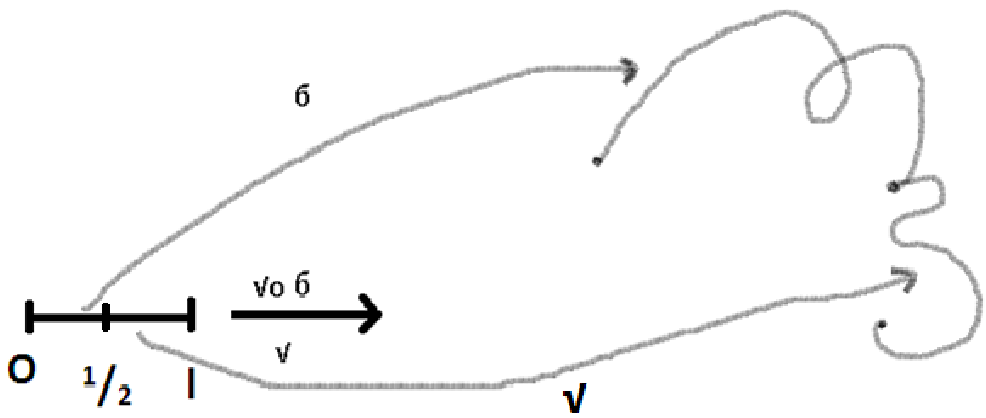
Theorem 3.7
The product of two locally connected space is locally connected
Proof
Let p and q be two locally connected spaces.
As locally connected there exists a connected neighborhood of $p \in$ **M** , $q \in$ **N** . The product space **M** x **N** has a connected neighborhood for p and q. i.e. For every pair $(p,q) \in$ **M** x **N** has a connected neighborhood because product of two topological space is topological space.

Any open subset O ϵ **M** x **N** , for (p,q) ϵ O There exists a connected neighborhood of O which contains (p,q) which is connected.
Hence the product of two locally connected space is locally connected.

Theorem 3.8
The product of two path connected space is path connected.
Proof
Let p and q are two path connected spaces. There exists a path γ and σ in p and q such that γ is a path between any two points in **M** , say p_1, p_2. σ is path between any two points in **N** say q_1, q_2. As the product spaces (p,q) ϵ **M** x **N** for all p_1, p_2 ϵ **M** , q_1, q_2 ϵ **N** if there exists a continuous map P(p,q)→p or q
i.e. $P_i(p_1 \times p_2 \times p_3 \ldots p_n) = p_i$ for i
By the definition of product of path
If γ and σ are paths in **M** such that the end point of γ is the beginning point of σ ,then we combine γ and σ to form a new path that fallows σ and then γ.

$$\gamma o\ \sigma = \begin{cases} \sigma(2S) & \text{if} \quad 0 \leq S \leq \frac{1}{2} \\ \gamma(2S-1) & \text{if} \quad \frac{1}{2} \leq S \leq 1 \end{cases}$$

The path γoσ is called the product of γ and σ



The product of the path 6 with the path V

This shows that the product of two path connected space is path connected space is the path connected.
i.e. There exists any path from one point to any point in **M** x **N**, $P(0) = p_1 \times q_1$ and $P(1) = p_2 \times q_2$ for all p_i ϵ **M** , q_i ϵ **N** in between P[0, 1] → **M** x **N**.

Remark:-
If lock down release within state i.e. each state is locally path connected then whole country is connected, so virus will spared fast which is out of control.
Applying lock down in the state or country is very essential for control of virus like covid-19.
The concept of cut point plays a very important role real word space –time manifold. In this paper we study the same cut point concept in connected real word space –time manifold
Cut points in some connected measure manifold have been studied by[10][11][2][6]. We already discussed about cut point and punctured points on Differential Manifold, Topological Manifold, Measure manifold and space time complete measure manifold in research articles[6][5][10][11].

IV. Compactness model on space-time manifold which is the Real world Manifold.

According to mathematical model, compactness is the one of the important property of topological manifold. Compactness property of topology play a vital role in controlling the spread of corona virus in space-time real world manifold.
The rate of spared of such corona virus in space –time real world manifold we have to maintain the following things
1) **Lockdown** : compactness and bounded
2)**Curfews**: boundedness

3) **Seals**: boundedness and compactness
4) **Quarantine:** Home, institute and city/village :- Disconnected space-time Measure manifold, punctured space(Dirichlets condition)
5) **Area restricted**:- path disconnectedness
6) **Isolations:** bounded
All the above mentioned social model are the topological properties . If the real world manifold is compact then it is easy to control the spread of corona virus.

Theorem:4.1
A compact surface with finitely many quarantine spaces is also compact.(the surface of real world is compact which contains quarantine spaces is also compact).
Proof:- If **M is** a real world manifold which is compact.
Let Q_i be the set of all quarantine spaces/points in **M**. A subset **I** of **M** is compact.
On real world manifold, any country is a subset of real world manifold. Which are subset of compact manifold they must be closed and bounded subset. It clearly indicates to follow rules of lock down, otherwise it fails to closed property .
By Bolzano Weierstrass property, is that, Every finite subset **I** of **M** has a limit point in **M**.
The neighborhood of$\{Qi\}$is closed point of **I** $\subseteq$**M** which are in totally in M, which is compact space **I** be subset which contains finitely many quarantine space. **M** having finite no. of quarantine spaces even **M** is compact.
Remark;4.2
A compact surface with infinitely many quarantine space is non compact.
It give idea, if lock down is not in the control the maximum quarantine space, its difficult to control.

Theorem:-4.3
Each country is closed subset of real –Manifold which is compact space then that country itself is compact.

Remark:-4.4
1) Each state **S** is closed subset of country **C** which is compact space then that state itself is compact.
2) "A closed subset of compact space is compact". It gives that **S is also compact.**

These topological property gives clear idea about need of compactness property in **all level(locally and globally), like** villages, city, district , state, country and world, we have to maintain closed property, then it will be compact. Compact space will easy to control any types of rule like curfew, lock down, quarantine, isolation social distances etc. These all social properties of lock down comes under closer property in space time real world Manifold **M**.

Definition :-4.5.(Lock down)
Let f(p) be a function we call it as lockdown function**. M** be a topological manifold we call this manifold is real world manifold
If f(p) satisfies Dirichlets Conditions , then
1) lockdown must be absolutely integrable over a period on world Manifold.
2) Lock down must have a finite number of conditions depends upon social need in the interval of lock down period.
3) Lock down must have a finite number of discontinuous between country to country, district to district, village to village etc.
4) F(p) must be bounded by conditions and rules.

Theorem:**-4.6**
A real world(manifold) is bounded then it satisfies Dirichlets conditions.

Theorem:-4.7:-
If real world manifold **M** contains finite number of cut point(road block) rules then **M** must have a finite number of discontinuous country/states / districts / city/ village.

Theorem:-4.8
A space is said to be a lock down space, if all the conditions and rules must be followed.

Theorem:-4.9
A compact real world manifold with finitely many quarantine spaces is also compact.

V. Cut points and punctured points

Each space is connected with other, i.e. each country /state/district/city/villages are connected with other. So there exits a path from each other by road/water/air in the real word manifold **M**. Lockdown is the important tool to disconnect the country /state/district/city/villages. Which can be disconnect by cut point. They may be one or more cut points may require to disconnect the space.

The real word manifold is path connected, then its connected. Connectedness is nature of world manifold,so its difficult to control the spread of any types of virus like covid-19. If we follows the rules of lock down is the main tool to control. In Lockdown all tools are the mathematical model. Among these tools cut points and punctured points are the tools which can be reduces the rate of spread of corona virus in the world. Those country are followed the rule of lock down , they are in the control situation. Relations of mathematical model and social rules are listed at the last page of this article.

Accordingly to Mathematical model, quarantine spaces are the puncture space .

Definition 5.1

A point **M** in a topological space is said to cut point if by removal of p, **M** becomes disconnected as separation.

Definition 5.2 Strong cut point

A point m of a space **M** is called a strong cut point if the removal of cut point from space **M**, **M** becomes separated sets which are connected

i.e. A connected space contains single cut point such point is called strong cut point.

In this paper this same concept will apply for real world space-time measure manifold as road block, lock down etc plays an important work to disconnect the space like village, city, state, country to control the spared of corona virus in the society.

Definition 5.3 Punctured point

A point p in topological space or surface M is called punctured point if **M** -{p} is connected.

Contentment zone. :- If corona positive case found in the space, suddenly isolate that space from other, i.e. which we call as punctured point / space.

Connectedness does not implies path connectivity, path connected space fail for **M** . Then **M** is not strongly connected

Lemma 5.4

Every strongly connected space is path connected converse is true for only path connected space with non cut points

5.5 Notations:-

We shall mean a topological space **M** contains at least two points.

Let **M** be topological space which is connected, p be a cut point of **M** where x is closed

Then Ct **M** – denote the set of all cut points of a space **M**, A/B from a separation of a space **M** by cut point p .

We say that each one of A and B is separating set of **M** , A separation A/B of **M** - {p} is denoted by Ap/Bq if the dependence of the separation on p is to be specified.

A_p* is issued for the set Ap ∪ {p} similarly for a connected subset Y of **M** – {p} , Ap(Y) is used to denote the separating subset of **M** – {p} containing Y.

A connected space with **M** = ct **M** is called a cut point space

Let a,b ϵ **M**, a point p ϵ ct **M** - {a,b} is said to be a separating point between a and b if for same separation Ap/Bp of **M** – {p}.

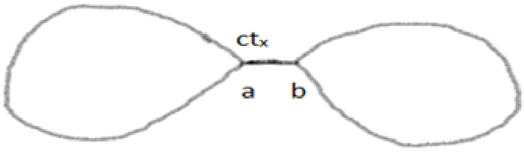
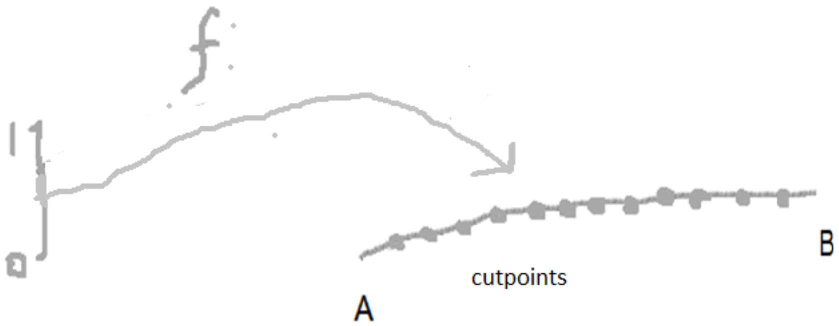


Figure. 1

- S(a,b) is used to denote the set of all separating points between a and b
- S | a ,b | Adjoin the point a and b to S (a,b)

A space **M** is called a space with end points if there exists a and b in **M** such that **M** =s| a ,b |

Theorem 5.6 .[4]
Let **M** is a connected space with end points then **M** has exactly two non cut points and every cut point of **M** is strong cut point.
Now redefine the path by using a cut point. The path in a connected space is the set of all cut points in **M** such that each preceding points is adjoin with next point which forms a continuous limits points of space.



Curves in R^n which is path with continous cutpoint

Curves in R^2 which is path with continuous cut points.

Lemma 5.7
Let **M** be a topological space which is connected and p, q є **M**, then
1) If p, q are the non cut points of **M** then there exacts a path between p and q
2) If pє **M** a set of cut points of **M** (from p to q) then there exists at least one path which covers cut points

Proof
Let **M** be a topological space which is connected by lemma 5.14 shows that there exits exactly two non cut points of **M**. by definition then exits a path between p and q ct p forms an open sub set of **M** which is connected then there is a path connected space. Therefore any two points p and q of **M** which are non cut points. **M** exits a path between them. Any point except cut point are adjoin with at least two neighborhood points which are open in **M** also in path this implies each path is connected which forms a dense in **M** which implies **M** is closed

$\Rightarrow$ **M** is closed
$\Rightarrow$ **M** is connected

Dr Haloli H G[2] introduced that, there exits path between two end points in **M** which are non cut points.That path passing through all cut points in **M**.

Theorem: 5.8
Let **M** is a strongly connected space with end points them there exists a path between them.
Proof
As **M** be a strongly connected space which contains p,q which are end points we have by lemma 5.4 every strongly connected space is connected and by Definition of strongly connected space show that there exist a path between p and q in **M** .

Definition 5.9 weakly connected spaces
A topological space **M** is said to be weekly connected if **M** exits a cut points and its neighborhood of **M**
Weekly connected topology is locally defined but strongly connected property is globally**.**

Theorem 5.10 [5]
Let **M** be a connected space than following are equivalent
i) **M** has exactly two non cut points.
ii) There exists a path between two non cut points in **M.**
iii) If **M** is weakly connected than each path contains at least are cut points.
iv)

Lemma 5.11

Let **M** be a path connected space. A point p ϵ **M** is a punctured point of space. Then there exists a path from any two points on **M** -{p}.

Theorem 5.12

Let **M** be a strongly connected space. Let P(p)
Be a punctured point (or space) in space **M**, Then
1) There exists a path between any two points in **M**
2) The space **M** becomes non converse .Space and connected Let **M** is strongly connected space P(p) be a punctured point(space)

Proof:-

By theorem 5.10 and lemma 5.7

There exists a path between any two point in **M**.

As the statement (1) if any point becomes punctured in space **M** Then there exist a path from any two point . Even punctured or non converse . Which is also continous.

Let p ϵ **M** be a punctured point of **M** i.e. **M** -{p} is a space without p in **M** i.e hole at **M** i.e. **M** is open convex also connected.

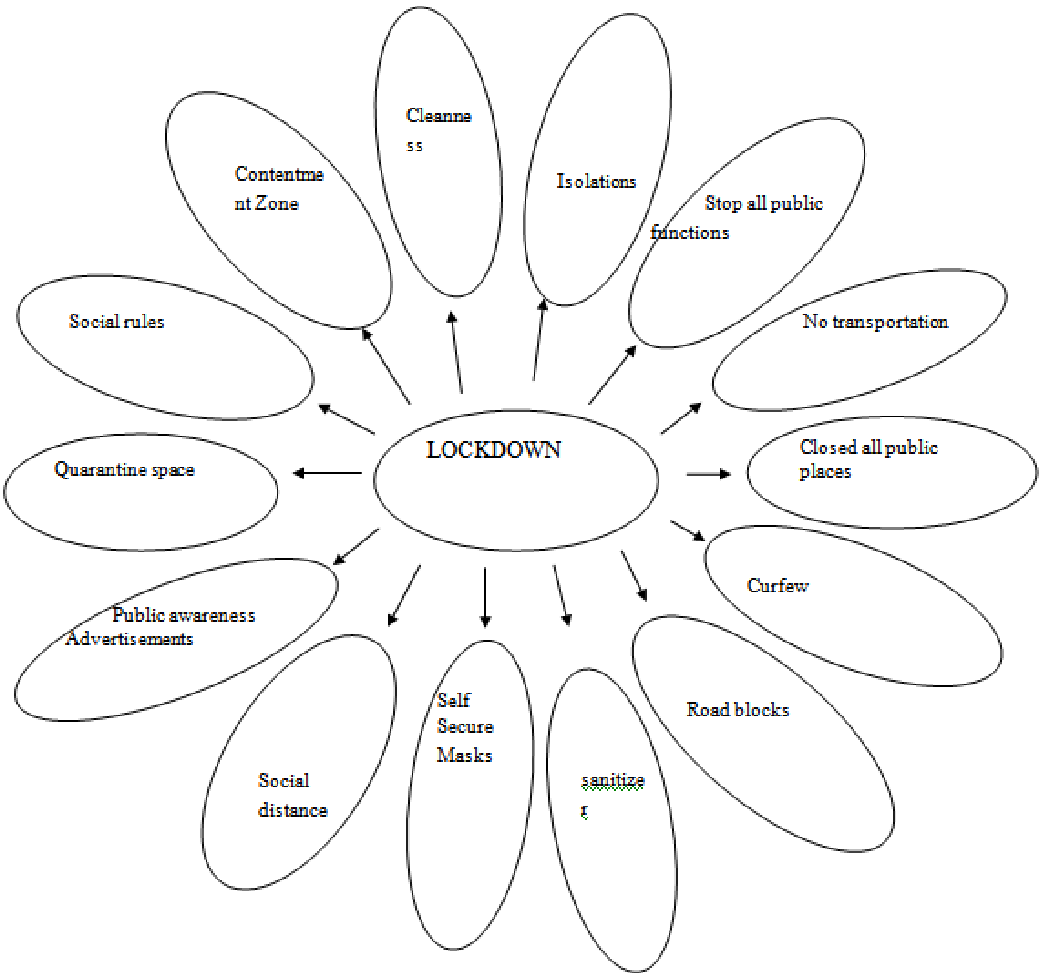


Fig. Different ways of lock down to control corona virus spread rate .

Comparative role of mathematical models
Lock down:- compactness , boundedness
Quarantine:-closed, boundedness, compactness
Social distance:-cut points
Curfew:- cut point punctured space , compactness, boundedness
Isolations :-boundedness, punctured spaces
Contentment zone:- Disconnected space

Stop of all public functions:- fail to homorphic functions
Close of all public places:-cut paths, punctured points
Road blocks:- cut paths
No transportation:- cut paths
Social rules:-connectedness, compactness, boundedness, cut point, punctured points
Self secure:-homomorphic functions, local charts
Public awareness advertisements:- Dirichlets conditions
Cleanness:-functions

VI. Conclusion :-

Real world is a Topological manifold, which satisfies the topological properties like connectedness, compactness, cut point and punctured points. These properties has vital role in the real world Manifold to control rate of spread of corona virus. These are not only properties but also they are models. Whole world is suffering from corona virus and to control this, we have to follows rules of lock down. These rules are the topological properties.

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