IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 16, Issue 5 Ser. I (Sep.– Oct. 2020), PP 01-11
www.iosrjournals.org

Inversion Formula For Generalised Integral Transform Of Several Variables.

PRASANTH, P* and VASUDEVAN NAMBISAN , T M**
*[*Asst. Professor, Govt. College, Kasaragod, Kerala. **Professor (Rtd), N.A.S.College, Kanhangad, Kerala]*

In this paper an inversion formula for integral transform of I- function of several complex variables has been established. Certain special cases are also given.

***Key words**: I- function, Mellin transform, Mellin-Barnes contour integral.*

Date of Submission: 29-08-2020 Date of Acceptance: 14-09-2020

I. Introduction

The I- function of r- variables defined and represented by Nambisan [9] et.al as:

$$\bar{I}^{0,n;(m_1,n_1);\dots\dots;(m_r,n_r)}_{P,Q:(p_1,q_1);\dots\dots;(p_r,q_r)}\left[\begin{matrix} z_1 \\ \cdot \\ \cdot \\ z_r \end{matrix}\left|\begin{matrix} (a_j,\alpha_j^{(1)},\dots,\alpha_j^{(r)},A_j)_{1,P}\,(c_j^{(1)},\gamma_j^{(1)},C_j^{(1)})_{1,p_1},\dots,(c_j^{(r)},\gamma_j^{(r)},C_j^{(r)})_{1,p_r} \\ \\ \\ (b_j,\beta_j^{(1)},\dots,\beta_j^{(r)},B_j)_{1,Q}:(d_j^{(1)},\delta_j^{(1)},D_j^{(1)})_{1,q_1};\dots,(d_j^{(r)},\delta_j^{(r)},D_j^{(r)})_{1,q_r} \end{matrix}\right.\right]$$

$$= \frac{1}{(2\pi i)^r}\int_{c_1}\dots\int_{c_r}\phi(s_1,\dots,s_r)\theta_1(s_1)\dots\theta_r(s_r)z_1^{s_1}\dots z_r^{s_r}\,ds_1\dots ds_r$$

(1.1)

Where

$$\phi(s_1\dots,s_r) = \frac{\prod_{j=1}^{n}\Gamma^{A_j}(1-a_j+\sum_{i=1}^{r}\alpha_j^{(i)}s_i)}{\prod_{j=n+1}^{P}\Gamma^{A_j}(a_j-\sum_{j=1}^{r}\alpha_j^{(i)}s_i)\prod_{j=1}^{Q}\Gamma^{B_j}(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}s_i)},\quad \text{i=1,2,....,r}$$

(1.2)

$$\theta_i(s_i) = \frac{\prod_{j=1}^{n_i}\Gamma^{C_j^{(i)}}(1-c_j^{(i)}+\gamma_j^{(i)}s_i)\prod_{j=1}^{m_i}\Gamma^{D_j^{(i)}}(d_j^{(i)}-\delta_j^{(i)}s_i)}{\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}\left(c_j^{(k)}-\gamma_j^{(i)}s_i\right)\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}s_i\right)}$$

i = 1,2,....,r

(1.3)

Also $z_i \neq 0$, i=1,2,....,r, an empty product is interpreted as unity.

The parameters $m_j, n_j, p_j, q_j, (j=1,\dots, r), n, p, q$ are non –negative integers such that $0 \le n \le p, q \ge 0, 0 \le n_j \le p_j, 0 \le m_j \le q_j, (j=1,2,\dots, r)$,not all zero simultaneously.

$\alpha_j^{(i)},(j=1,\dots,P,i=1,\dots,r)$, $\beta_j^{(i)},(j=1,\dots,Q,i=1,\dots,r)$, $\gamma_j^{(i)},(j=1,\dots,p_i,i=1,\dots,r)$, $\delta_j^{(i)},(j=1,\dots,q_i,i=1,\dots,r)$ are assumed to be positive quantities.
$a_j,(j=1,2,\dots,p)$, $b_j,(j=1,2,\dots,q)$, $c_j^{(i)},(j=1,\dots,p_i,i=1,\dots,r)$, $d_j^{(i)},(j=1,\dots,q_i,i=1,\dots,r)$ are complex numbers. All the singularities of $\Gamma^{D_j^{(i)}}(d_j^{(i)}-\delta_j^{(i)}s_i), j=1,\dots,m_i$ lie to the right and $\Gamma^{C_j^{(i)}}(1-c_j^{(i)}+\gamma_j^{(i)}s_i), j=1,\dots,n_i$ are to the left of c_i
The I- function of r- variables is analytic by Braaksma, [1],
if

$$\psi_k=\sum_{j=1}^{p}A_j\alpha_j^{(k)}-\sum_{j=1}^{q}B_j\beta_j^{(k)}+\sum_{j=1}^{p_k}C_j^{(k)}\gamma_j^{(k)}-\sum_{j=1}^{q_k}D_j^{(k)}\delta_j^{(k)}\le 0, k=1,2,\dots,r.$$

(1.4)
The integral (1.3) converges absolutely if

$\left|\arg z_k\right|<\frac{1}{2}\Delta_k\pi, k=1,\dots,r$, where

$$\Delta_k=-\sum_{j=n+1}^{p}A_j\alpha_j^{(k)}-\sum_{j=1}^{q}B_j\beta_j^{(k)}+\sum_{j=1}^{m_k}D_j^{(k)}\delta_j^{(k)}-\sum_{j=m_k+1}^{q_k}D_j^{(k)}\delta_j^{(k)}+\sum_{j=1}^{n_k}\gamma_j^{(k)}-\sum_{j=n_k+1}^{p_k}C_j^{(k)}\gamma_j^{(k)}>0.$$

and if

$$\left|\arg z_k\right|<\frac{1}{2}\Delta_k\pi, \Delta_k\ge 0, k=1,\dots,r,$$

then integral (1.1) converges absolutely under the following conditions.
i) $\psi_k=0,\Omega_k<-1$, where ψ_k given by (1.4) and for k= 1,2,…,r

$$\Omega_k=\sum_{j=1}^{p}[\tfrac{1}{2}-\mathrm{Re}(a_j)]A_j-\sum_{j=1}^{q}[\tfrac{1}{2}-\mathrm{Re}(b_j)]B_j+\sum_{j=1}^{p_k}[\tfrac{1}{2}-\mathrm{Re}(c_j^{(k)})]C_j^{(k)}-\sum_{j=1}^{q_k}[\tfrac{1}{2}-\mathrm{Re}(d_j^{(k)})]D_j^{(k)}$$

$\psi_k\ne 0$, with $s_k=\sigma_k+it_k,(\sigma_k,t_k)$ are real numbers, (k=1, 2,…, r) are chosen that $\left|t_k\right|\to\infty$, $\Omega_k+\sigma_k\psi_k<-1$.

If

$C_j^{(i)}=1,(j=1,\dots,n_i),i=1,\dots,r)$, $D_j^{(i)}=1,(j=1,\dots,m_i),i=1,\dots,r)$ and n = 0, in (1.3), the corresponding function will be denoted by

$$\bar{I}_1[z_1\dots z_r]=I_{P,Q:(p_1,q_1);\dots\dots;(p_r,q_r)}^{0,0;(m_1,n_1);\dots\dots;(m_r,n_r)}\left[\begin{matrix}z_1\\ \cdot\\ \cdot\\ z_r\end{matrix}\left|\begin{matrix}(a_j,\alpha_j^{(1)},\dots,\alpha_j^{(r)},A_j)_{1,P}(c_j^{(1)},\gamma_j^{(1)},1)_{1,n_1}(c_j^{(1)},\gamma_j^{(1)},C_j^{(1)})_{n_1+1,,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)},1)_{1,n_r}(c_j^{(r)},\gamma_j^{(r)},C_j^{(r)})_{n_r+1,p_r}\\ \\ \\ (b_j,\beta_j^{(1)},\dots,\beta_j^{(r)},B_j)_{1,Q}:(d_j^{(1)},\delta_j^{(1)},1)_{1,m_1}(d_j^{(1)},\delta_j^{(1)},D_j^{(1)})_{m_1+1,,q_1};\dots;(d_j^{(r)},\delta_j^{(r)},1)_{1,m_r}(d_j^{(r)},\delta_j^{(r)},D_j^{(r)})_{m_r+1,,q_r}\end{matrix}\right.\right]$$

$$= \frac{1}{(2\pi i)^r} \int_{c_1} \ldots \int_{c_r} \phi(s_1, \ldots, s_r) \bar{\theta}_1(s_1) \ldots \bar{\theta}_r(s_r) z_1^{s_1} \ldots z_r^{s_r} ds_1 \ldots ds_r$$

(1.5)

Where $\phi(s_1 ..., s_r) = \dfrac{1}{\prod_{j=1}^{P} \Gamma^{A_j}(a_j - \sum_{j=1}^{r} \alpha_j^{(i)} s_i) \prod_{j=1}^{Q} \Gamma^{B_j}(1 - b_j + \sum_{i=1}^{r} \beta_j^{(i)} s_i)}$

(1.6)

$$\bar{\theta}_i(s_i) = \frac{\prod_{j=1}^{n_i} \Gamma(1 - c_j^{(i)} + \gamma_j^{(i)} s_i) \prod_{j=1}^{m_i} \Gamma(d_j^{(i)} - \delta_j^{(i)} s_i)}{\prod_{j=n_i+1}^{p_i} \Gamma^{C_j^{(i)}}\left(c_j^{(k)} - \gamma_j^{(i)} s_i\right) \prod_{j=m_i+1}^{q_i} \Gamma^{D_j^{(i)}}\left(1 - d_j^{(i)} + \delta_j^{(i)} s_i\right)}$$

(1.7)

The integral (1.5) converges absolutely if $|\arg z_k| < \frac{1}{2} \Delta_k' \pi, k = 1, \ldots, r$, where

$$\Delta_k' = -\sum_{j=n+1}^{p} A_j \alpha_j^{(k)} - \sum_{j=1}^{q} B_j \beta_j^{(k)} + \sum_{j=1}^{m_k} \delta_j^{(k)} - \sum_{j=m_k+1}^{q_k} D_j^{(k)} \delta_j^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \sum_{j=n_k+1}^{p_k} C_j^{(k)} \gamma_j^{(k)} > 0 .$$

$$|\arg z_k| < \frac{1}{2} \Delta_k \pi, \Delta_k \geq 0, k = 1, \ldots, r$$,

then integral (1.5) converges absolutely under the following conditions.

i) $\psi_k' = 0, \Omega_k' < -1$, where for k= 1,….,r

$$\psi_k' = \sum_{j=1}^{p} A_j \alpha_j^{(k)} - \sum_{j=1}^{q} B_j \beta_j^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} + \sum_{j=n_k+1}^{p_k} C_j^{(k)} \gamma_j^{(k)} - \sum_{j=1}^{m_k} \delta_j^{(k)} - \sum_{j=m_i+1}^{q_k} D_j^{(k)} \delta_j^{(k)} \leq 0,$$

and

$$\Omega_k' = \sum_{j=1}^{p} [\tfrac{1}{2} - \mathrm{Re}(a_j)] A_j - \sum_{j=1}^{q} [\tfrac{1}{2} - \mathrm{Re}(b_j)] B_j + \sum_{j=1}^{n_k} \gamma_j^{(k)} + \sum_{j=n_k+1}^{p_k} C_j^{(k)} \gamma_j^{(k)} - \sum_{j=1}^{m_k} [\tfrac{1}{2} - \mathrm{Re}(d_j^{(k)})] - \sum_{j=m_k+1}^{q_k} [\tfrac{1}{2} - \mathrm{Re}(d_j^{(k)})],$$

$\psi_k' \neq 0$, with $s_k = \sigma_k + it_k, (\sigma_k, t_k)$ are real numbers, k=1,2,…,r are chosen that $|t_k| \to \infty$,

$\Omega_k' + \sigma_k \psi_k' < -1$

PRATHIMA & NAMBISAN , [11, p 25-30].

$$\int_0^\infty \ldots \int_0^\infty x_1^{\rho_1 - 1} \ldots x_r^{\rho_r - 1} \bar{I}_1[s_1 x_1^{\lambda_1}, \ldots, s_r x_r^{\lambda_r}] \times \bar{I}_1'[t_1 x_1^{\mu_1}, \ldots, t_r x_r^{\mu_r}\} dx_1 \ldots dx_r$$

$$= \frac{1}{\mu_1 \ldots \mu_r} t_1^{\frac{-\rho_1}{\mu_1}} \ldots t_r^{\frac{-\rho_r}{\mu_r}}$$

$$I^{0,0:m_1+n_1',n_1+m_1';\ldots\ldots\ldots;m_r+n_r',n_r+m_r'}_{p+q',q+p':p_1+q_1',q_1+p_1';\ldots;p_r+q_r',q_r+p_r'} \left[\begin{matrix} s_1 t_1^{-\frac{\lambda_1}{\mu_1}} \\ \cdot \\ s_r t_r^{-\frac{\lambda_r}{\mu}} \end{matrix} \middle| \begin{matrix} C : C_1;\ldots;C_r \\ \\ D : D_1;\ldots;D_r \end{matrix} \right] \quad (1.8)$$

Where

$$C = (a_j, \alpha_j^{(1)}, \ldots, \alpha_j^{(r)}; A_j)_{1,p} (1 - b_j' - \frac{\rho_1}{\mu_1} \beta_j'^{(1)} - \ldots - \frac{\rho_r}{\mu_r} \beta_j'^{(r)}, \frac{\lambda_1}{\mu_1} \beta_j'^{(1)}, \ldots, \frac{\lambda_r}{\mu_r} \beta_j'^{(r)}, B_j')_{1,q'}$$

$$C_i = (c_j^{(i)}, \gamma_j^{(i)}, 1)_{1,n_i} (1 - d_j'^{(i)} - \frac{\rho_i}{\mu_i} \delta_j'^{(i)}, \frac{\lambda_i}{\mu_i} \delta_j'^{(i)}, 1)_{1,m_i'},$$

$$(c_j^{(i)}, \gamma_j^{(i)}, C_j^{(i)})_{n_i+1,p_i} (1 - d_j'^{(1)} - \frac{\rho_i}{\mu_i} \delta_j'^{(1)}, \frac{\lambda_i}{\mu_i} \delta_j'^{(1)}, D_j'^{(i)})_{m_1+1,q_{1i}'}$$

$$D = (b_j, \beta_j^{(1)}, \ldots, \beta_j^{(r)}; B_j)_{1,q} (1 - a_j' - \frac{\rho_1}{\mu_1} \alpha_j'^{(1)} - \ldots - \frac{\rho_r}{\mu_r} \alpha_j'^{(r)}, \frac{\lambda_1}{\mu_1} \alpha_j'^{(1)}, \ldots, \frac{\lambda_r}{\mu_r} \alpha_j'^{(r)}, A_j')_{1,p'}$$

$$D_i \equiv (d_j^{(i)}, \delta_j^{(i)}, 1)_{1,m_i} (1 - c_j'^{(i)} - \frac{\rho_i}{\mu_i} \gamma_j'^{(i)}, \frac{\lambda_i}{\mu_i} \gamma_j'^{(i)}, 1)_{1,n_i'},$$

$$(d_j^{(i)}, \delta_j^{(i)}, D_j^{(i)})_{m_i+1,q_i} (1 - c_j'^{(1)} - \frac{\rho_i}{\mu_i} \gamma_j'^{(i)}, \frac{\lambda_i}{\mu_i} \gamma_j'^{(i)}, D_j'^{(i)})_{m_1+1,q_{1i}'}$$

j

The integral (1.8) is valid under the following set of conditions.

$\lambda_i > 0, \mu_i > 0, i = 1, \ldots, r$

$$-\lambda_i \min_{1 \le j \le m_i} \mathrm{Re}(\frac{d_j^{(i)}}{\delta_j^{(i)}}) - \mu_i \min_{1 \le j \le m_i'} \mathrm{Re}(\frac{d_j'^{(i)}}{\delta_j'^{(i)}}) < \mathrm{Re}(\rho_i) < \lambda_i \min_{1 \le j \le n_i} \mathrm{Re}(\frac{1 - c_j^{(i)}}{\gamma_j^{(i)}}) + \mu_i \min_{1 \le j \le n_i'} (\frac{1 - c_j'^{(i)}}{\gamma_j'^{(i)}}), i = 1, \ldots, r$$

,

$$\psi_{1i}'' \le 0, \Delta_{1i}'' > 0, |\arg s_i| < \frac{1}{2} \Delta_{1i}'' \pi, \psi_{2i}'' \le 0, \Delta_{2i}'' > 0, |\arg t_i| < \frac{1}{2} \Delta_{2i}'' \pi$$

Where

$$\psi_{1i}'' = \sum_{j=1}^{p} A_j \alpha_j^{(i)} - \sum_{j=1}^{q} B_j \beta_j^{(i)} + \sum_{j=1}^{n_k} \gamma_j^{(i)} + \sum_{j=n_k+1}^{p_k} C_j^{(i)} \gamma_j^{(i)} - \sum_{j=1}^{m_k} \delta_j^{(i)} - \sum_{j=m_i+1}^{q_k} D_j^{(i)} \delta_j^{(i)}$$

$$\Delta_{1i}'' = -\sum_{j=1}^{p} A_j \alpha_j^{(i)} + \sum_{j=1}^{n_i} \gamma_j^{(i)} + \sum_{j=1}^{m_k} \delta_j^{(i)} - \sum_{j=n_k+1}^{p_k} C_j^{(i)} \gamma_j^{(i)} - \sum_{j=1}^{q} B_j \beta_j^{(i)} - \sum_{j=m_i+1}^{q_i} D_j^{(i)} \delta_j^{(i)}$$

$$\psi_{2i}^{//} = \sum_{j=1}^{p^/} A_j^/ \alpha_j^{/(i)} - \sum_{j=1}^{q^/} B_j^/ \beta_j^{/(i)} + \sum_{j=1}^{n^/_k} \gamma_j^{/(i)} + \sum_{j=n_k^/+1}^{p_k^/} C_j^{/(i)} \gamma_j^{/(i)} - \sum_{j=1}^{m_k^/} \delta_j^{/(i)} - \sum_{j=m_i^/+1}^{q_k^/} D_j^{/(i)} \delta_j^{/(i)}$$

$$\Delta_{2i}^{//} = -\sum_{j=1}^{p^/} A_j^/ \alpha_j^{/(i)} + \sum_{j=1}^{n^/_i} \gamma_j^{/(i)} + \sum_{j=1}^{m_k^/} \delta_j^{/(i)} - \sum_{j=n_k+1}^{p_k^/} C_j^{/(i)} \gamma_j^{/(i)} - \sum_{j=1}^{q^/} B_j^/ \beta_j^{/(i)} - \sum_{j=m_i^/+1}^{q_i^/} D_j^{/(i)} \delta_j^{/(i)}$$

PRASANTH [7, P-176,177]

If

$$f(p_1,\ldots,p_r) = \int_0^\infty \ldots \int_0^\infty K[u_1(p_1x_1)^{\lambda_1},\ldots,(p_rx_r)^{\lambda_r}]\, f(x_1,\ldots,x_r)\,dx_1\ldots dx_r \quad , \tag{1.9}$$

then

$$f(x_1,\ldots,x_r) = \frac{1}{2\pi i}\int_{\sigma_1-i\infty}^{\sigma_1+\infty} \ldots \int_{\sigma_r-i\infty}^{\sigma_r+\infty} x_1^{-t_1}\ldots x_r^{-t_r}\, \frac{\psi_2(t_1,\ldots,t_r)}{\psi_1(t_1,\ldots,t_r)}\,dt_1\ldots dt_r \ . \tag{1.10}$$

Where

$$\psi_1(t_1,\ldots,t_r) = \int_0^\infty \ldots \int_0^\infty p_1^{-t_1}\ldots p_r^{-t_r} K[u_1 p_1^{\lambda_1},\ldots,u_r p_r^{\lambda_r}]\,dp_1\ldots dp_r \tag{1.11}$$

$$\psi_2(t_1,\ldots,t_r) = \int_0^\infty \ldots \int_0^\infty p_1^{-t_1}\ldots p_r^{-t_r} \phi(p_1,\ldots,p_r)\,dp_1\ldots dp_r \tag{1.12}$$

Provided,

i) $\lambda_1,\ldots,\lambda_r$ are positive real numbers

ii) $f(x_1,\ldots,x_r)$ is sectionally continuous for $x_1,\ldots,x_r > 0$

iii) The generalized integral transform (1.9) of $|f(x_1,\ldots,x_r)|$ exists,

iv) The multiple integrals involved in (1.11) and (1.12) are absolutely convergent.

REED [12, P, 56]

If

$$M[f(x_n)] = F(s_n) = n\int_0^\infty g(x_n) \prod_{k=1}^{n} \left\{x_k^{s_k-1} d(s_k)\right\} \tag{1.13}$$

, then

$$M^{-1}[F(s_n)] = g(x_n) = \frac{1}{(2\pi i)^n}\, n\int_{-i\infty}^{i\infty} F(s_n) \prod_{k=1}^{n} \left\{x_k^{-s_k} d(s_k)\right\} \tag{1.14}$$

II. Main Result

If

$$\phi(p_1,\dots,p_r)=\int_0^\infty\dots\int_0^\infty \bar{I}_1[u_1(p_1x_1)^{\lambda_1},\dots,u_r(p_rx_r)^{\lambda_r}]\bar{I}_1^{/}[v_1(p_1x_1)^{\mu_1},\dots,(v_r(p_rx_r)^{\mu_r}]f(x_1,\dots,x_r)dx_1\dots dx_r$$

(2.1)

Then

$$f(x_1,\dots,x_r)=\frac{1}{(2\pi i)^r}\int_{\sigma_1-i\infty}^{\sigma_1+\infty}\dots\int_{\sigma_r-i\infty}^{\sigma_r+\infty} x_1^{-t_1}\dots x_r^{-t_r}\ \frac{\psi_2(t_1,\dots,t_r)}{\psi_1(t_1,\dots,t_r)}dt_1\dots dt_r$$

(2.2)

Where

$$\psi_2(t_1,\dots,t_r)=\int_0^\infty\dots\int_0^\infty p_1^{-t_1}\dots p_r^{-t_r}\phi(p_1,\dots,p_r)dp_1\dots dp_r$$

(2.3)

$$\psi_1(t_1,\dots,t_r)=\frac{p_i^{\lambda_1}\dots p_r^{\lambda_r}}{\mu_1\dots\mu_r}v_1^{\frac{-(1-t_1)}{\mu_1}}\dots v_r^{\frac{-(1-t_r)}{\mu_r}}$$

$$I^{\,0,0:(m_1+n_1^/,n_1+m_1^/);\dots;(m_r+n_r^/,n_r+m_r^/)}_{\,p+q^/,q+p^/:(p_1+q_1^/,q_1+p_1^/);\dots;(p_r+q_r^/,q_r+p_r^/)}\left[\begin{array}{l|l} p_1^{\lambda_1}u_1v_1^{\frac{-\lambda_1}{\mu_1}} & C;C_1,\dots,C_r \\ . & \\ . & \\ p_r^{\lambda_r}u_rv_r^{-\frac{\lambda_r}{\mu_r}} & D:D_1,\dots,D_r\end{array}\right]$$

(2.4)

Where

$$C=(a_j,\alpha_j^{(1)},\dots,\alpha_j^{(r)};A_j)_{1,p}(1-b_j^/-\frac{(1-t_1)}{\mu_1}\beta_j^{/(1)}-\dots-\frac{(1-t_r)}{\mu_r}\beta_j^{/(r)},\frac{\lambda_1}{\mu_1}\beta_j^{/(1)},\dots,\frac{\lambda_r}{\mu_r}\beta_j^{/(r)},B_j^{\,/})_{1,q^/}$$

$$C_i=(c_j^{(i)},\gamma_j^{(i)},1)_{1,n_i}(1-d_j^{/(i)}-\frac{(1-t_i)}{\mu_i}\delta_j^{/(i)},\frac{\lambda_i}{\mu_i}\delta_j^{/(i)},1)_{1,m_i^/},$$

$$(c_j^{(i)},\gamma_j^{(i)},C_j^{(i)})_{n_i+1,p_i}(1-d_j^{/(i)}-\frac{(1-t_i)}{\mu_i}\delta_j^{/(i)},\frac{\lambda_i}{\mu_i}\delta_j^{/(i)},D_j^{/(i)})_{m_1+1,q_{1i}^/}$$

,i=1,2,...,r

$$D=(b_j,\beta_j^{(1)},\dots,\beta_j^{(r)};B_j)_{1,q}(1-a_j^/-\frac{(1-t_1)}{\mu_1}\alpha_j^{/(1)}-\dots-\frac{(1-t_r)}{\mu_r}\alpha_j^{/(r)},\frac{\lambda_1}{\mu_1}\alpha_j^{/(1)},\dots,\frac{\lambda_r}{\mu_r}\alpha_j^{/(r)},A_j^{\,/})_{1,p^/}$$

$$D_i\equiv(d_j^{(i)},\delta_j^{(i)},1)_{1,m_i}(1-c_j^{/(i)}-\frac{(1-t_i)}{\mu_i}\gamma_j^{/(i)},\frac{\lambda_i}{\mu_i}\gamma_j^{/(i)},1)_{1,n_i^/},$$

$$(d_j^{(i)},\delta_j^{(i)},D_j^{(i)})_{m_i+1,q_i}(1-c_j^{/(1)}-\frac{(1-t_i)}{\mu_i}\gamma_j^{/(i)},\frac{\lambda_i}{\mu_i}\gamma_j^{/(i)},D_j^{/(i)})_{m_1+1,q_{1i}^/}$$

i=1,2,...,r

Provided,

i) $\lambda_i, \mu_i, (i = 1, ..., r)$ are positive real numbers

ii) $f(x_1, ..., x_r)$ is sectionally continuous for $x_1, ..., x_r > 0$

iii) The generalized integral transform (2.1) of $|f(x_1, ..., x_r)|$ exists,

iv) The multiple integrals involved in (2.3) is absolutely convergent.

$$-\lambda_i \min_{1\le j\le m_i} \mathrm{Re}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right) - \mu_i \min_{1\le j\le m_i'} \mathrm{Re}\left(\frac{d_j^{/(i)}}{\delta_j^{/(i)}}\right) < \mathrm{Re}(1-t_i) < \lambda_i \min_{1\le j\le n_i} \mathrm{Re}\left(\frac{1-c_j^{(i)}}{\gamma_j^{(i)}}\right) + \mu_i \min_{1\le j\le n_i'}\left(\frac{1-c_j^{/(i)}}{\gamma_j^{/(i)}}\right), i = 1, \ldots, r$$

$$A_i \le 0, \Delta_i > 0, \left|\arg u_i p_i^{\lambda_i}\right| < \frac{1}{2}\Delta_i \pi, A_i' \le 0, \Delta_i' > 0, \left|\arg v_i p_i^{\mu_i}\right| < \frac{1}{2}\Delta_i' \pi, i = 1, \ldots, r$$

Where

$$A_i = \sum_{j=1}^{p} A_j \alpha_j^{(i)} - \sum_{j=m+1}^{q} B_j \beta_j^{(i)} + \sum_{j=1}^{p_i} C_j^{(i)} \gamma_j^{(i)} - \sum_{j=n+1}^{q_i} D_j^{(i)} \delta_j^{(i)}$$

$$\Delta_i = -\sum_{j=1}^{p} A_j \alpha_j^{(i)} + \sum_{j=1}^{n_i} C_j^{(i)} \gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i} C_j^{(i)} \gamma_j^{(i)} - \sum_{j=1}^{q} B_j \beta_j^{(i)} + \sum_{j=1}^{m_i} D_j^{(i)} \delta_j^{(i)} - \sum_{j=m_i+1}^{q_i} D_j^{(i)} \delta_j^{(i)}, i = 1, \ldots, r$$

$$A_i' = \sum_{j=1}^{p'} A_j' \alpha_j^{/(i)} - \sum_{j=m+1}^{q'} B_j' \beta_j^{/(i)} + \sum_{j=1}^{p_i'} C_j^{/(i)} \gamma_j^{/(i)} - \sum_{j=n+1}^{q_i'} D_j^{/(i)} \delta_j^{/(i)}$$

$$\Delta_i' = -\sum_{j=1}^{p'} A_j' \alpha_j^{(i)} + \sum_{j=1}^{n_i'} C_j^{/(i)} \gamma_j^{/(i)} - \sum_{j=n_i'+1}^{p_i'} C_j^{/(i)} \gamma_j^{/(i)} - \sum_{j=1}^{q'} B_j' \beta_j^{/(i)} + \sum_{j=1}^{m_i'} D_j^{/(i)} \delta_j^{/(i)} - \sum_{j=m_i'+1}^{q_i'} D_j^{/(i)} \delta_j^{/(i)}, i = 1, \ldots, r$$

PROOF:

From (2.1) and (2.3),

$$\psi_2(t_1, \ldots, t_r) = \int_0^\infty \ldots \int_0^\infty p_1^{-t_1} \ldots . p_r^{-t_r} \left\{ \int_0^\infty \ldots \int_0^\infty \bar{I}_1[u_1(p_1 x_1)^{\lambda_1}, \ldots, u_r(p_r x_r)^{\lambda_r}] \times \right.$$

$$\left. \bar{I}'_1[v_1(p_1 x_1)^{\mu_1}, ..., v_r(p_r x_r)^{\mu_r}] f(x_1, ..., x_r) dx_1 .... dx_r \right\} dp_1 .... dp_r$$

(2.5)

Changing the order of $(p_1, ..., p_r)$ integrals and $(x_1, ..., x_r)$ integrals [which is justified due to the absolute convergence of these integrals] and replacing $(p_i x_i)$ by ξ_i, (i =1, 2…, r) in (2.5), to get:

$$\psi_2(t_1, ..., t_r) = \int_0^\infty .... \int_0^\infty x_1^{t_1-1} \ldots x_r^{t_r-1} f(x_1, \ldots, x_r) \left\{ \int_0^\infty .... \int_0^\infty \xi_1^{-t_1} \ldots \xi_r^{-t_r} \bar{I}[u_1 \xi_1^{\lambda_1}, \ldots, u_r \xi_r^{\lambda_r}] \bar{I}'[v_1 \xi_1^{\mu_1}, \ldots, v_r \xi_r^{\mu_r}] d\xi_1 \ldots d\xi_r \right\} \times$$

$$dx_1, ..., dx_r . \quad (2.6).$$

On evaluating $(\xi_1, ..., \xi_r)$ integrals using (2.4), (2.6) becomes;

$$\psi_2(t_1, \ldots, t_r) = \psi_1(t_1, \ldots, t_r) \int_0^\infty \ldots \int_0^\infty x_1^{t_1-1} \ldots x_r^{t_r-1} f(x_1, \ldots, x_r) dx_1 \ldots dx_r .$$

Now using (1.2) for r- variables, the above result is completely established under the stated conditions.

SPECIAL CASES:

When r=2,

If

$$\phi(p_1, p_2) = \int_0^\infty . \int_0^\infty I[u_1(p_1x_1)^{\lambda_1}, u_2(p_2x_2)^{\lambda_2}] I^{/}[v_1(p_1x_1)^{\mu_1}, (v_2(p_2x_2)^{\mu_2}] f(x_1, .x_2) dx_1.dx_2$$

(2.7)

then

$$f(x_1, x_2) = \frac{1}{(2\pi i)^2} \int_{\sigma_1 - i\infty}^{\sigma_1 + \infty} \int_{\sigma_2 - i\infty}^{\sigma_2 + \infty} x_1^{-t_1} x_1^{-t_1} \frac{\psi_2(t_1, t_2)}{\psi_1(t_1, t_2)} dt_1 dt_2$$

(2.8)

where

$$\psi_2(t_1, t_2) = \int_0^\infty \int_0^\infty p_1^{-t_1} p_2^{-t_2} \phi(p_1, p_2) dp_1 dp_2$$

(2.9)

$$\psi_1(t_1, t_2) = \frac{p_i^{\lambda_1} p_2^{\lambda_2}}{\mu_1 \mu_2} v_1^{\frac{-(1-t_1)}{\mu_1}} v_2^{\frac{-(1-t_2)}{\mu_2}}$$

$$I^{0,0:(m_1+n_1^{/}, n_1+m_1^{/});(m_2+n_2^{/}, n_2+m_2^{/})}_{p+q^{/}, q+p^{/}:(p_1+q_1^{/}, q_1+p_1^{/});(p_2+q_2^{/}, q_2+p_2^{/})} \left[\begin{matrix} p_1^{\lambda_1} u_1 v_1^{\frac{-\lambda_1}{\mu_1}} \\ p_2^{\lambda_2} u_2 v_2^{-\frac{\lambda_r}{\mu_r}} \end{matrix} \middle| \begin{matrix} C; C_1, C_2 \\ D : D_1, D_2 \end{matrix} \right]$$

(2.10)
Where

$$C = (a_j, \alpha_j, A_j \xi_j)_{1,p} (1 - b_j^{/} - \frac{(1-t_1)}{\mu_1} \beta_j^{//} - \frac{(1-t_2)}{\mu_2} B_j^{/}, \frac{\lambda_1}{\mu_1} \beta_j^{/}, \frac{\lambda_2}{\mu_2} B_j^{/}, \eta_j^{\ /})_{1,q^{/}}$$

$$C_1 = (c_j, C_j, 1)_{1,n_1} (1 - d_j^{/} - \frac{(1-t_1)}{\mu_{i1}} D_j^{/}, \frac{\lambda_1}{\mu_1} D_j^{/}, 1)_{1,m_1^{/}}$$

$$(c_j, C_j, U_j)_{n_i+1, p_i} (1 - d_j^{/} - \frac{(1-t_1)}{\mu_1} D_j^{/}, \frac{\lambda_1}{\mu_1} D_j^{/}, V_j^{/})_{m_1+1, q_{11}^{/}}$$

$$D = (b_j, \beta_j, B_j \eta_j)_{1,q} (1 - a_j^{/} - \frac{(1-t_1)}{\mu_1} \alpha_j^{\ /} - \frac{(1-t_2)}{\mu_2} A_j^{/}, \frac{\lambda_1}{\mu_1} \alpha_j^{/}, \frac{\lambda_2}{\mu_2} A_j^{/}, \eta_j^{\ /})_{1,p^{/}}$$

$$D_1 \equiv (d_j, D_j, 1)_{1,m_1} (1 - c_j^{/} - \frac{(1-t_1)}{\mu_1} C_j^{/}, \frac{\lambda_1}{\mu_1} C_j^{/}, 1)_{1,n_1^{/}},$$

$$(d_j, D_j, V_j)_{m_1+1, q_1} (1 - c_j^{/} - \frac{(1-t_1)}{\mu_1} C_j^{/}, \frac{\lambda_1}{\mu_1} C_j^{/}, V_j^{/})_{m_1+1, q_1^{/}}$$

$$C_2 = (c_j, C_j, 1)_{1,n_2} (1 - d_j^{/} - \frac{(1-t_2)}{\mu_2} D_j^{/}, \frac{\lambda_1}{\mu_1} D_j^{/}, 1)_{1,m_2^{/}}$$

$$(c_j, C_j, U_j)_{n_{i2}+1, p_i} (1 - d_j' - \frac{(1-t_2)}{\mu_2} D_j', \frac{\lambda_1}{\mu_1} D_j', V_j')_{m_2+1, q_{21}'}$$

$$D_2 = (d_j, D_j, 1)_{1, m_2} (1 - c_j' - \frac{(1-t_2)}{\mu_2} C_j', \frac{\lambda_1}{\mu_1} C_j', 1)_{1, n_2'},$$

$$(d_j, D_j, V_j)_{m_2+1, q_2} (1 - c_j' - \frac{(1-t_2)}{\mu_2} C_j', \frac{\lambda_2}{\mu_2} C_j', V_j')_{m_2+1, q_2'}$$

Provided,

i) $\lambda_1 and \lambda_2$ are positive real numbers

ii) $f(x_1, x_2)$ is sectionally continuous for $x_1, x_2 > 0$,

iii) The generalized integral transform (2.1) of $|f(x_1, x_2)|$ exists,

iv) The multiple integrals involved in (2.3) is absolutely convergent.

$$-\lambda_1 \min_{1 \le j \le m_1} \mathrm{Re}(\frac{d_j}{D_j}) - \mu_1 \min_{1 \le j \le m_1'} \mathrm{Re}(\frac{d_j'}{D_j'}) < \mathrm{Re}(1 - t_1) < \lambda_1 \min_{1 \le j \le n_1} \mathrm{Re}(\frac{1 - c_j}{C_j}) + \mu_1 \min_{1 \le j \le n_1'} (\frac{1 - c_j'}{C_j'}),$$

$$-\lambda_2 \min_{1 \le j \le m_2} \mathrm{Re}(\frac{d_j}{D_j}) - \mu_2 \min_{1 \le j \le m_2'} \mathrm{Re}(\frac{d_j'}{D_j'}) < \mathrm{Re}(1 - t_2) < \lambda_2 \min_{1 \le j \le n_2} \mathrm{Re}(\frac{1 - c_j}{C_j}) + \mu_2 \min_{1 \le j \le n_2'} (\frac{1 - c_j'}{C_j'}),$$

$$A_i \le 0, \Delta_i > 0, \left|\arg u_i p_i^{\lambda_i}\right| < \frac{1}{2} \Delta_i \pi, A_i' \le 0, \Delta_i' > 0, \left|\arg v_i p_i^{\mu_i}\right| < \frac{1}{2} \Delta_i' \pi, i = 1,2$$

Where

$$A_1 = \sum_{j=1}^{p} \alpha_j - \sum_{j=m+1}^{q} \beta_j + \sum_{j=1}^{p_1} C_j - \sum_{j=n+1}^{q_1} D_j, \quad A_2 = \sum_{j=1}^{p} \alpha_j - \sum_{j=m+1}^{q} \beta_j + \sum_{j=1}^{p_2} C_j - \sum_{j=n+1}^{q_2} D_j$$

$$\Delta_1 = -\sum_{j=1}^{p} \alpha_j + \sum_{j=1}^{n_1} C_j - \sum_{j=n_1+1}^{p_1} C_j - \sum_{j=1}^{q} \beta_j + \sum_{j=1}^{m_1} D_j - \sum_{j=m_1+1}^{q_1} D_j$$

$$\Delta_2 = -\sum_{j=1}^{p} \alpha_j + \sum_{j=1}^{n_1} C_j - \sum_{j=n_2+1}^{p_2} C_j - \sum_{j=1}^{q} \beta_j + \sum_{j=1}^{m_2} D_j - \sum_{j=n_2+1}^{q_2} D_j,$$

$$A_1' = \sum_{j=1}^{p'} \alpha_j' - \sum_{j=m+1}^{q'} \beta_j' + \sum_{j=1}^{p_1'} C_j' - \sum_{j=n+1}^{q_1'} D_j' \quad A_2' = \sum_{j=1}^{p'} \alpha_j' - \sum_{j=m+1}^{q'} \beta_j' + \sum_{j=1}^{p_2'} C_j' - \sum_{j=n+1}^{q_2'} D_j'$$

$$\Delta_1' = -\sum_{j=1}^{p'} \alpha_j' + \sum_{j=1}^{n_i'} C_j' - \sum_{j=n_1'+1}^{p_1'} C_j' - \sum_{j=1}^{q'} \beta_j' + \sum_{j=1}^{m_1'} D_j'^{(i)} - \sum_{j=m_1+1}^{q_1'} D_j'$$

$$\Delta_2' = -\sum_{j=1}^{p'} \alpha_j' + \sum_{j=1}^{n_i'} C_j' - \sum_{j=n_1'+1}^{p_1'} C_j' - \sum_{j=1}^{q'} \beta_j' + \sum_{j=1}^{m_2'} D_j'^{(i)} - \sum_{j=m_2+1}^{q_2'} D_j'$$

III. Main Result-2

If

$$\phi(p_1,...,p_r)=\int_0^\infty...\int_0^\infty \bar{I}_1[u_1(\frac{x_1}{p_1})^{\lambda_1},....,u_r(\frac{x_r}{p_r})^{\lambda_r}]\bar{I}_1^{/}[v_1(\frac{x_1}{p_1})^{\mu_1},....,(v_r(\frac{x_r}{p_r})^{\mu_r}]f(x_1,.....,x_r)dx_1.....dx_r$$

(2.11)

Then

$$f(x_1,...,x_r)=\frac{1}{(2\pi i)^r}\int_{\sigma_1-i\infty}^{\sigma_1+\infty}....\int_{\sigma_r-i\infty}^{\sigma_r+\infty}x_1^{-t_1}...x_r^{-t_r}\frac{\psi_2(t_1,...,t_r)}{\psi_1(t_1,....,t_r)}dt_1.....dt_r$$

(2.12)

Where

$$\psi_2(t_1,...,t_r)=\int_0^\infty...\int_0^\infty p_1^{t_1-2}....p_r^{t_r-2}\phi(p_1,....,p_r)dp_1....dp_r$$

(2.13)

$$\psi_1(t_1,...,t_r)=\frac{p_i^{-\lambda_1}....p_r^{-\lambda_r}}{\mu_1...\mu_r}v_1^{\frac{(1-t_1)}{\mu_1}}...v_r^{\frac{(1-t_r)}{\mu_r}}$$

$$I^{0,0:(m_1+m_1^{/},n_1+n_1^{/});..........;(m_r+m_r^{/},n_r+n_r^{/})}_{p+p^{/},q+q^{/}:(p_1+p_1^{/},q_1+q_1^{/});.....;(p_r+p_r^{/},q_r+q_r^{/})}\left[\begin{array}{c|c} p_1^{-\lambda_1}u_1v_1^{\frac{\lambda_1}{\mu_1}} & C;C_1,....,C_r \\ . & \\ . & \\ p_r^{-\lambda_r}u_rv_r^{\frac{\lambda_r}{\mu_r}} & D:D_1,...,D_r \end{array}\right]$$

(2.14)

Where

$$C=(a_j,\alpha_j^{(1)},...,\alpha_j^{(r)};A_j)_{1,p}(a_j^{/}+\frac{(t_1-1)}{\mu_1}\alpha_j^{/(1)}+....+\frac{(t_r-1)}{\mu_r}\alpha_j^{/(r)},\frac{\lambda_1}{\mu_1}\alpha_j^{/(1)},....,\frac{\lambda_r}{\mu_r}\alpha_j^{/(r)},A_j^{/})_{1,p^{/}}$$

$$C_i=(c_j^{(i)},\gamma_j^{(i)},1)_{1,n_i}(c_j^{/(i)}+\frac{(t_i+1)}{\mu_i}\gamma_j^{/(i)},\frac{\lambda_i}{\mu_i}\gamma_j^{/(i)},1)_{1,p_i^{/}},$$

$$(c_j^{(i)},\gamma_j^{(i)},C_j^{(i)})_{n_i+1,p_i}(c_j^{/(1)}+\frac{(t_i+1)}{\mu_i}\gamma_j^{/(1)},\frac{\lambda_i}{\mu_i}\gamma_j^{/(1)},C_j^{/(i)})_{n_1+1,p_{1i}^{/}}$$

,i=1,2,...,r

$$D=(b_j,\beta_j^{(1)},...,\beta_j^{(r)};B_j)_{1,q}(b_j^{/}+\frac{(t_1-1)}{\mu_1}\beta_j^{/(1)}+....+\frac{(t_r-1)}{\mu_r}\beta_j^{/(r)},\frac{\lambda_1}{\mu_1}\beta_j^{/(1)},....,\frac{\lambda_r}{\mu_r}\beta_j^{/(r)},B_j^{/})_{1,q^{/}}$$

$$D_i\equiv(d_j^{(i)},\delta_j^{(i)},1)_{1,m_i}(d_j^{/(i)}+\frac{(t_i-1)}{\mu_i}\delta_j^{/(i)},\frac{\lambda_i}{\mu_i}\delta_j^{/(i)},1)_{1,m_i^{/}},$$

$$(d_j^{(i)},\delta_j^{(i)},D_j^{(i)})_{m_i+1,q_i}(d_j^{/(1)}+\frac{(t_i-1)}{\mu_i}\delta_j^{/(i)},\frac{\lambda_i}{\mu_i}\delta_j^{/(i)},D_j^{/(i)})_{m_1+1,q_{1i}^{/}}$$

i=1,2,...,r

Provided,

i) $\lambda_i, \mu_i, (i = 1,2,.., r)$ are positive real numbers

ii) $f(x_1,...., x_r)$ is sectionally continuous for $x_1,...., x_r > 0$,

iii) The generalized integral transform (2.11) of $|f(x_1,....,x_r)|$ exists,

iv) The multiple integrals involved in (2.13) is absolutely convergent.

$$-\lambda_i \min_{1\le j\le m_i} \mathrm{Re}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right) - \mu_i \min_{1\le j\le m_i^{/}} \mathrm{Re}\left(\frac{d_j^{/(i)}}{\delta_j^{/(i)}}\right) < \mathrm{Re}(1-t_i) < \lambda_i \min_{1\le j\le n_i} \mathrm{Re}\left(\frac{1-c_j^{(i)}}{\gamma_j^{(i)}}\right) + \mu_i \min_{1\le j\le n_i^{/}}\left(\frac{1-c_j^{/(i)}}{\gamma_j^{/(i)}}\right), i = 1,\ldots,r$$

$$A_i \le 0, \Delta_i > 0, \left|\arg\left(\frac{u_i}{p_i^{\lambda_i}}\right)\right| < \frac{1}{2}\Delta_i \pi, A_i^{/} \le 0, \Delta_i^{/} > 0, \left|\arg\left(\frac{v_i}{p_i^{\mu_i}}\right)\right| < \frac{1}{2}\Delta_i^{/}\pi, i = 1,\ldots,r$$

Where

$$A_i = \sum_{j=1}^{p} A_j \alpha_j^{(i)} - \sum_{j=m+1}^{q} B_j \beta_j^{(i)} + \sum_{j=1}^{p_i} C_j^{(i)} \gamma_j^{(i)} - \sum_{j=n+1}^{q_i} D_j^{(i)} \delta_j^{(i)}$$

$$\Delta_i = -\sum_{j=1}^{p} A_j \alpha_j^{(i)} + \sum_{j=1}^{n_i} C_j^{(i)} \gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i} C_j^{(i)} \gamma_j^{(i)} - \sum_{j=1}^{q} B_j \beta_j^{(i)} + \sum_{j=1}^{m_i} D_j^{(i)} \delta_j^{(i)} - \sum_{j=m_i+1}^{q_i} D_j^{(i)} \delta_j^{(i)}, i = 1,\ldots,r$$

$$A_i^{/} = \sum_{j=1}^{p^{/}} A_j^{/} \alpha_j^{/(i)} - \sum_{j=m+1}^{q^{/}} B_j^{/} \beta_j^{/(i)} + \sum_{j=1}^{p_i^{/}} C_j^{/(i)} \gamma_j^{/(i)} - \sum_{j=n+1}^{q_i^{/}} D_j^{/(i)} \delta_j^{/(i)}$$

$$\Delta_i^{/} = -\sum_{j=1}^{p^{/}} A_j^{/} \alpha_j^{(i)} + \sum_{j=1}^{n_i^{/}} C_j^{/(i)} \gamma_j^{/(i)} - \sum_{j=n_i^{/}+1}^{p_i^{/}} C_j^{/(i)} \gamma_j^{/(i)} - \sum_{j=1}^{q^{/}} B_j^{/} \beta_j^{/(i)} + \sum_{j=1}^{m_i^{/}} D_j^{/(i)} \delta_j^{/(i)} - \sum_{j=m_i^{/}+1}^{q_i^{/}} D_j^{/(i)} \delta_j^{/(i)}, i = 1,\ldots,r$$

Proof is similar to that of Result -1.

References:

[1]. Braaksma, B.L.J (1964): Asymptotic expansions and analytic continuations for a class of Barnes integrals, Compositio Math.15.239-341.

[2]. Erdelyi, (1954): Tables of integral transforms, I, II, Mc, Graw Hill, New York.

[3]. I.S Ansari, F. Yilmaz, M,S.Alouni and O. Kucur: New results on the sum of Gamma random variables with application to the performance of Wireless communication system over Nakagami-m Fadding Channels, arxiv:1202-2576v4 cs IT, (2012).

[4]. Joshi,N (1986): The generalized complex inversion theorem for integral transforms; Vijnana parishad Anusandan Patrika, Vol-29, No-2.

[5]. K, Arjun Rathie (1997): A new generalization of generalized hyper geometric functions, Le Mathematiche,, Vol.LII. Fasc.II ,297-310.

[6]. Mittal, P.K and Gupta, K C (1972): An integral involving generalized function of two variables. Proc.Indian. Acad.Sci. Sect. A. 75, 117-123.

[7]. Prasanth ,P [2008]: Generalised Integral transforms, Ph.D Thesis , Kannur University.

[8]. Prathima, J, Vasudevan Nambisan,T M, Santhakumari (2014) :A study of I- function of several- complex variables, International Journal of Engineering Mathematics, Vol-2014, Article ID 931395, Hindawi Publishing Corporation.

[9]. Prathima, J, Vasudevan Nambisan,T M [2014]:Multidimensional integral transform involving I- function of several- complex variables as kernel, IOSR, Journal of Mathematics, Vol-10,issue 2, Ver,IV [March-April-2014],p-73-80.

[10]. Prathima, J, Vasudevan Nambisan,T M [2013]:Multidimensional Mellin transform involving I- function of several- complex variables, International Journal of Mathematics and Statistic invention, Vol-1,issue 2,,p-25-30.

[11]. Reed, I.S, (1944): The Mellin type of double integral, Duke. Math.J.11, 565-572

[12]. Santhakumari, K, Vasudevan Nambisan, TM, Arjun K Rathie (2014): A study of the I- function of two variables, LE MATEMATICHE, Vol.LXIX (2014). Fasc. Pp 285-305.

[13]. Srivastava, Gupta, K.C, and Goyal, S.P(1982): H-function of one and two variables; South Asian publishers , New Delhi.