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New Characterization to the Theory of Semi-Montel and Montel Spaces

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I. Introduction

The basic definition of a semi-Montel space is "A locally convex Hausdorff space E is said to be a semi-Montel space if every bounded subset of E is relatively compact".
Here, we take the new definition of semi-Montel space as follows :
"A locally convex space E is to be semi-Montel space if every bounded subset of E has compact closure".
On the basis of the above definition, we give a new characterization to the theory of semi-Montel and Montel space.

Some Basic Definition and Notations :

(1) Topological Vector Space : A set E on which a structure of vector space over K and a topology are defined is a topological vecter space if

(a) The map (x, y) → x + y from E × E into E is continuous.

(b) The map (λ, x)→λx from K x E into E is continuous.

(2) Locally convex spaces :A topological vector space E is said to be a locally convex topological vector space or simply locally convex space or a convex space, if there is a fundamental system of convex nhds of the origin in E.

(3) Semi-Montel space :A locally convex space E is said to be semi-Montel space if every bounded subset of E has compact closure.

(4) Infra barrelled space : A locallyconvex space X is said to be infrabarelled if every bornivourous barrel in X is a nhd. of origin.

(5) Montel Spaces : An infrabarrelled semi-montel space is called a montel space.

(6) Semi-reflexive Spaces :A locally convex Hausdortt space E is said to be semi-reflexive if the canonical imbedding from E into its bidual E" is onto.

(7) Reflexive spaces :A locally convex Hausdorff space, E is said to be reflexive, if the canonical imbedding from E into its bidual E" is an isomorphism when we equip E" with topology B(E", E').

Main Characterization of Semi-Montel and Montel Spaces :

(1) Every Semi-Montel space E is semi-reflexive.

Proof : Let E be a Semi-Montel space. If A be a bounded subset of E, then its closure $\overline{A}$ is compact. Thus $\overline{A}$ is σ(E, E') closed, σ(E, E') – bounded and σ(E, E') – compact. By definition, E is semi-reflexive.

(2) Every Montel space E is reflexive.

Proof : Every Montel space E is an infrabarrelled Semi-Montel space and consequently and by (1) an infrabarrelled semi-reflexive space and hence reflexive.

(3) Every Montel space is barrelled.

Proof : Let E be a Montel space. Then by (2), E is reflexive. We know that reflexive space is always barrelled. Then E is barrelledspace. Then every Montel space is barrelled.

(4) Let E be a semi-Montel space with topology, T. If B is a bounded subset of E, then the topology induced on B by T is the same as the topology induced by σ (E, E').

Proof : Let E be a semi-Montel space and B a bounded subset of E. Then closure $\overline{B}$ is compact. Since σ (E, E') is closure than T, then $\overline{B}$ is σ (E, E') –compact. Then $\overline{B}$ is the same for every topology of the dualpair. Since B is a bounded subset of E, then B is also σ (E, E') – bounded. By (1), E is semi-reflexive, therefore, B is σ (E, E') – compact. Also $\overline{B}$ is σ (E, E') – compact. Hence the topology T on E is the same as the topology induced by σ (E, E').

(5) A closed subspace F of a semi-Montel space E is a Semi-Montel space.

Proof : Let A be a bounded subset of F. Then, A is also a bounded subset of E. So, $\overline{A}$is compact in E. Since F is closed, then $\overline{A}$is also compact in F. Hence F is a Semi-Montel space.

(6) A locally convex space E is Semi-Montel iff K(E', E) &β (E', E) coincide on E'.

Proof : Let E be a semi-Montel space. Then by (1), E is Semi -reflexive. Then E is quasi-complete for the topology (E, E'). Let U be a precompact subset of E. Then the balanced, convex hull V of U is precompact. The closure $\overline{V}$ of precompact set V is precompact. Since precompact sets are bounded, therefore $\overline{V}$ is balanced, convex, closed and bounded subset of F. Since E is Semi-Montel space, then by definition, since V is bounded so $\overline{V}$ is compact. So $\overline{V}$ is both precompact and compact subset of E. Then topology β(E', E) coincides with topology K(E', E). In general K(E', E) is closer than λ(E', E). Since each compact subset of E is σ(E, E')-compact, therefore, K(E' E) is closer than the MackeytopologyT(E', E). Also K(E', E) is finer than σ(E', E). Then the dual of E' equipped with the topology K(E', E) is the space E. Again E is semi-reflexive, so β(E', E) is compatible with duality between E and E' and β(E', E) = T(E', E). Hence K(E', E), &β(E', E) coincide on E'. And thus K(E', E) = λ(E', E) = T(E', E) = β(E', E).

Conversely, let K(E' E) = β(E', E) = λ(E', E). Then precompact, compact and σ (E, E')-bounded sets in E coincide. If A is precompact set in E then $\overline{A}$ is precompact and so $\overline{A}$ is compact. Since bounded sets are the same for every topology of the dual pair, so A is bounded. So every bounded subset of E has compact closure $\overline{A}$. Hence E is semi-Montel space.

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