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State Feedback Control for Uncertain Systems with Input Saturation

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Abstract: *The problem of state feedback control for a class of uncertain systems with input saturation is considered in this paper. Based on the Lyapunov stability theory, the stability condition and the state feedback controller design method are obtained by using the linear matrix inequality approach. By introducing the matrix into Lyapunov functional, the proposed conditions are less conservative than the previous results. Finally, a numerical example is given to demonstrate the validity of the results.*

Keywords: *Input saturation; Uncertain; Linear matrix inequality.*

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I. Introduction

Saturation exists widely in various power systems. If the saturation limit is not considered, the system performance will be degraded or even unstable in serious cases. In the actual engineering control process, the control input often needs to meet certain conditions, and input saturation is the most common constraint control, so the research on actuator saturation control has very important practical significance.

Fuller first proposed the saturation system in 1960s, and adopted the strategy of feedback calculation and tracking to make the system quickly exits the saturation region [1]. Especially in recent decades, the problem of input saturation control has been widely concerned by many scholars [2−4]. In [5], Hu et al proposed a convex combination method for discrete-time linear systems with input saturation. By introducing auxiliary matrix, the stability condition is transformed into linear matrix inequality, and the stability condition and controller design method are obtained. For linear systems with input saturation, Zuo et al gave the estimation of the expanded region of attraction by using the linear matrix inequality method [6]. Then, Zhou et al introduced the design method of saturated systems into the saturated networked control systems. For example, the reference [7] studied the output feedback stabilization of saturated networked systems. However, the Lyapunov functions designed in the literature are lack of proper parameter matrix, and the results are conservative. In addition, the influence of external disturbance or uncertainty on the system is not considered. Because of this, based on the previous studies, we obtain the sufficient conditions and the state feedback controller for a class of uncertain systems with actuator saturation in this paper.

II. Problem Formulation

Consider the following uncertain system with input saturation

$$\begin{aligned}\dot{x}(t) &= (A+\Delta A(t))x(t)+(B+\Delta B(t))sat(u(t)) \\ x(t) &= \phi(t)\end{aligned} \quad (1)$$

where $x(t)\in \mathrm{R}^{n}$ is the system state, $u(t)\in \mathrm{R}^{m}$ is the control input, $A\in \mathrm{R}^{n\times n}$, $B\in \mathrm{R}^{n\times m}$ are constant matrices, $\phi(t)=[\phi_1(t)\ \ \phi_2(t)\ \ \cdots\ \ \phi_n(t)]^T\in \mathrm{R}^{n}$ is the initial function, $sat(u(t))=[sat(u_1(t)), sat(u_2(t)),\cdots, sat(u_m(t))]$ is the saturation function, where

$$sat(u_i(t))=\begin{cases}\underline{u}_i & u_i(t)\le \underline{u}_i<0\\ u_i(t) & \underline{u}_i\le u_i(t)\le \bar{u}_i\\ \bar{u}_i & 0<\bar{u}_i\le u_i(t)\end{cases}$$

$\Delta A(t), \Delta B(t)$ are the uncertainties satisfying

$$[\Delta A(t), \Delta B(t)] = DF(t)[E_1, E_2] \quad (2)$$

where $F(t)$ satisfies $F^T(t)F(t)\le I$.

The following state feedback controller will be designed as

$$u(t) = 2Kx(t) \tag{3}$$

where $K \in \mathrm{R}^{m\times n}$ is a constant matrix. Inserting the above controller（3）into （1）

$$\begin{aligned} \dot{x}(t) &= \bar{A}(t)x(t) + \bar{B}(t)\eta(t) \\ x(t) &= \phi(t) \end{aligned} \tag{4}$$

where

$$\begin{aligned} \bar{A}(t) &= A + BK + \Delta A(t) + \Delta B(t)K \\ \bar{B}(t) &= B + \Delta B(t) \\ \eta(t) &= sat(2Kx(t)) - Kx(t) \end{aligned} \tag{5}$$

and $\eta(t)$ satisfies

$$\eta^T(t)\eta(t) \le x^T(t)K^T Kx(t) \tag{6}$$

Lemma1 [4] The LMI $\begin{bmatrix} Y(x) & W(x) \\ * & R(x) \end{bmatrix} > 0$ is equivalent to

$$R(x) > 0, Y(x) - W(x)R^{-1}(x)W^T(x) > 0$$

where $Y(x) = Y^T(x), R(x) = R^T(x)$ depend on x.

Lemma2 [6] For the given constant matrix Y, D and E with appropriate dimension, where Y is symmetric matrix，then $Y + DEF + E^T F^T D^T < 0$ for matrix F satisfying $F^T F \le I$, if and only if there is a constant $\varepsilon > 0$, such that

$$Y + \varepsilon DD^T + \varepsilon^{-1}E^T E < 0$$

III. Main Results

Theorem1 If there exist a constant $\varepsilon > 0$， positive-definite matrices $P \in \mathrm{R}^{n\times n}$ and matrix $K \in R^{m\times n}$ such that the following matrix inequalities holds

$$\Theta = \begin{bmatrix} \bar{A}^T(t)P + P\bar{A}(t) + \varepsilon K^T K & P\bar{B}(t) \\ * & -\varepsilon I \end{bmatrix} < 0 \tag{7}$$

with the controller（3），the closed system（4）is asymptotically stable.

Proof The Lyapunov function is selected as follows

$$V(t) = x^T(t)Px(t)$$

$P \in \mathrm{R}^{n\times n}$ is a positive-definite matrix.

Along the solution of system (4) we have

$$\begin{aligned} \dot{V}(t) &= 2x^T(t)P\dot{x}(t) \\ &= x^T(t)(P\bar{A}(t) + \bar{A}^T(t)P)x(t) + 2x^T(t)P\bar{B}(t)\eta(t) \\ &= \Phi^T(t)\begin{bmatrix} P\bar{A}(t) + \bar{A}^T(t)P & P\bar{B}(t) \\ * & 0 \end{bmatrix}\Phi(t) \end{aligned} \tag{8}$$

where

$$\Phi(t) = \begin{bmatrix} x(t) \\ \eta(t) \end{bmatrix}$$

with（6）, we can obtain

$$0 \le \Phi^T(t)\begin{bmatrix} \varepsilon K^T K & 0 \\ * & -\varepsilon I \end{bmatrix}\Phi(t)$$

where ε is an arbitrarily small positive number.

Inserting the above inequality into (8), we obtain

$$\dot{V}(t) \le \Phi^T(t)\Theta\Phi(t)$$

where

$$\Theta = \begin{bmatrix} \bar{A}^T(t)P + P\bar{A}(t) + \varepsilon K^T K & P\bar{B}(t) \\ * & -\varepsilon I \end{bmatrix}$$

With Lyapunov stability theorem and the condition(7), we know that the closed system（4）is asymptotically stable.

Theorem2 If there exist constants $\bar{\varepsilon}, \varepsilon_1 > 0$，positive-definite matrices $X \in \mathrm{R}^{n\times n}$ and matrix $\bar{K} \in R^{m\times n}$ such that the following linear matrix inequalities holds

$$\begin{bmatrix} AX + B\bar{K} + X^T A^T + \bar{K}^T B^T + \varepsilon_1 DD^T & \bar{\varepsilon} B & \bar{K}^T & (E_1 X + E_2 \bar{K})^T \\ * & -\bar{\varepsilon} I & 0 & (\bar{\varepsilon} E_3)^T \\ * & * & -\bar{\varepsilon} I & 0 \\ * & * & * & -\varepsilon_1 I \end{bmatrix} < 0 \qquad (9)$$

with the controller $u(t) = 2\bar{K}X^{-1}x(t)$，the closed system（4）is asymptotically stable.

Proof With lemma1, we know that the inequality (7) is equivalent to

$$\begin{bmatrix} \bar{A}^T(t)P + P\bar{A}(t) & P\bar{B}(t) & \varepsilon K^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0$$

On both sides of the above inequality, multiply left by right and multiply by $diag\{P^{-1}, \varepsilon^{-1}I, \varepsilon^{-1}I\}$, we obtain

$$\begin{bmatrix} P^{-1}\bar{A}^T(t) + \bar{A}(t)P^{-1} & \varepsilon^{-1}\bar{B}(t) & P^{-1}K^T \\ * & -\varepsilon^{-1}I & 0 \\ * & * & -\varepsilon^{-1}I \end{bmatrix} < 0$$

Inserting (5) into the above inequality, we obtain

$$\begin{bmatrix} \begin{matrix} AP^{-1} + \Delta A(t)P^{-1} + BKP^{-1} \\ +\Delta B(t)KP^{-1} + (AP^{-1} + \Delta A(t)P^{-1} \\ +BKP^{-1} + \Delta B(t)KP^{-1})^T \end{matrix} & \varepsilon^{-1}B + \varepsilon^{-1}\Delta B(t) & P^{-1}K^T \\ * & -\varepsilon^{-1}I & 0 \\ * & * & -\varepsilon^{-1}I \end{bmatrix} < 0$$

i.e.

$$\Sigma + \begin{bmatrix} \Delta A(t)P^{-1} + \Delta B(t)KP^{-1} + (\Delta A(t)P^{-1} + \Delta B(t)KP^{-1})^T & \varepsilon^{-1}\Delta B(t) & 0 \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix} < 0$$

where

$$\Sigma = \begin{bmatrix} AP^{-1} + BKP^{-1} + (AP^{-1} + BKP^{-1})^T & \varepsilon^{-1}B & P^{-1}K^T \\ * & -\varepsilon^{-1}I & 0 \\ * & * & -\varepsilon^{-1}I \end{bmatrix}$$

Inserting (2) into the above inequality, and let $X = P^{-1}, \bar{K} = KP^{-1}, \bar{\varepsilon} = \varepsilon^{-1}$, we have

$$\Sigma + \begin{bmatrix} D \\ 0 \\ 0 \end{bmatrix} F(t)\begin{bmatrix} E_1X + E_2\bar{K} & \varepsilon E_2 & 0 \end{bmatrix} + \begin{bmatrix} (E_1X + E_2\bar{K})^T \\ (\varepsilon E_2)^T \\ 0 \end{bmatrix} F^T(t)\begin{bmatrix} D^T & 0 & 0 \end{bmatrix} < 0$$

With lemma2， we know that if $\varepsilon_1 > 0$,the following inequality holds

$$\Sigma + \varepsilon_1 \begin{bmatrix} D \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} D^T & 0 & 0 \end{bmatrix} + \varepsilon_1^{-1} \begin{bmatrix} (E_1X + E_2\bar{K})^T \\ (\varepsilon E_2)^T \\ 0 \end{bmatrix} \begin{bmatrix} E_1X + E_2\bar{K} & \varepsilon E_2 & 0 \end{bmatrix} < 0$$

With lemma 1， we know that the above inequality is equivalent to (9).

IV. Numerical example

Consider the closed-loop system (4), where

$A = \begin{bmatrix} -2 & 0 \\ 1 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0.5 \\ 2 & -1 \end{bmatrix}$, $D = \begin{bmatrix} 0.2 & 0 \\ 0 & -0.1 \end{bmatrix}$, $F(t) = \begin{bmatrix} 0.5\cos t & 0 \\ 0 & \sin t \end{bmatrix}$, $E_1 = \begin{bmatrix} 1 & 0.2 \\ 0 & -0.1 \end{bmatrix}$,

$E_2 = \begin{bmatrix} -0.01 & 1 \\ 0 & 0.1 \end{bmatrix}$.

Solving the linear matrix inequality (9), we obtain

$X = \begin{bmatrix} 1.4256 & 0.5897 \\ 0.5897 & 0.2324 \end{bmatrix}$, $\bar{Q} = \begin{bmatrix} 2.2468 & 0.6757 \\ 0.6757 & 1.8735 \end{bmatrix}$, $\bar{K} = \begin{bmatrix} -4.8906 & 0.4357 \\ 0.4357 & 1.9578 \end{bmatrix}$

The state feedback controller is designed as

$$u(t) = 2\bar{K}X^{-1}x(t) = \begin{bmatrix} -1.6736 & 0.2596 \\ -0.5389 & 1.8764 \end{bmatrix} x(t),$$

From theorem 1, we know that the states of the closed-loop system is asymptotically stable.

V. Conclusion

In this paper, the asymptotic stability condition and the design method of state feedback control for a class of uncertain systems with input saturation are given. By introducing a parameter matrix into Lyapunov function, the conservatism of the asymptotic stability condition is reduced. On this basis, the design method of state feedback controller is obtained.

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