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Impact of Heat transfer on peristaltic transport of a Dusty Fluid with temperature gradient heat source in porous medium

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Abstract:
The effect of heat transfer on the peristaltic transport of a Dusty fluid with temperature gradient heat source in a two dimensional flexible channel under long wave length approximation has been studied. A perturbation method of solution is obtained in terms of wall slope parameter and closed form of expressions has been derived for stream function, temperature and heat transfer coefficient. The effects of elastic parameters and pertinent parameters on the coefficient of heat transfer have been computed numerically. It is observed that Heat transfer coefficient of fluid phase and particle phase decreases for larger values of Stanton number.

Keywords: *Peristaltic transport, Heat transfer, Dusty fluid and Temperature gradient heat source.*

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I. Introduction

The study of the mechanism of peristalsis, in both physiological and mechanical situations, has become the object scientific research. From fluid mechanical point of view peristaltic motion is defined as the flow of generated by a wave traveling along the walls of an elastic tube. In physiology it may be described as a progressive wave of area contraction or expansion along a length of a distensible tube containing fluid provided with transverse and muscular fibers. It consists in narrowing and transverse shortening of a portion of the tube which then relaxes while the lower portion becomes shortened and narrowed. The mechanism of peristalsis occur for urine transport from kidney to bladder through the ureter, movement of chime in the gastro-intestinal tract, the movement of spermatozoa in the ducts efferent's of the mail reproductive tract, movement of ovum in the fallopian tube, vasomotion in small blood vessels, the food mixing and motility in the intestines, blood flow in cardiac chambers etc. Also bio-medical instruments such as heart-lung machine use peristalsis to pump blood while mechanical devices like roller pumps use this mechanism to pump and other corrosive fluids. The problem of the mechanism of peristalsis transport has attracted the attention of many investigators. Fung and Yih[1], Shapiro and Jaffrin et al. [2] have studied peristaltic pumping with long wavelength at low Reynolds number. Mitra et.al [3] investigated the influence of wall properties and Poiseuillie flow in peristalsis.

The study of two-phase flows finds applications in many branches of Engineering, Environmental, Physical Sciences, etc. A few examples of such flows in diverse fields are the flow of dissolved micro molecules of fibre suspensions in paper making, flow of blood through arteries, propulsion and combustion in rockets, dispersion and fall out of pollutants in air, erosion of material due to continuous impingement of suspended particles in air etc. Frederick [4] studied two phase fluid-solid flow. In order to develop a mathematical theory of blood flow in arteries, Alihasan Nayfeh [5] considered blood as binary system of plasma (liquid phase) and blood cells a (solid phase). Saffman [6] dusty fluid serves as a better model to describe blood as a binary system. Solid-particle motion in two dimensional peristaltic flows has been discussed by Hung et.al [7]. Kaimal, M.R.[8] investigated peristaltic pumping of a Newtonian fluid with particles suspended in it at low Reynolds number under long wave length approximation. Nag, S.K. [9] studied the two-dimensional flow of unbounded dusty fluid induced by the sinusoidal transverse motion of an infinite wall. Radhakrishnamacharya [10] studied the pulsatile flow of a fluid containing small solid particles through a two-dimensional constricted channel, Srinivasacharya et al. [11] studied the effects of wall properties of Peristaltic Transport of a Dusty Fluid, Dust velocity shear driven rotational waves and associated vertices in a non-uniform dusty plasma has been investigated by Shukla et al.[12], Rashmi [13] studied unsteady flow of a dusty fluid between two oscillating plates under varying constant pressure gradient.

The interaction of peristalsis and heat transfer has become highly relevant and significant in several industrial processes also thermo dynamical aspects of blood become significant in process like haemodialysis and oxygenation when blood is drawn out of the body. Keeping these things in view, Raghunath Rao *et.al*.[14] investigated the effect of heat transfer on peristaltic transport of Viscoelastic fluid in a channel with wall properties. Srinivasulu and Radhakrishnamacharya [15] studied the influence of wall properties on peristaltic transport with heat transfer.

Flow through porous media has attracted considerable attention in recent years due to its potential application in nearly all fields of engineering and biomechanics. In chemical industries it has been used to achieve an effective mixing and process, and in the study of fixed bed reactors packed with granular material. In petroleum industries porous medium is used for oil recovery, filtration, and for cleaning oil spills, in nuclear reactors. It is of interest in many problems such as radioactive and reclaimed sewage waste disposal into aquifers, the movements of fertilizers in the soil and the leaching of salts from soil in agriculture, the transition zone between salt water and fresh water in coastal aquifers. Study of flow through a porous medium is also of immense use in biomedical problems to understand the transport processes in lungs and kidneys, to investigate intervertebral disc tissues, cartilage and bones etc. Most of the tissues in the body (e.g. bone, cartilage, muscle) are deformable porous media. The proper functioning of such materials depends crucially on the flow of blood, nutrients and so forth through them. Porous- medium models are used to understand various medical conditions (such as tumor growth) and treatments (such as injections). Recently, A. A.Khan et.al [16] studied peristaltic flow of second-grade dusty fluid through a porous medium in an asymmetric channel and Tariq et al. [17] studied peristaltic transport of a second-grade dusty fluid in a tube.

The present research aimed is to investigate the effect of heat transfer on peristalsis for the flow of a Dusty fluid with temperature gradient heat source and wall properties in porous medium in a flexible channel under long wavelength approximation. A perturbation method of solution is obtained in terms of wall slope parameter and closed form of expressions has been derived for temperature distribution and heat transfer coefficient. The effects of elasticity parameters and pertinent parameters on temperature distribution and heat transfer coefficient have been computed numerically.

II. Formulation of the Problem

Consider a peristaltic flow of a Dusty fluid in flexible channel of width *d*, the walls of the channel are assumed to be flexible and are taken as a stretched membrane on which traveling sinusoidal waves of moderate amplitude are imposed.

The geometry of the flexible walls are represented by

$$y = \eta(x,t) = d + a\ Sin\frac{2\pi}{\lambda}(x - ct) \tag{2.1}$$

Where,'*d*' is the mean half width of the channel, '*a*' is the amplitude of the peristaltic wave, '*c*' is the wave velocity, 'λ' is the wave length and *t* is the time.

The equations governing the two-dimensional flow of a dusty viscous fluid in Fluid Phase are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.2}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - kN_0\left(u^p - u\right) - \frac{\mu}{k_1}u \tag{2.3}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - kN_0\left(v^p - v\right) - \frac{\mu}{k_1}v \tag{2.4}$$

$$\rho C_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + kN_0C_s(T^p - T) + Q\frac{\partial T}{\partial y} \tag{2.5}$$

The equations governing the two-dimensional flow of a dusty viscous fluid in Particle Phase are

$$\frac{\partial u^p}{\partial x} + \frac{\partial v^p}{\partial y} = 0 \tag{2.6}$$

$$\frac{\partial u^p}{\partial t} + u^p \frac{\partial u^p}{\partial x} + v^p \frac{\partial u^p}{\partial y} = \frac{k}{m}(u - u^p) \tag{2.7}$$

$$\frac{\partial v^p}{\partial t} + u^p \frac{\partial v^p}{\partial x} + v^p \frac{\partial v^p}{\partial y} = \frac{k}{m}(v - v^p) \tag{2.8}$$

$$\frac{\partial T^p}{\partial t} + u^p \frac{\partial T^p}{\partial x} + v^p \frac{\partial T^p}{\partial y} = \frac{k}{m}(T - T^p) \tag{2.9}$$

Where u, v are the fluid velocity components, u^p, v^p are the dust velocity components, *'p'* is the fluid pressure, 'ρ' is the density of the fluid, μ is the coefficient of viscosity, m is the mass of the dust particles, T & T^P are the temperatures of the fluid and particle phase respectively, N_0 is the number density of the particles, k_1 is the permeability of the porous medium, k is the stokes resistance coefficient. The particles are assumed to be uniform in size and uniformly distributed in the fluid so that N_0 remains a constant.
The governing equation of motion of the flexible wall may be expressed as

$$L(\eta) = p - p_0 \tag{2.10}$$

Where *'L'* is an operator, which is used to represent the motion of stretched membrane with damping forces such that

$$L \equiv -T^* \frac{\partial^2}{\partial x^2} + m^* \frac{\partial^2}{\partial t^2} + C \frac{\partial}{\partial t} \tag{2.11}$$

Here T^* is the elastic tension in the membrane, m^* is the mass per unit area and C is the coefficient of viscous damping forces, p_0 is the pressure on the outside surface of the wall due to tension in the muscles. For simplicity, we assume $p_0 = 0$. The horizontal displacement will be assumed zero. Hence the boundary conditions for the fluid are

$$\left.\begin{aligned} u &= 0 \\ u^p &= 0 \end{aligned}\right\} at \quad y = \pm \eta = \pm \left[d + a \, Sin \frac{2\pi}{\lambda}(x - ct)\right] \tag{2.12}$$

Continuity of stresses requires that at the interfaces of the walls and the fluid p must be same as that which acts on the fluid at $y = h_1$ & $y = h_2$. The use of 'x' momentum equation the dynamic boundary conditions at flexible walls are

$$\frac{\partial}{\partial x} L(\eta) = \frac{\partial p}{\partial x} = \rho \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) + k N_0 \left(u^p - u \right) - \frac{\mu}{k_1} u \tag{2.13}$$

The conditions on temperature are

$$\left.\begin{aligned} T &= T_0 \\ T^p &= T_0 \end{aligned}\right\} \text{at} \quad y = -\eta \quad , \qquad \left.\begin{aligned} T &= T_1 \\ T^p &= T_1 \end{aligned}\right\} \text{at} \quad y = \eta \tag{2.14}$$

In view of the incompressibility of the fluid and two-dimensionality of the flow, we introduce the Stream function 'ψ' such that

$$\left.\begin{aligned} u &= \frac{\partial \psi}{\partial y} \quad and \quad v = -\frac{\partial \psi}{\partial x} \\ u^p &= \frac{\partial \phi}{\partial y} \quad and \quad v^p = -\frac{\partial \phi}{\partial x} \end{aligned}\right\} \tag{2.15}$$

Introducing non-dimensional variables

$$x' = \frac{x}{\lambda},\ y' = \frac{y}{d}, t' = \frac{ct}{\lambda},\ \eta' = \frac{\eta}{d},\ \psi' = \frac{\psi}{cd}, \phi' = \frac{\phi}{(kd^2/m)}, \theta = \frac{T - T_0}{T_1 - T_0}, \theta^p = \frac{T^p - T_0}{T_1 - T_0} \tag{2.16}$$

in equations of motion and the conditions (1) – (6) & (8) – (11) and eliminating p , we finally get (after dropping primes)

$$R\,\delta\left\{\frac{\partial}{\partial t}\left(\nabla^2\psi\right) + \frac{\partial\psi}{\partial y}\left(\frac{\partial}{\partial x}\left(\nabla^2\psi\right)\right) - \frac{\partial\psi}{\partial x}\left(\frac{\partial}{\partial y}\left(\nabla^2\psi\right)\right)\right\} = \nabla^4\psi + \frac{R\,\alpha}{\zeta}\left(\frac{1}{\zeta}\nabla^2\phi - \nabla^2\psi\right) - \frac{1}{Da}\nabla^2\psi \tag{2.17}$$

$$\delta\left\{\zeta\frac{\partial}{\partial t}\left(\nabla^2\phi\right) + \frac{\partial\phi}{\partial y}\left(\frac{\partial}{\partial x}\left(\nabla^2\phi\right)\right) - \frac{\partial\phi}{\partial x}\left(\frac{\partial}{\partial y}\left(\nabla^2\phi\right)\right)\right\} = \zeta\frac{\partial}{\partial t}\left(\nabla^2\psi\right) - \nabla^2\phi \tag{2.18}$$

$$R\,\delta\left(\frac{\partial\theta}{\partial t} + \frac{\partial\psi}{\partial y}\left(\frac{\partial\theta}{\partial x}\right) - \frac{\partial\psi}{\partial x}\left(\frac{\partial\theta}{\partial y}\right)\right) = \frac{1}{P_r}\left(\frac{\partial^2\theta}{\partial y^2} + \delta^2\frac{\partial^2\theta}{\partial x^2}\right) + \frac{R\,\alpha\,\gamma}{\zeta}\left(\theta^p - \theta\right) + S_t\frac{\partial\theta}{\partial y} \tag{2.19}$$

$$\delta\left(\zeta\frac{\partial\theta^p}{\partial t} + \frac{\partial\theta}{\partial y}\left(\frac{\partial\theta^p}{\partial x}\right) - \frac{\partial\theta}{\partial x}\left(\frac{\partial\theta^p}{\partial y}\right)\right) = -\left(\theta^p - \theta\right) \tag{2.20}$$

$$\frac{\partial\psi}{\partial y} = 0,\ \frac{\partial\phi}{\partial y} = 0 \quad at \quad y = \pm\eta \tag{2.21}$$

$$\frac{\partial}{\partial y}(\nabla^2\psi) - R\,\delta\left(\frac{\partial^2\psi}{\partial y\,\partial t} + \frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\,\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}\right) + \frac{R\,\alpha}{\zeta}\left(\frac{1}{\zeta}\frac{\partial\phi}{\partial y} - \frac{\partial\psi}{\partial x}\right) - \frac{1}{Da}\frac{\partial\psi}{\partial y}$$
$$= \left(E_1\frac{\partial^3}{\partial x^3} + E_2\frac{\partial^3}{\partial x\,\partial t^2} + E_3\frac{\partial^2}{\partial x\,\partial t}\right)\eta \quad at \quad y = \pm\eta \tag{2.22}$$

$$\theta = 0 \quad on \quad y = -\eta,\quad \theta = 1 \quad on \quad y = \eta \tag{2.23}$$

Where $\nabla^2 = \frac{\partial^2}{\partial y^2} + \delta^2\frac{\partial^2}{\partial x^2}$

The non-dimensional parameters are

$R = \frac{c\,d}{\upsilon}$ (Reynolds number), $S_t = \frac{Q}{\rho C_p c}$ (Stanton number), $P_r = \frac{c_p\rho\nu}{k}$ (Prandtl number),

$\alpha = \frac{N_0 m}{\rho}$ (Dust Concentration parameter), $\zeta = \frac{c\ m}{k\ d}$ (Relaxation time),

$D_a = \frac{k_1}{d^2}$ (Darcy number), $\varepsilon = \frac{a}{d}$, and $\delta = \frac{d}{\lambda}$ are geometric parameters

$E_1 = -\frac{T\ d^3}{\lambda^3\rho\,\upsilon\,c}$ (The rigidity of the wall), $E_2 = \frac{m\ c\ d^3}{\lambda^3\rho\,\upsilon}$ (The stiffness of the wall)

$E_3 = \frac{C\ d^3}{\lambda^2\rho\,\nu}$ (The damping nature of the wall)

III. Method of Solution

We seek perturbation solution in terms of small parameter δ as follows:

$$\psi = \psi_0 + \delta\,\psi_1 + \delta^2\psi_2 + \ldots.. \tag{3.1}$$

$$\theta = \theta_0 + \delta\theta_0 + \delta^2\theta_0 + \ldots\ldots \tag{3.2}$$

Substituting equations (22) & (23) in equations (17) to (21) and collecting the coefficients of various powers of δ

The zeroth order equations are

$$\frac{\partial^4\psi_0}{\partial y^4}+\frac{R\alpha}{\zeta}\left(\frac{1}{\zeta}\frac{\partial^2\phi_0}{\partial y^2}-\frac{\partial^2\psi_0}{\partial y^2}\right)-\frac{1}{Da}\frac{\partial^2\psi_0}{\partial y^2}=0 \tag{3.3}$$

$$\zeta\frac{\partial^2\psi_0}{\partial y^2}-\frac{\partial^2\phi_0}{\partial y^2}=0 \tag{3.4}$$

$$\frac{1}{P_r}\frac{\partial^2\theta_0}{\partial y^2}+\frac{R\alpha\gamma}{\zeta}\left(\theta_0{}^p-\theta_0\right)+S_t\frac{\partial\theta_0}{\partial y}=0 \tag{3.5}$$

$$\theta_0{}^p-\theta_0=0 \tag{3.6}$$

The corresponding boundary conditions are

$$\frac{\partial\psi_0}{\partial y}=0 \quad and \quad \frac{\partial\phi_0}{\partial y}=0 \quad at \quad y=\pm\eta \tag{3.7}$$

$$\frac{\partial^3\psi_0}{\partial y^3}+\frac{R\alpha}{\zeta}\left(\frac{1}{\zeta}\frac{\partial\phi_0}{\partial y}-\frac{\partial\psi_0}{\partial x}\right)-\frac{1}{Da}\frac{\partial\psi_0}{\partial y}=\left(E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial x\partial t^2}+E_3\frac{\partial^2}{\partial x\partial t}\right)\eta \quad at \quad y=\pm\eta \tag{3.8}$$

$$\left.\begin{array}{l}\theta_0=0 \quad at \quad y=-\eta\\ \theta_0=1 \quad at \quad y=\eta\end{array}\right\} \tag{3.9}$$

Zeroth-order problem

On solving the equations (3.3) - (3.5) subject to the conditions (3.7) - (3.9), we get

$$\psi_0=C_1y+C_2Sinh\,My \tag{3.10}$$

$$\phi_0=\zeta\left(C_1y+C_2Sinh\,My\right) \tag{3.11}$$

$$\theta_0=C_5+C_6e^{-Ay}=\theta_0{}^p \tag{3.12}$$

The first order equations are

$$R\left(\frac{\partial}{\partial t}\left(\frac{\partial^2\psi_0}{\partial y^2}\right)+\frac{\partial\psi_0}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial^2\psi_0}{\partial y^2}\right)\right)-\frac{\partial\psi_0}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial^2\psi_0}{\partial y^2}\right)\right)\right)= \tag{3.13}$$

$$\frac{\partial^4\psi_1}{\partial y^4}+\frac{R\alpha}{\zeta}\left(\frac{1}{\zeta}\frac{\partial^2\phi_1}{\partial y^2}-\frac{\partial^2\psi_1}{\partial y^2}\right)-\frac{1}{Da}\frac{\partial^2\psi_1}{\partial y^2}$$

$$\zeta\left(\frac{\partial}{\partial t}\left(\frac{\partial^2\phi_0}{\partial y^2}\right)+\frac{\partial\phi_0}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial^2\phi_0}{\partial y^2}\right)\right)-\frac{\partial\phi_0}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial^2\phi_0}{\partial y^2}\right)\right)\right)=\zeta\frac{\partial^4\psi_1}{\partial y^4}-\frac{\partial^2\phi_1}{\partial y^2} \tag{3.14}$$

$$R\left(\frac{\partial\theta_0}{\partial t}+\frac{\partial\psi_0}{\partial y}\left(\frac{\partial\theta_0}{\partial x}\right)-\frac{\partial\psi_0}{\partial x}\left(\frac{\partial\theta_0}{\partial y}\right)\right)=\frac{1}{P_r}\frac{\partial^2\theta_1}{\partial y^2}+\frac{R\alpha\gamma}{\zeta}\left(\theta_1{}^p-\theta_1\right)+S_t\frac{\partial\theta_1}{\partial y} \tag{3.15}$$

$$\zeta\frac{\partial\theta_0{}^p}{\partial t}+\frac{\partial\phi_0}{\partial y}\left(\frac{\partial\theta_0{}^p}{\partial x}\right)-\frac{\partial\phi_0}{\partial x}\left(\frac{\partial\theta_0{}^p}{\partial y}\right)=-\left(\theta_1{}^p-\theta_1\right) \tag{3.16}$$

The corresponding boundary conditions are

$$\frac{\partial\psi_1}{\partial y}=0 \quad and \quad \frac{\partial\phi_1}{\partial y}=0 \quad at \quad y=\pm\eta \tag{3.17}$$

$$\frac{\partial^3\psi_1}{\partial y^3}+\frac{R\alpha}{\zeta}\left(\frac{1}{\zeta}\frac{\partial\phi_1}{\partial y}-\frac{\partial\psi_1}{\partial x}\right)-\frac{1}{Da}\frac{\partial\psi_1}{\partial y}=R\left(\frac{\partial^2\psi_0}{\partial t\partial y}+\frac{\partial\psi_0}{\partial y}\frac{\partial^2\psi_0}{\partial x\partial y}-\frac{\partial\psi_0}{\partial x}\frac{\partial^2\psi_0}{\partial y^2}\right) \tag{3.18}$$

$$at \quad y=\pm\eta$$

$$\left.\begin{array}{l}\theta_1=0 \quad at \quad y=-\eta\\ \theta_1=0 \quad at \quad y=\eta\end{array}\right\} \tag{3.19}$$

First-order problem

On solving the equation (3.13) & (3.15) subject to the conditions (3.17) - (3.19), we obtain

$$\psi_1 = C_3\, y + C_4 Sinh\, My + \left(\frac{L_1}{2M^3} - \frac{5L_2}{4M^4}\right) y\; Cosh\, My + \frac{L_2}{4M^3} y^2\; Sinh\, My \tag{3.20}$$

$$\phi_1 = \zeta\left[C_3\, y + C_4 Sinh\, My + \left(\frac{L_1}{2M^3} - \frac{5L_2}{4M^4}\right) y\; Cosh\, My + \frac{L_2}{4M^3} y^2\; Sinh\, My\right] - \zeta^2\left[(a_4 + a_3C_1 + 2a_1C_2)\; Sinh\, My - M\; a_1C_2\; Cosh\, My\right] \tag{3.21}$$

$$\theta_1 = C_7 + C_8\; e^{-Ay} + \frac{N_1}{A} y - \left(\frac{N_2}{A} + \frac{N_4}{A^2}\right) y\; e^{-Ay} + \frac{N_3}{2\left(M^2 + MA\right)} e^{My} + \frac{N_4}{2\left(M^2 - MA\right)} e^{-My} - \frac{N_4}{2A} y^2\; e^{-Ay} + (N_5 + N_6)\frac{1}{2\left(M^2 - MA\right)} e^{-(A-M)y} + (N_5 - N_6)\frac{1}{2\left(M^2 + MA\right)} e^{-(A+M)y} \tag{3.22}$$

$$\theta_1^{\,p} = C_7 + C_8\; e^{-Ay} + \frac{N_1}{A} y - \left(\frac{N_2}{A} + \frac{N_4}{A^2}\right) y\; e^{-Ay} + \frac{N_3}{2\left(M^2 + MA\right)} e^{My} + \frac{N_4}{2\left(M^2 - MA\right)} e^{-My} - \frac{N_4}{2A} y^2\; e^{-Ay} + (N_5 + N_6)\frac{1}{2\left(M^2 - MA\right)} e^{-(A-M)y} + (N_5 - N_6)\frac{1}{2\left(M^2 + MA\right)} e^{-(A+M)y} - \zeta\begin{pmatrix} a_6 + C_1a_5 + (a_8 + C_1a_7)e^{-Ay} + C_2a_5M\; Cosh\, My + C_2a_7M\; e^{-Ay} Cosh\, My + \\ C_6a_1Ay\; e^{-Ay} + C_6a_3A\; e^{-Ay} Sinh\, My \end{pmatrix} \tag{3.23}$$

The heat transfer coefficient Z of fluid phase is given by

$$Z = Z_0 + \delta Z_1 + \ldots\ldots. \tag{3.24}$$

Where
$$Z_0 = \frac{\partial\eta}{\partial x}.\frac{\partial\theta_0}{\partial y} \tag{3.25}$$

$$Z_1 = \frac{\partial\theta_0}{\partial x} + \frac{\partial\eta}{\partial x}.\frac{\partial\theta_1}{\partial y} \tag{3.26}$$

The heat transfer coefficient Z of particle phase is given by

$$Z^p = Z_0^{\,p} + \delta Z_1^{\,p} + \ldots\ldots. \tag{3.27}$$

Where
$$Z_0^{\,p} = \frac{\partial\eta}{\partial x}.\frac{\partial\theta_0^{\,p}}{\partial y} \tag{3.28}$$

$$Z_1^{\,p} = \frac{\partial\theta_0^{\,p}}{\partial x} + \frac{\partial\eta}{\partial x}.\frac{\partial\theta_1^{\,p}}{\partial y} \tag{3.29}$$

Where

$\eta(x,t) = 1 + \varepsilon\; Sin\; 2\pi\;(x-t)$, $G = -\varepsilon\left(8\pi^3 Cos\; 2\pi\,(x-t)(E_1+E_2) - 4\pi^2 Sin\;(2\pi\,(x-t)E_3\right)$,

$N_1 = R\; P_r(1+\alpha\;\gamma)(a_6 + C_1a_5)$, $N_2 = R\; P_r(1+\alpha\;\gamma)(a_8 + C_1a_7)$, $N_3 = R\; P_r\; C_2\; a_5(1+\alpha\;\gamma)$,

$N_4 = S_t\; R^2\; P_r^{\,2}\; C_6\; a_1(1+\alpha\;\gamma)$, $N_5 = M\;\; R\; P_r\; C_1\; a_7(1+\alpha\;\gamma)$, $N_6 = S_t\; R^2\; P_r^{\,2}\; C_6\; a_3(1+\alpha\;\gamma)$,

$L_1 = R\; M^2\left((1+\alpha)a_4 + (1+\alpha\;\zeta)C_2a_3\right)$, $L_2 = -R\; M^3C_2\; a_1(1+\alpha\;\zeta)$

$a_1 = C_{1x}$, $a_2 = C_{1t}$, $a_3 = C_{2x}$, $a_4 = C_{2t}$, $a_5 = C_{5x}$, $a_6 = C_{5t}$, $a_7 = C_{6x}$, $a_8 = C_{6t}$

$A = RS_tP_r$, $M = \frac{1}{\sqrt{D_a}}$, $C_1 = -\frac{G}{M^2}$, $C_2 = \frac{G}{M^3 Cosh\; M\eta}$,

$$C_3 = \left(\frac{L_1}{M^3} - \frac{L_2}{M^4} - \frac{Ra_4}{M} - \frac{RC_1 a_3}{M} - \frac{RC_2 a_1}{M}\right) Cosh\ M\eta + \left(\frac{L_2}{M^3} + RC_2 a_1\right)\eta\ Sinh\ M\eta$$
$$- \frac{Ra_2}{M^2} - \frac{RC_1 a_1}{M^2} - RC_2 a_3 ,$$

$$C_4 = \left(\frac{Ra_2}{M^3} + \frac{RC_1 a_1}{M^3} + RC_2 a_3\right) Sech\ M\eta - \left(\frac{L_1}{2M^3} + \frac{L_2}{4M^4} + \frac{RC_2 a_1}{M}\right)\eta\ Tanh\ M\eta$$
$$- \frac{L_2}{4M^3}\eta^2 + \frac{9L_2}{4M^5} - \frac{3L_1}{2M^4} + \frac{Ra_4}{M^2} + \frac{RC_1 a_3}{M^2} \frac{RC_2 a_1}{M^2}$$

$$C_5 = \frac{1}{2}\left(1 + Coth\ A\eta\right),\quad C_6 = -\frac{1}{2}\left(1 + Csch\ A\eta\right),$$

$$C_7 = \frac{N_4}{2A}\eta^2 Cosh\ A\eta - \left(\frac{N_2}{A} + \frac{N_4}{A^2}\right)\eta\ Sinh\ A\eta - \left(\frac{N_4}{2\left(M^2 - MA\right)} + \frac{N_3}{2\left(M^2 + MA\right)}\right) Cosh\ A\eta$$
$$- \left(N_5 + N_6\right)\frac{1}{2\left(M^2 - MA\right)} Cosh\ (A - M)\eta - \left(N_5 - N_6\right)\frac{1}{2\left(M^2 + MA\right)} Cosh\ (A + M)\eta$$
$$- \frac{N_1}{A}\eta\ Coth\ A\eta - \frac{N_4}{2A}\eta^2 + \left(\frac{N_4}{2\left(M^2 - MA\right)} + \frac{N_3}{2\left(M^2 + MA\right)}\right) Coth\ A\eta\ Sinh\ M\eta$$
$$+ \left(N_5 + N_6\right)\frac{1}{2\left(M^2 - MA\right)} Sinh\ (A - M)\eta Coth\ A\eta + \left(\frac{N_2}{A} + \frac{N_4}{A^2}\right)\eta\ Coth\ A\eta\ Cosh\ A\eta$$
$$+ \left(N_5 - N_6\right)\frac{1}{2\left(M^2 + MA\right)} Sinh\ (A + M)\eta\ Coth\ A\eta$$

$$C_8 = \frac{N_1}{A}\eta\ Csch\ A\eta + \frac{N_4}{2A}\eta^2 - \left(\frac{N_4}{2\left(M^2 - MA\right)} + \frac{N_3}{2\left(M^2 + MA\right)}\right)\frac{Sinh\ M\eta}{Sinh\ A\eta}$$
$$+ \left(N_5 + N_6\right)\frac{1}{2\left(M^2 - MA\right)}\frac{Sinh\ (A - M)\eta}{Sinh\ A\eta} - \left(\frac{N_2}{A} + \frac{N_4}{A^2}\right)\eta\ Coth\ A\eta$$
$$+ \left(N_5 - N_6\right)\frac{1}{2\left(M^2 + MA\right)}\frac{Sinh\ (A + M)\eta}{Sinh\ A\eta}$$

IV. Results and Discussions

In this research paper, we studied the effect of heat transfer coefficient Z of fluid phase for the different values of E_1, E_2, E_3, R, p_r, D_a, S_t, α, and γ. The heat transfer coefficient Z is shown in Figures (1) to (9), we observed that the heat transfer coefficient Z enhances with increase in E_1 in fig.(1), higher values of E_2, Z increases in the flow region fig. (2). In fig.(3) heat transfer coefficient decreases with increase in E_3. The enhancement of Z in the central region is large in comparative that near the boundary. Heat transfer coefficient decreases for larger values of R in fig (4). Z decreases with increase in p_r. in fig. (5). Z increases with increase in D_a and the enhancement of Z is large near the boundary in fig.(6). Heat transfer coefficient decreases for larger values of S_t in fig (7). Z increases with increase in α, and γ from fig. (8) & fig.(9) respectively. We also studied the effect of heat transfer coefficient Z^P of particle phase for the values of S_t, and ζ . We observed that Z^P decreases with increase in S_t, and ζ in (10) & fig.(11).

V. Figures

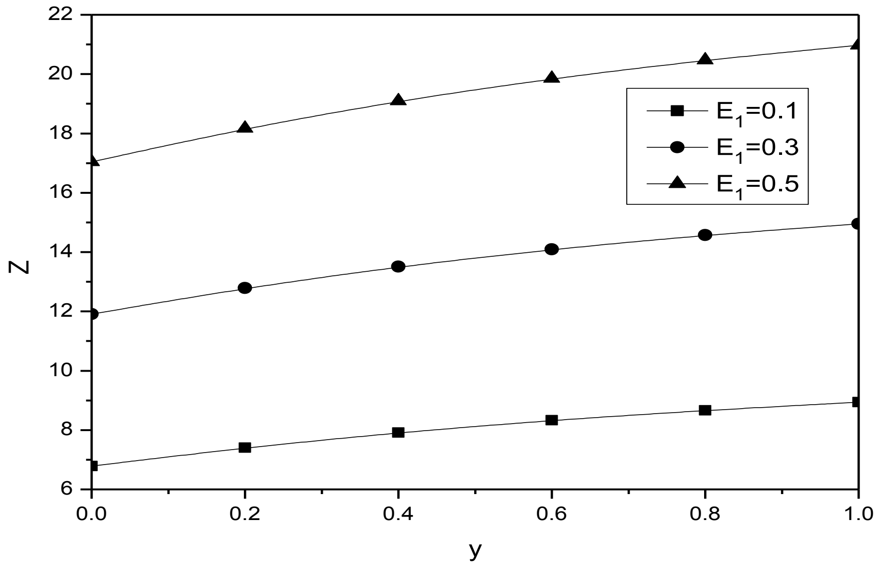

Fig.1-Effect of the rigidity of the wall E_1 on Z for
$\gamma = 1$, $\alpha = 1$, $\varepsilon = 0.01$, $\delta = 0.01$, R =1, $D_a = 1000$, S_t =1, $P_r = 1$, $E_2 = 0.2$, $E_3 = 0.3$

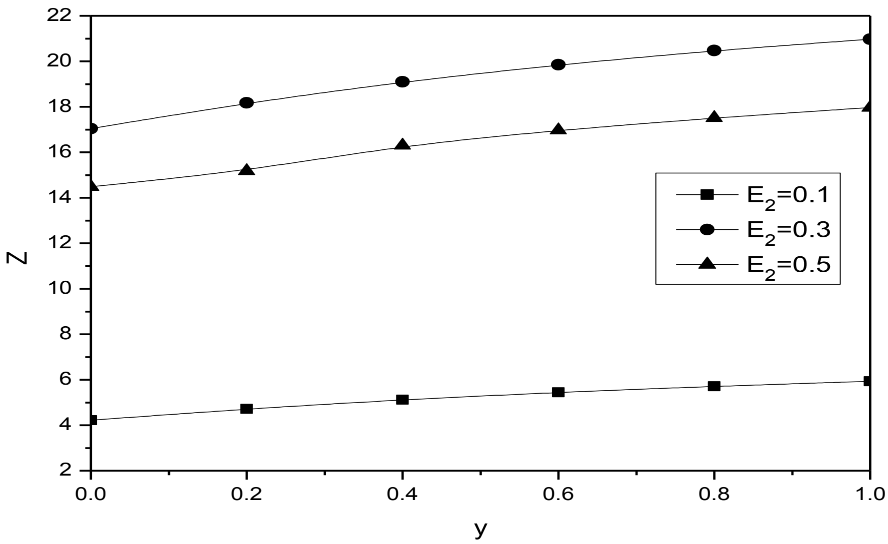

Fig.2-Effect of the stiffness of the wall E_2 on Z for
$\gamma = 1$, $\alpha = 1$, $\varepsilon = 0.01$, $\delta = 0.01$, R =1, $D_a = 1000$, S_t =1, $P_r = 1$, $E_1 = 0.1$, $E_3 = 0.3$

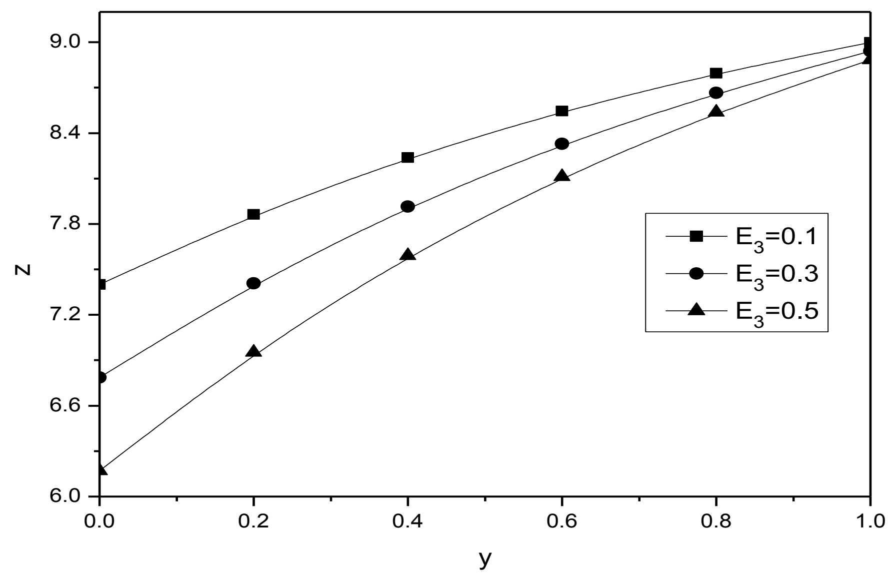

Fig.3-Effect of the damping nature of the wall E_3 on Z for
$\gamma = 1$, $\alpha = 1$, $\varepsilon = 0.01$, $\delta = 0.01$, R =1, $D_a = 1000$, S_t =1, $P_r = 1$, $E_1 = 0.1$, $E_2 = 0.2$

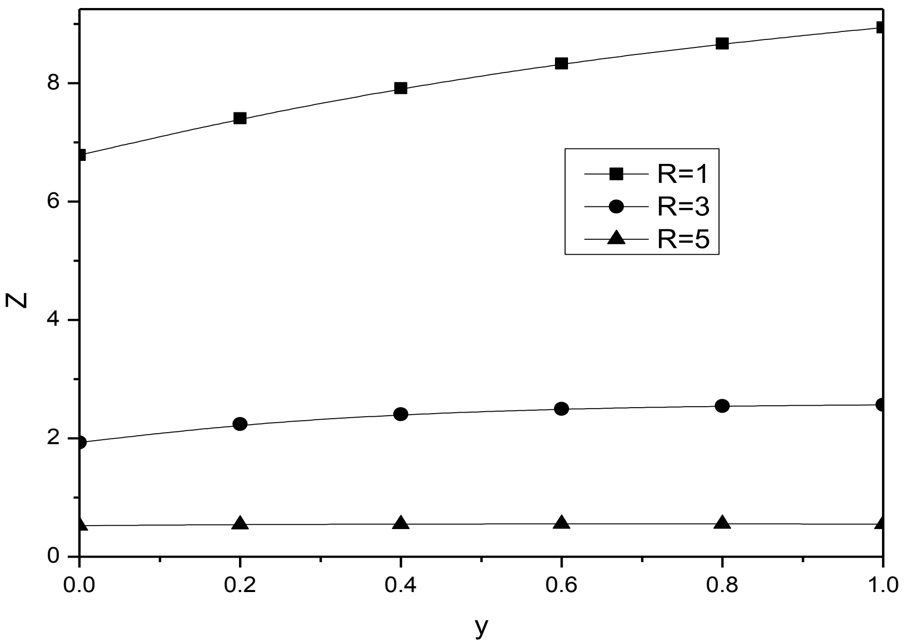

Fig.4-Effect of the Reynolds number R on Z for
$\gamma = 1$, $\alpha = 1$, $\varepsilon = 0.01$, $\delta = 0.01$, $D_a = 1000$, S_t =1, $P_r = 1$, $E_1 = 0.1$, $E_2 = 0.2$, $E_3 = 0.3$

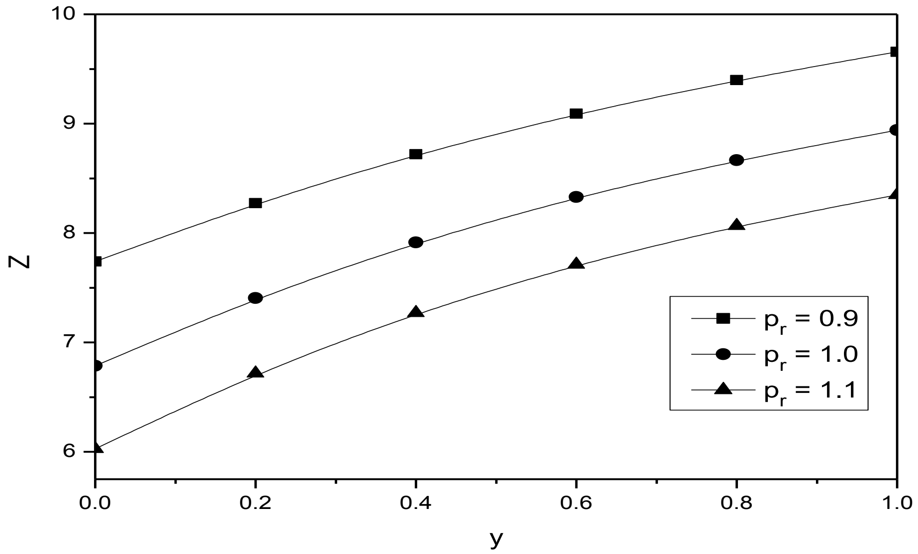

Fig.5 - Effect of Prandtl number p_r on Z for
$\gamma = 1$, $\alpha = 1$, $\varepsilon = 0.01$, $\delta = 0.01$, R =1, $D_a = 1000$, $S_t =1$, $E_1 = 0.1$, $E_2 = 0.2$, $E_3 = 0.3$

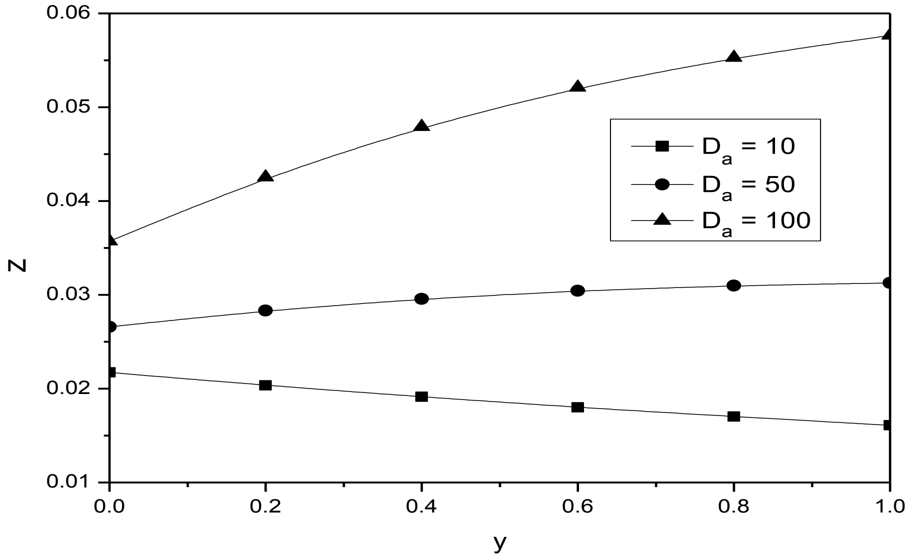

Fig.6 - Effect of the Darcy number D_a on Z for
$\gamma = 1$, $\alpha = 1$, $\varepsilon = 0.01$, $\delta = 0.01$, R =1, $S_t =1$, $P_r = 1$, $E_1 = 0.1$, $E_2 = 0.2$, $E_3 = 0.3$

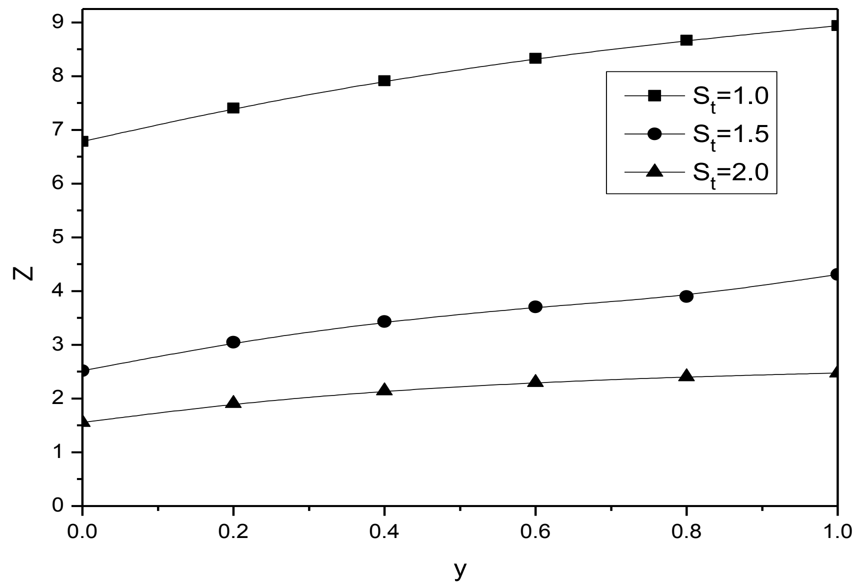

Fig.7 - Effect of the Stanton number S_t on Z for
$\gamma = 1$, $\alpha = 1$, $\varepsilon = 0.01$, $\delta = 0.01$, R =1, $D_a = 1000$, $P_r = 1$, $E_1 = 0.1$, $E_2 = 0.2$, $E_3 = 0.3$

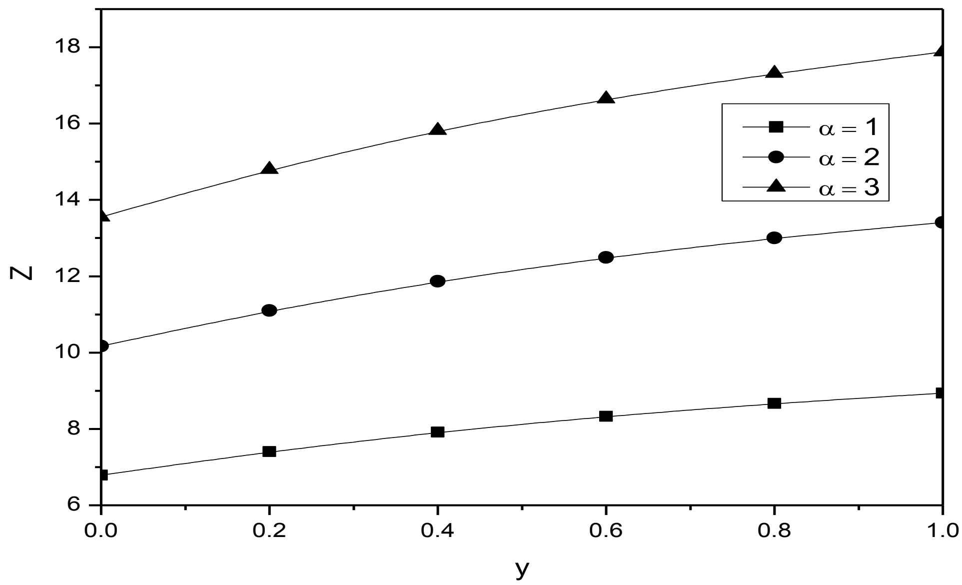

Fig.8 - Effect of the Dust Concentration parameter α on Z for
$\gamma = 1$, $\varepsilon = 0.01$, $\delta = 0.01$, R =1, $D_a = 1000$, S_t =1, $P_r = 1$, $E_1 = 0.1$, $E_2 = 0.2$, $E_3 = 0.3$

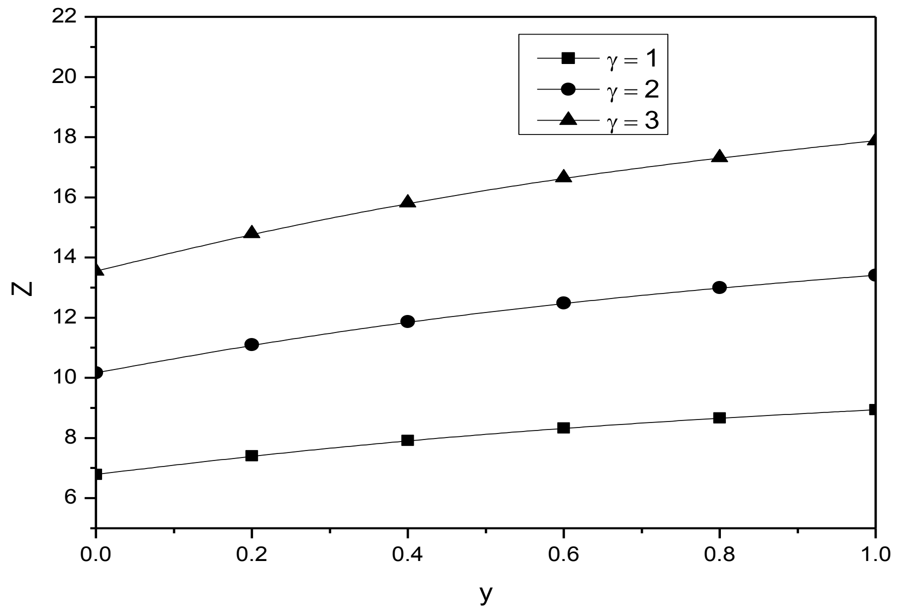

Fig.9 - Effect of the Coefficient of Specific heats γ on Z for
$\alpha = 1$, $\varepsilon = 0.01$, $\delta = 0.01$, $R = 1$, $D_a = 1000$, $S_t = 1$, $P_r = 1$, $E_1 = 0.1$, $E_2 = 0.2$, $E_3 = 0.3$

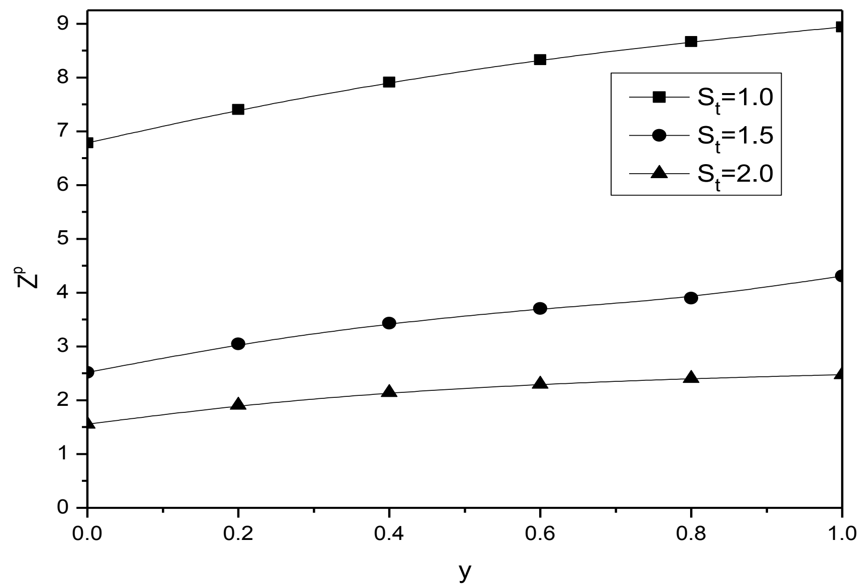

Fig.10- Effect of the Stanton number S_t on Z^P for
$\gamma = 1$, $\varepsilon = 0.01$, $\delta = 0.01$, $R = 1$, $D_a = 1000$, $S_t = 1$, $P_r = 1$, $E_1 = 0.1$, $E_2 = 0.2$, $E_3 = 0.3$

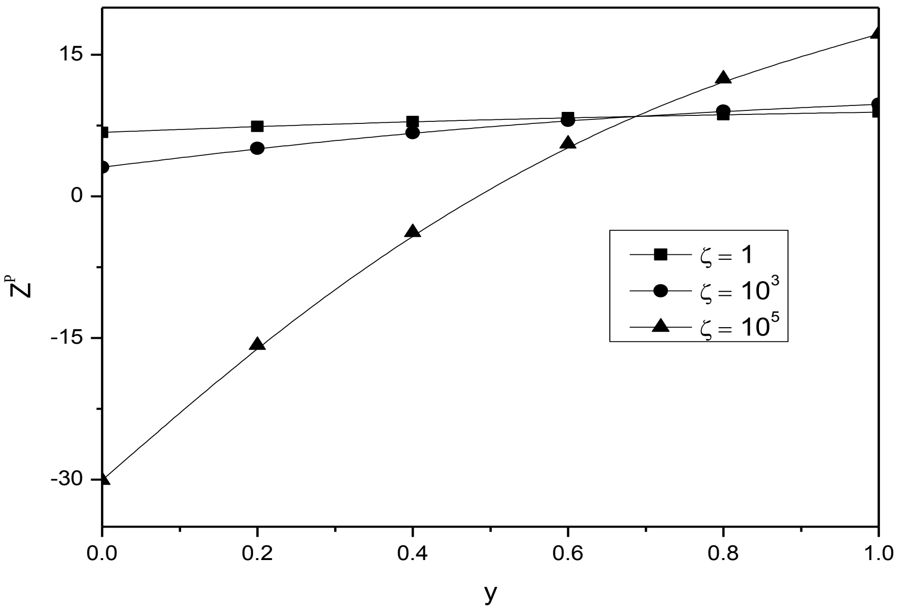

Fig.11 - Effect of the Relaxation time ζ on Z^P for
$\gamma = 1$, $\alpha = 1$, $\varepsilon = 0.01$, $\delta = 0.01$, $R = 1$, $D_a = 1000$, $S_t = 1$, $P_r = 1$, $E_1 = 0.1$, $E_2 = 0.2$, $E_3 = 0.3$

VI. Conclusions

In the present paper we have discussed the peristaltic transport of a Dusty fluid with pressure gradient heat source in a flexible channel. The governing equations of motion are solved analytically using long wave length approximation. Furthermore, the effects of elastic parameters and pertinent parameters on the coefficient of heat transfer have been computed numerically. We conclude the following observations:

1. Heat transfer coefficient Z enhances with increase in E_1, α, and γ.
2. Z increases for higher values of E_2.
3. Heat transfer coefficient decreases with increase in E_3, p_r.
4. Z increases with increase in D_a and the enhancement of Z is large near the boundary.
5. Heat transfer coefficient decreases for larger values of R and S_t.
6. Heat transfer coefficient Z^P of particle phase decreases with increase in S_t, and ζ.

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