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Axioms of Probability - Discrete Models Covering Conditional Probability

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Abstract
In this paper we suggest an extended axiomatic approach to probability so as to cover the notion of conditional probability - erstwhile requiring a separate definition.
Keywords *Axioms for Probability Function, Conditional Probability*

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I. Introduction

Role of Axioms in algebraic structures is akin to that of genes in biology. Thus, while no development can afford to defy the presumed axioms, these ought to be sufficient to derive any results thought to be relevant to the contextual algebra. Probability Theory being no exception, we need additional axiom to cover the notion of Conditional Probability - erstwhile requiring a *separate* definition, [2, 3, 1].

II. Three essential entities in the context of a Random Experiment

Depending upon the purpose of investigation, one decides the mode of observing the outcome(s) of a Random Phenomenon / Experiment, R. The set of possible outcomes is referred to be the Sample Space, S. The individual elements O of S will be assumed to conform to the following requirements:

- Each O is a *possible outcome* of R
- On the performance of R, the outcome will be one and only one of O and so, O's are *mutually exclusive*

Further, the *Power Set* $\xi = \{E | E \in \phi, S\}$ provides set theoretic expression to *possible events* and is called the *Event Space*. Further, O as an element of S represents what is observable as per S.

The probability measure function $P(O|S,R)$ essentially involves three arguments, O, S and R. In view of the inherent difference in their nature, it is preferred to separate these by different symbols ($|$ and $,$). Correspondingly, $P(E|S,R)$ is defined as a function of E, S and R.

Evaluation of $P(O|S,R)$ as well as $P(E|S,R)$ is of immense importance in dealing with uncertainties concerning the outcome of R. Following are plausible axioms to govern the assignment of numerical values to $P(O|S,R)$, $P(E|S,R)$ and $P(O|E,R)$:

Axiom 1: S comprises of at least one element
Axiom 2: $0 < P(O|S,R) < 1$, $O \in S$ along with $P(O|S,R) = 0$, if $O \notin S$ and $P(S|S,R) = 1$
Axiom 3: If E is a proper, non-null sub-set of S then $P(E|S,R) = \sum_{O \in E} P(O|S,R)$

This will yield:

$$P(E_1 \cup E_2 | S,R) = P(E_1|S,R) + P(E_2|S,R) - P(E_1 \cap E_2|S,R)$$

Axiom 4:

$$P(O|E,R) \propto P(O|S,R)$$
$$= 0 \text{ if } O \notin E \text{ and } 1 \text{ if } O = E \text{ as per Axiom 2.}$$

Equivalently $P(O|E,R) = k \cdot P(O|S,R)$ where k is a constant for given E and all $O \in E$. Application of Axioms 2 & 3 will yield

$$P(O|E,R) = \frac{P(O|S,R)}{P(E|S,R)}.$$

Further, for $F \in E$, $F = F \cap E$. This in turn comfortably yields the following commonly used formula for the “conditional" probability, viz.,

$$P(F|E,R) = \frac{P(F \cap E|S,R)}{P(E|S,R)}$$

and the sequential formula, more commonly named the multiplication rule

$$P(E_1 \cap E_2|S,R) = P(E_1|S,R) \times P(E_2|E_1,S,R).$$

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