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Hydromagnetic Free Convection Unsteady Turbulent Fluid Flow Over A Vertical Infinite Heat Absorbing Plate

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Abstract
Analysis of hydromagnetic free convection turbulent fluid flow over an infinite vertical plate was carried out. The fluid flow was modeled using conservation equations of energy and momentum. The governing equations were then non-dimensionalised giving rise to non-dimensional parameters. The approximate numerical solution for the non-linear partial differential equations are determined by use of the finite difference method and solved using MATLAB computer software. Thereafter the solutions presented in graphs. The various non-dimensional parameters are analysed of their effects on the velocities and temperature profiles. Hydromagnetics is important as it has applications in areas like meteorology and astrophysics, biological, environmental, aerospace and aeronautical engineering among others. It is evident from the results that the primary velocity increases with decreasing magnetic parameter (M), increases with increase in Hall parameter and also increases with increase in Grashoff number. Also, the secondary velocity increases with decreasing magnetic parameter (M) and decreases with increasing Hall parameter. It is also found that the temperature profile decreases with decreasing magnetic parameter (M), decreases with increasing Hall parameter and increases with decrease in Prandtl number.

Keywords: *Unsteady, Hydromagnetic, Turbulent, Free convection, Vertical plate, Finite difference*

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Nomenclature

u,v,w	*Cartesian velocity components* (ms^{-1})
x,y,z	*Cartesian coordinate variables*
t	*Time (s)*
p	*Pressure* (Nm^{-2})
T	*Temperature (K)*
μ	*Dynamic viscosity* ($Kgm^{-2}s$)
v	*Kinematic viscosity* (m^2s^{-1})
k	*Thermal conductivity* ($wm^{-1}k^{-1}$)
∇	*Gradient operator,* $\nabla = \mathrm{i}\frac{\partial}{\partial \mathrm{x}} + j\frac{\partial}{\partial \mathrm{y}} + k\frac{\partial}{\partial \mathrm{z}}$
ρ	*Density* (Kg/m^3)
m	*Mass of the fluid particle (Kg)*
E	*Electric field* (Vm^{-1})*, internal energy (J)*
Gr	*Thermal Grashoff number*
g	*Acceleration due to gravity* (ms^{-2})
Pr	*Prandtl number*
Pr_t	*Turbulent Prandtl number*
M	*Magnetic parameter*
H_o	*External applied transverse magnetic field intensity*(wbm^{-2})
ϕ	*Viscous dissipative rate*
α	*Thermal diffusivity*
β	*Thermal expansion coefficient* (K^{-1})
C_p	*Specific heat at constant pressure* ($JKg^{-1}K^{-1}$)
$u', v' w'$	*Fluctuating components of velocity*
$\overline{u},\ \overline{v},\ \overline{w}$	*Mean velocities*
a	*Acceleration* (m/s^2)
Q	*Heat (J)*
W	*Work (J)*

$\frac{D}{Dt}$	*Material derivative given by* $\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$
$\frac{\partial}{\partial t}$	*Partial derivative with respect to time*
σ	*Electrical conductivity* ($\Omega^{-1}m^{-1}$)
L	*Characteristic length (m)*
μ_e	*Electron permeability (H/m)*
B	*Magnetic flux density* (wbm^{-2})
H	*Magnetic field intensity* (wbm^{-2})
J	*Current density vector*

I. Introduction

Laminar flow is a fluid flow where the particles of a fluid move parallel to each other and there is no mixing between the adjacent layers of the fluid. On the other hand, turbulent flow is a fluid flow in which the fluid particles do not move parallel to each other and the adjacent layers of the fluid cross each other. A boundary layer of fluid refers to the immediate vicinity of the boundary surface where the effects of viscosity are significant. The region in which flow adjusts from zero velocity at the wall to a maximum in the main stream of the flow is called boundary layer. When there is fluid flow over a surface a thermal boundary layer must develop if there is bulk temperature difference. Convection is a process in which energy is transferred through a fluid when there is motion of bulk fluid [1]. The theoretical investigation of fluid flow is referred to as computational fluid dynamics. The use of computational software, applied mathematics and physics to picture out in what manner fluid flows and how this fluid affects objects as it flows is known as computational fluid dynamics. A field of study in which magnetofluids is carried out is called hydromagnetics or magnetohydrodynamics (MHD). The study of magnetohydrodynamics is significant in areas like meteorology and astrophysics, biological, environmental, aerospace and aeronautical engineering among others [5].

Wang *et al,* [8], studied simulation of turbulent flow around a surface-mounted finite square cylinder. In their study they used the Reynolds Stress Model and the Detached Eddy Simulation. They evaluated the performance of the two methods and they found that both models reproduced successfully large scale vortex structure in the wake of finite wall mounted body.

Mukuna *et.al.* [5] considered a mathematical model of hydromagnetic turbulent boundary layer fluid flow past a vertical infinite cylinder with Hall current. They used prandtl mixing length hypothesis to resolve Reynolds stresses due to turbulence in conservation equations and these equations solved using finite difference method.

Rency [6] investigated an incompressible turbulent fluid flow past a vertical semi-infinite rotating plate in the presence of a strong inclined constant magnetic field. The governing equations were resolved using finite difference method to obtain numerical approximate solutions. Velocity and temperature profiles were presented graphically. The effects of non-dimensional parameters and the angle inclined by the magnetic field on the flow changes were analysed. It was then noted that an increase in Eckert number and rotational parameter led to a decrease in the primary velocity.

Umamaheswar M. *et al* [7] analysed numerical solutions to unsteady MHD convection flow. The flow was of a well-Known-Newtonian visco-elastic second order Rivlin-Erickson fluid past a semi-infinite vertical plate. The plate was impulsively started under the influence of magnetic field whose transverse was uniform. The findings were that, an increase in magnetic field parameter, prandtl number and schimdt number led to a decrease in the velocity. Besides, Grasshoff number, Soret number and visco-elastic parameter increased with increase in velocity.

Kwanza *et.al.* [1] did some work on MHD stokes problem for a dissipative fluid which is heat generating with ion-slip and hall current, mass diffusion and radiation absorption. The influence on the rate of mass transfer, concentration, velocity skin friction, the rate of heat transfer and temperature for the various parameters were analyzed.

Diaz-Daniel *et al,* [2], investigated numerically the statistics of the wall shear stress fluctuations for a turbulent boundary layer and their relation to the velocity fluctuations outside of the near-wall region. They obtained flow data from a Direct Numerical Simulation of a zero pressure-gradient turbulent boundary layer using the high-order flow solver Incompact3D. The model suggested that the wall shear stress fluctuations may induce a high slope in the turbulence energy spectra of streamwise velocities.

Mayaka *et.al.* [4] investigated the turbulent fluid flow problem of a conducting fluid past a porous vertical infinite plate which is in a rotating system. They accounted for the joule's heating, mass transfer and Hall current. Turbulence was approximated using Prandtl mixing length hypothesis and mathematical formulation was constructed for the same. Partial differential equations which were obtained were resolved using forward time central space finite difference method. The results of difference equations were then solved iteratively by use of computer program in MATLAB. The solutions were presented graphically and they discussed the influence on velocity and temperature profiles for the various non-dimensional parameters. There was a profound effect on the primary velocities, secondary velocities, temperature and even the concentration profiles due to mass transfer, rotation, joule's heating and Hall current. The Hall parameter and the rotational parameter were found to enhance the secondary velocities while inhibiting the primary velocities.

II. Mathematical Model

We are considering a two-dimensional flow for this study. The infinite vertical plate is taken to be along the x-axis and the horizontal is the y-axis while the z+-axis is taken normal to the plate. The fluid is assumed to be incompressible and viscous. A strong magnetic field of uniform strength H_0 is applied normal to the direction of the flow. The induced magnetic field is considered negligible hence H = (0, 0, H_0), as shown in the figure below. The temperature of the plate and the fluid are assumed to be the same initially. At time t*>0 the plate is stationary and the fluid starts moving impulsively in its plane with velocity U_o and at the same time the temperature of the plate is instantaneously raised to T_w^* which is maintained constant later on.

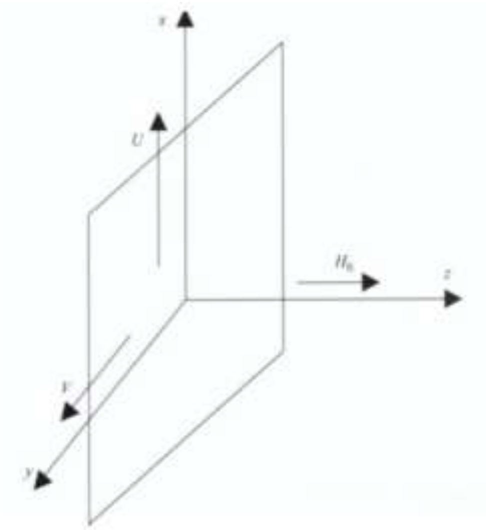


Figure 1: Schematic diagram for the fluid flow

The above flow is governed by the following equations:

$$\frac{\partial \overline{U^*}}{\partial t} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} + v\frac{\partial^2 \overline{U^*}}{\partial z^2} - \frac{\partial(\overline{u'w'})}{\partial z} + \rho g + \mathbf{J} \times \boldsymbol{B} \qquad (1)$$

$$\frac{\partial \overline{V^*}}{\partial t} = v\frac{\partial^2 V^*}{\partial z^2} - \frac{\partial(\overline{u'w'})}{\partial z} + \mathbf{J} \times \boldsymbol{B} \qquad (2)$$

$$\frac{\partial \overline{T^*}}{\partial t} = \frac{k}{\rho C_p}\frac{\partial^2 \overline{T^*}}{\partial z^2} - \frac{\partial(\overline{w'T'})}{\partial z} \qquad (3)$$

The boundary and initial conditions are:

$$\left.\begin{array}{l} t^* < 0 : \ \overline{U^*} = 0, \overline{V^*} = 0, \ \overline{T^*} = \overline{T_\infty^*} \ everywhere \\ t^* \geq 0 : \ \overline{U^*} = 0, \overline{V^*} = 0, \ \overline{T^*} = \overline{T_w^*} \ \text{ at } Z = 0 \\ \overline{U^*} \to U_o, \ \overline{V^*} \to 0, \overline{T^*} \to \overline{T_\infty^*}, as\ Z \to \infty \end{array}\right\} \qquad (4)$$

Solving the electromagnetic force term $\mathbf{J} \times \mathbf{B}$ in equations (1) and (2)

$$(\mathbf{J} \times \mathbf{B})_{x+} = \frac{\sigma\mu^2{}_o H^2{}_0(mV^* - U^*)}{1+m^2} \qquad (5)$$

$$(\mathbf{J} \times \mathbf{B})_{y+} = \frac{-\sigma\mu^2{}_o H^2{}_0(mU^* + V^*)}{1+m^2} \qquad (6)$$

Substituting equations (5) and (6) into (1) and (2) respectively yields:

$$\frac{\partial U^*}{\partial t} = V\frac{\partial^2 U^*}{\partial z^{*2}} - \frac{\partial u^* w^*}{\partial z^*} + g\beta(T^* - T^*_\infty) + \frac{\sigma\mu^2{}_o H^2{}_0(mV^* - U^*)}{1+m^2} \quad (7)$$

$$\frac{\partial V^*}{\partial t} = V\frac{\partial^2 V^*}{\partial z^{*2}} - \frac{\partial v^* w^*}{\partial z^*} + g\beta(T^* - T^*_\infty) - \frac{\sigma\mu^2{}_o H^2{}_0(mU^* + V^*)}{1+m^2} \quad (8)$$

The equations (7) and (8) are the x and y components of the momentum equations respectively.

The following scaling variables are applied to non-dimensionalize equations (3), (7) and (8)

$$t = \frac{t^* U_0^2}{v};\ z = \frac{z^* U_o}{v};\ U = \frac{U^*}{U_o};\ V = \frac{V^*}{U_o}\ \theta = \frac{T^* - T_w^*}{T_w^* - T_\infty^*}; \quad (9)$$

Using the above scaling variables equations (3), (7) and (8) yields:

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial z^2} - \frac{\partial \overline{uw}}{\partial z} + Gr\theta + \frac{M^2(mV - U)}{1+m^2} \quad (10)$$

$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial z^2} - \frac{\partial \overline{vw}}{\partial z} - \frac{M^2(mU + V)}{1+m^2} \quad (11)$$

$$Pr\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial z^2} - Pr\left(\frac{\partial \overline{wT}}{\partial z}\right) \quad (12)$$

Boundary and initial conditions

$$\left.\begin{array}{l} t < 0, U = 0, V = 0, \theta = 0, \text{ everywhere} \\ t \geq 0, U = 0, V = 0, \theta = 1, \text{ at } z = 0 \\ U \to 1, V \to 0, , \theta \to 0, \text{ as } z \to \infty \end{array}\right\} \quad (13)$$

Using Prandtl Mixing length hypothesis and turbulent Prandtl number to resolve turbulent stress results in:

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial z^2} + 2k^2 z\left(\frac{\partial U}{\partial z}\right)^2 + 2k^2 z^2\left(\frac{\partial^2 U}{\partial z^2}\right)\left(\frac{\partial U}{\partial z}\right) + Gr\theta + \frac{M^2(mU + V)}{1+m^2} \quad (14)$$

$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial z^2} + 2k^2 z\left(\frac{\partial V}{\partial z}\right)^2 + 2k^2 z^2\left(\frac{\partial^2 V}{\partial z^2}\right)\left(\frac{\partial V}{\partial z}\right) - \frac{M^2(mV - U)}{1+m^2} \quad (15)$$

Given that the turbulent prandtl number is given by $Pr_t = \frac{\varepsilon_M}{\varepsilon_H}$

Where $\varepsilon_M = -2k^2 z^2 \frac{\partial \overline{u}}{\partial z}$ then: then $\overline{wT} = -\frac{2k^2 z^2 \varepsilon_H}{\varepsilon_M}\frac{\partial \overline{u}}{\partial z}\frac{\partial \theta}{\partial z}$, substituting this in equation (12) yields:

$$Pr\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial z^2} - Pr\left(\frac{2k^2 z^2}{Pr_t}\frac{\partial \overline{u}}{\partial z}\frac{\partial \theta}{\partial z}\right) \quad (16)$$

III. Finite Difference Scheme

The equivalent finite difference scheme for equations (14), (15) and (16) are respectively:

$$\frac{U(i,j+1) - U(i,j)}{\Delta t} = \frac{U(i+1,j) - 2U(i,j) + U(i-1,j)}{(\Delta z)^2} + 0.32i\Delta z\left(\frac{U(i+1,j) - U(i,j)}{\Delta z}\right)^2 + 0.32(i\Delta z)^2\left(\frac{U(i+1,j) - 2U(i,j) + U(i-1,j)}{(\Delta z)^2}\right)\left(\frac{U(i+1,j) - U(i,j)}{\Delta z}\right) + Gr\,\theta(i,j) + M^2\frac{mU(i,j) + V(i,j)}{1+m^2} \quad (17)$$

$$\frac{V(i,j+1) - V(i,j)}{\Delta t} = \frac{V(i+1,j) - 2V(i,j) + V(i-1,j)}{(\Delta z)^2} + 0.32i\Delta z\left(\frac{V(i+1,j) - V(i,j)}{\Delta z}\right)^2 + 0.32(i\Delta z)^2\left(\frac{V(i+1,j) - 2V(i,j) + V(i-1,j)}{(\Delta z)^2}\right)\left(\frac{V(i+1,j) - V(i,j)}{\Delta z}\right) + M^2\frac{mV(i,j) - U(i,j)}{1+m^2} \quad (18)$$

$$Pr\frac{\theta(i,j+1) - \theta(i,j)}{\Delta t} = \frac{\theta(i+1,j) - 2\theta(i,j) + \theta(i-1,j)}{(\Delta z)^2} + 0.32(i\Delta z)^2\frac{Pr}{Pr_t}\left\{\left(\frac{U(i+1,j) - U(i,j)}{\Delta z}\right)\left(\frac{\theta(i+1,j) - \theta(i,j)}{\Delta z}\right)\right\} \quad (19)$$

Where i and j refer to z and t respectively. The values for *k* have been substituted as 0.4 and z have been substituted with $i\Delta z$.

The boundary and initial conditions 13 now take the form:

$$\left.\begin{array}{l} U(i,j) = 0;\ V(i,j) = 0; \theta(i,j) = 0 \text{ everywhere for } \theta < 0 \\ \theta \geq 0; U(i,j) = 0;\ V(i,j) = 0; \theta(i,j) = 1 \text{ for } i = 0 \\ U(i,j) = 1;\ V(i,j) = 0; \theta(i,j) = 0 \text{ for } i = \infty \end{array}\right\} \quad (20)$$

Using the boundary and initial conditions (20) we compute values for consecutive grid points for primary and secondary velocities and temperature, that is $U(i,j+1)$; $V(i,j+1)$ and $\theta(i,j+1)$:

$U(i,j+1) =$
$U(i,j) +$

$$\Delta t\left\{\frac{U(i+1,j)-2U(i,j)+U(i-1,j)}{(\Delta z)^2}+0.32i\Delta z\left(\frac{U(i+1,j)-U(i,j)}{\Delta z}\right)^2+0.32(i\Delta z)^2\left(\frac{U(i+1,j)-2U(i,j)+U(i-1,j)}{(\Delta z)^2}\right)\left(\frac{U(i+1,j)-U(i,j)}{\Delta z}\right)+ Gr\,\theta(i,j)+M^2\frac{mU(i,j)+V(i,j)}{1+m^2}\right\} \quad (21)$$

$$V(i,j+1) = V(i,j)+ \Delta t\left\{\frac{V(i+1,j)-2V(i,j)+V(i-1,j)}{(\Delta z)^2}+0.32i\Delta z\left(\frac{V(i+1,j)-V(i,j)}{\Delta z}\right)^2+0s.32(i\Delta z)^2\left(\frac{V(i+1,j)-2V(i,j)+V(i-1,j)}{(\Delta z)^2}\right)\left(\frac{V(i+1,j)-V(i,j)}{\Delta z}\right)+\right.$$

$$M2mVi,j-Ui,j1+m2 \quad (22)$$

$$\theta(i,j+1) = \theta(i,j)+\frac{\Delta t}{Pr}\left[\frac{\theta(i+1,j)-2\,\theta(i,j)+\theta(i-1,j)}{(\Delta z)^2}+0.32(i\Delta z)^2\frac{Pr}{Pr_t}\left\{\left(\frac{U(i+1,j)-U(i,j)}{\Delta z}\right)^2\left(\frac{\theta(i+1,j)-\theta(i,j)}{\Delta z}\right)\right\}\right] \quad (23)$$

IV. DISCUSSION OF RESULTS

The numerical results obtained from the finite difference scheme are presented in figures 2, 3 and 4. The trends of various fluid flow parameters are discussed and explained as they were observed upon varying various non-dimensional parameters.
The negative values of Gr in this case means that the plate is at a lower temperature than its surrounding fluid hence absorbing heat from the fluid.

4.1 Primary velocity

From figure 2 it is observed that:

i) When there is decrease in Magnetic parameter results in increase in the primary velocity profile. The presence of magnetic field in an electrically conducting fluid introduces a force which acts against the flow if the magnetic field is applied hence the effect in the primary velocity.
ii) An increase in Hall parameter leads to increase in the primary velocity
iii) Finally an increase in Grashoff parameter decreases the primary velocity. The Grashoff number shows the relative effect of the buoyancy force to the viscous force in the boundary layer.

4.2 Secondary velocity

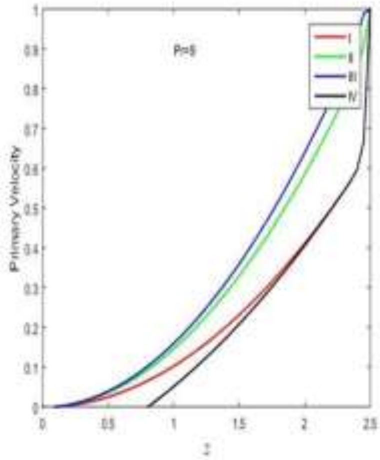
From figure 3 it is observed that:

i) For a decrease in Magnetic parameter there is a resulting increase in the secondary velocity. The presence of magnetic field in an electrically conducting fluid introduces a force which acts against the flow if the magnetic field is applied hence the change in the secondary velocity.
ii) When the Hall parameter is increased, there is a decrease in the secondary velocity.
iii) An increase in the Grashoff number doesnot affect the secondary velocity.

4.3 Temperature

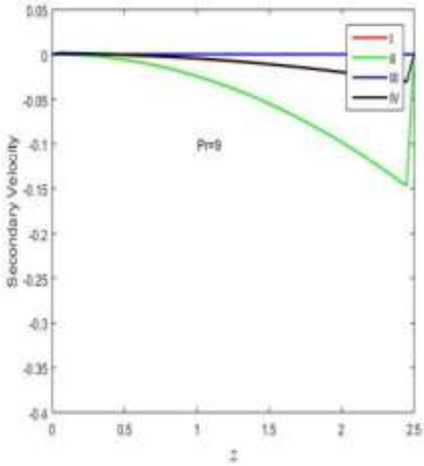
From figure 4 it is observed that:

i) Increase Hall parameter decreases the temperature profiles.
ii) Decrease in Prandtl number increases the temperature profiles. Physically, decrease in Prandtl number leads to an increase in thermal boundary layer and rise in the average temperature within boundary layer
iii) For a decrease in Magnetic parameter there is a resulting decrease in the temperature profiles.



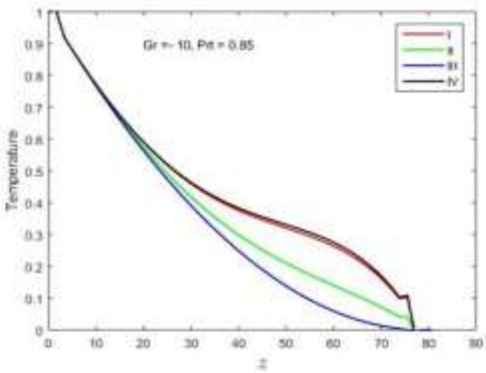
	M^2	m	Gr
I	50	0.1	-10
II	50	2	-10
III	5	0.1	-10
IV	50	0.1	-1000

Figure 2: Primary velocity profiles



	M^2	m	Gr
I	50	0.1	-10
II	50	2	-10
III	5	0.1	-10
IV	50	0.1	-1000

Figure 3: Secondary velocity



	M^2	Pr	M
I	50	9	1
II	50	9	2
III	5	9	1
IV	50	1	1

Figure 4: Temperature profiles

V. Conclusion

It is concluded that the velocity profiles increases with decreasing magnetic parameter (M) while the temperature profile is directly proportional to magnetic parameter (M). The primary velocity increases with increase in Hall parameter and also increases with increase in Grashoff number. On the other hand, increase in Hall parameter decreases both the secondary velocity and temperature profiles. It is also observed that an increase in Prandtl number leads to decrease in temperature profiles.

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