IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 17, Issue 5 Ser. I (Sep. – Oct. 2021), PP 01-07
www.iosrjournals.org

Divide Row and Subtract One Assignment Method for Solving Assignment Problem

Humayra Dil Afroz[1], Dr. Mohammad Anwar Hossen[2]
[1](Department of Mathematics,Comilla University,Bangladesh)
[2](Department of Mathematics,Comilla University,Bangladesh)

***Abstract*:**
Background: *Assignment problem is of great importance in mathematics and is also discuss in real physical world. In this paper we attempt to bring in a new effective method for solving assignment problem with algorithm and solution steps. We experiment a numerical example by using this method and enumerate by existing two methods. Moreover we assimilate the optimal solutions among this new method and two existing methods. The new proposed method is a systematic process, easy to apply for solving assignment problem.*
Result: *The optimum solution of proposed method is same as the optimum solutions of existing method which is 41.*
Conclusion: *New proposed method is different from two existing methods and is also effective for solving assignment problem.*
Keywords: *Assignment problem, Hungarian assignment method (HA-method), Matrix one's assignment method (MOA-method), Proposed method, Optimization.*

Date of Submission: 20-08-2021 Date of Acceptance: 05-09-2021

I. Introduction

The assignment problem is one of the main problems while assigning task to the worker. It is one of the fundamental combinatorial optimization problems in the branch of optimization or operation research in Mathematics. It is one of the special cases of transportation problems. This is particularly important in the theory of decision making. This problem finds numerous applications in production planning, telecommunication, VLSI design, economics etc. In a normal case of assignment problem where the objective is to assign the available resources to the activity going on so as to get the minimum cost or maximum total benefits of allocation.

In this paper we developed a solution method for assignment problem. The corresponding method has been formulated and numerical example has been considered to illustrate the method. Finally we compare the optimal solutions among new method and two existing methods.

II. Formulation Of Assignment Problem

Each assignment problem has a matrix associated with it. Generally the row contains the objects or people we wish to assign and the column comprise the jobs or tasks we want them assigned to. Consider a problem of assignment of n resources to m activities so as to minimize the overall cost or time in such a way that each resource can associate with one and only one job. The cost matrix (C_{ij}) is given as under:

Table 1.1 Approach of Assignment Problem

		Activity				
		A_1	A_2	.....	A_n	Available
Resource	R_1	C_{11}	C_{12}	.....	C_{1n}	1
	R_2	C_{21}	C_{22}	.....	C_{2n}	1
	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
	R_n	C_{n1}	C_{n2}	.....	C_{nn}	1
Required		1	1	.....	1	

Divide Row and Subtract One Assignment Method for Solving Assignment Problem

The cost matrix is same as that of a transportation problem except that availability at each of the resource and the requirement at each of the destinations is unity.

Let x_{ij} denote the assignment of i^{th} resource to j^{th} activity, such that

$$x_{ij} = \begin{cases} 1 \ ; & \text{if resource i is assigned to activity j} \\ 0 \ ; & \text{otherwise} \end{cases}$$

Then the mathematical formulation of the assignment problem is

Minimize $z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}\, x_{ij}$

Subject to the constraints

$\sum_{i=1}^{n} x_{ij} = 1$ and $\sum_{j=1}^{n} x_{ij} = 1 \ : \ x_{ij} = 0$ or 1

For all $i = 1,2, \ldots \ldots n$ and $j = 1,2, \ldots \ldots n$.

III. New Approach for Solving Assignment Problem

In this section we introduce a new approach for solving Assignment problem with the help of HA-method and MOA-method but different from them. This new method is easy process to solve Assignment problem. Also an example is solved by this method and the result is compared to HA-method and MOA-method.

Now we consider the assignment matrix where c_{ij} is the cost of assigning ith job to jth machine.

	1	2	3	...	n
1	C_{11}	C_{12}	C_{13}	.....	C_{1n}
2	C_{21}	C_{22}	C_{23}	.....	C_{2n}
.	.	.	.	.....	.
.	.	.	.	.....	.
.	.	.	.	.....	.
n	C_{n1}	C_{n2}	C_{n3}	.....	C_{nn}

Proposed Method: Divide Row and Subtract One Assignment Method

The proposed algorithm of proposed method is as follows:

Step 1: Find the smallest number (cost) of each column. Divide each row by smallest number of its column.

Step 2: Now find the smallest number (cost) of each row. Divide each row by its smallest number.

Step 3: Then subtract 1 from every number (except zero) and we get zero in every row.

Now make assignment in terms of zeros. If there are some rows and columns without assignment, then we cannot get the optimum solution. Then we go to the next step.

Step 4: Draw the minimum number of lines passing through all ones by using the following procedure:

I. Mark (√) rows that do not have assignments.
II. Mark (√) columns that have crossed zeros in that marked rows.
III. Mark (√) rows that have assignments in marked columns.
IV. Repeat (b) and (c) till no more rows or columns can be marked.
V. Draw straight lines through all unmarked rows and marked columns.

If the number of lines drawn is equal to the number of rows or columns, then the current solution is optimal solution. Otherwise go to next step.

Step 5: Select the smallest number of the reduced matrix not covered by the lines. Subtract this smallest number from all numbers not covered by a straight line and this same smallest number is added to every number including zeros lying at the intersection of any two lines. Other numbers covered by lines remain unchanged. Again make assignment in terms of zeros.

Step 6: If optimal solution is not found, then repeat steps (4) and (5) successively till an optimum solution is obtained.

IV. Comparison Of Existing Methods With Proposed Method

1. Solve the following assignment problem using Proposed Method.

The departmental head has four subordinates and four tasks to be performed. The subordinates differ in efficiency and tasks differ in their intrinsic difficulty. His estimates of times each man would take to perform. Each task is given below:

	Tasks			
Subordinates	1	2	3	4
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

How should the tasks be allocated to subordinates so as to minimize the total man-hours?

Solution:

Step 1: Select the minimum element of each column. Then divide each row by minimum element of its column.

Divide Row and Subtract One Assignment Method for Solving Assignment Problem

	1	2	3	4
A	1	3.25	2.13	1.38
B	0.68	1.47	0.21	1.37
C	9.5	4.75	4.5	3.75
D	1.9	2.6	2.4	1

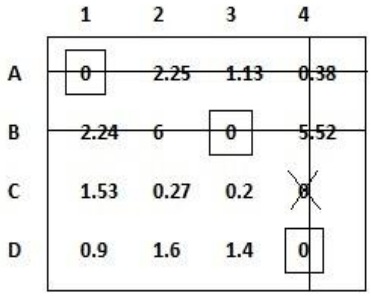
Step 2: Select the minimum element of each row. Then divide each row by its minimum element

	1	2	3	4
A	1	3.25	2.13	1.38
B	3.24	7	1	6.52
C	2.53	1.27	1.2	1
D	1.9	2.6	2.4	1

Step 3: Subtract 1 from all elements (except zeros).

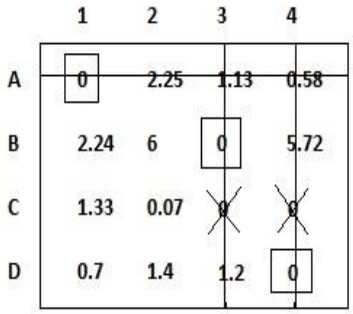
	1	2	3	4
A	0	2.25	1.13	0.38
B	2.24	6	0	5.52
C	1.53	0.27	0.2	0
D	0.9	1.6	1.4	0

Step 4: Then make initial assignment.

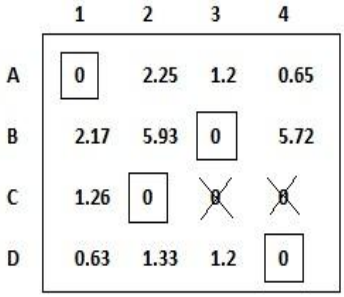


Divide Row and Subtract One Assignment Method for Solving Assignment Problem

Here, 2nd column and 3rd row do not have any assignment. Thus the solution is not optimum and we go to next.
Step 4: Select the smallest number of the reduced matrix not covered by the lines. Subtract this smallest number from all numbers not covered by a straight line and this same smallest number is added to every number including zeros lying at the intersection of any two lines. Other numbers covered by lines remain unchanged. Again make assignment in terms of zeros.



Here, 2nd column and 3rd row do not have any assignment again. Thus the solution is not optimum and we go to next.
Step 5: Repeat step (4) and the reduced matrix as follows.



Here we see that all zeros are either assigned or crossed out. That is, the total assigned zero's is 4 which is equal to the number of rows or columns. And the solution is (1,1), (2,3), (3,2), (4,4).
So the total minimum man-hours = 8 + 4 + 19 + 10 = 41.
Hence the departmental head should assign the task to his subordinates as given above and the total minimum man-hours required are 41.

Divide Row and Subtract One Assignment Method for Solving Assignment Problem
2. Solve the following assignment problem using Hungarian Assignment Method.
The departmental head has four subordinates and four tasks to be performed. The subordinates differ in efficiency and tasks differ in their intrinsic difficulty. His estimates of times each man would take to perform. Each task is given below:

	Tasks			
Subordinates	1	2	3	4
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

How should the tasks be allocated to subordinates so as to minimize the total man-hours?

Solution:

Step 1: Select the minimum element in each row and subtract this element from every element in that row.

	1	2	3	4
A	0	18	9	3
B	9	24	0	22
C	23	4	3	0
D	9	16	14	0

Step 2: Select the minimum element in each column and subtract this element from every element in that column.

	1	2	3	4
A	0	14	9	3
B	9	20	0	22
C	23	0	3	0
D	9	12	14	0

Step 3: Now make initial assignment.

Divide Row and Subtract One Assignment Method for Solving Assignment Problem

	1	2	3	4
A	[0]	14	9	3
B	9	20	[0]	22
C	23	[0]	3	~~0~~
D	9	12	14	[0]

Here we see that all zeros are either assigned or crossed out. That is, the total assigned zero's is 4 which is equal to the number of rows or columns. And the solution is (1,1), (2,3), (3,2), (4,4).

So the total minimum man-hours = 8 + 4 + 19 + 10 = 41.

Hence the departmental head should assign the task to his subordinates as given above and the total minimum man-hours required are 41.

3. Solve the following assignment problem using Matrix One's Assignment Method.
The departmental head has four subordinates and four tasks to be performed. The subordinates differ in efficiency and tasks differ in their intrinsic difficulty. His estimates of times each man would take to perform. Each task is given below:

	Tasks			
Subordinates	1	2	3	4
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

How should the tasks be allocated to subordinates so as to minimize the total man-hours?
Solution:
Step 1: Find the minimum cost of each row and then divide each element of each row of the matrix by its minimum cost.

Divide Row and Subtract One Assignment Method for Solving Assignment Problem

	1	2	3	4
A	1	3.25	2.13	1.38
B	3.25	7	1	6.5
C	2.53	1.27	1.2	1
D	1.9	2.6	2.4	1

Step 2: Find the minimum cost of each column and then divide each element of the column by its minimum cost.

	1	2	3	4
A	1	2.56	2.13	1.38
B	3.25	5.51	1	6.5
C	2.53	1	1.2	1
D	1.9	2.05	2.4	1

Step 3: Now make initial assignment.

	1	2	3	4
A	[1]	2.56	2.13	1.38
B	3.25	5.51	[1]	6.5
C	2.53	[1]	1.2	~~1~~
D	1.9	2.05	2.4	[1]

Hence we can assign the ones and the solution is (1,1), (2,3), (3,2), (4,4) and So the total minimum man-hours = 8 + 4 + 19 + 10 = 41.
Hence the departmental head should assign the task to his subordinates as given above and the total minimum man-hours required are 41.

Table 1.2: Comparison of Optimal Values of three Methods

Problem	HA-Method	MOA-Method	Proposed Method	Optimum
01	**41**	**41**	**41**	**41**

Therefore we conclude that this new Proposed Method is effective for solving assignment problem.

Divide Row and Subtract One Assignment Method for Solving Assignment Problem

V. Conclusion

In this paper, we developed a new method for solving Assignment problem. Initially, we explained the proposed algorithm and showed the efficiency of it by numerical example. And we get the optimal solution which is same as the optimal solutions of HA-method and MOA-method. Therefore this paper introduces a different approach which is easy to solve Assignment problem.

References

[1]. D.F. Votaw, 1952, A. Orden, The perssonel assignment problem, Symposium on Linear Inequalities and Programming, SCOOP 10, US Air Force, pp. 155-163.
[2]. H.W. Kuhn, 1955, The Hungarian method for the assignment problem, Naval Research Logistics Quarterly 2 (1&2) 83-97 (original publication).
[3]. M.S. Bazarra, John J. Jarvis, Hanif D. Sherali, 2005, Linear programming and network flows
[4]. B.S. Goel, S.K. Mittal, 1982, Operations Research, Fifty Ed., 2405-2416.
[5]. Hamdy A. Tsaha, 2007, Operations Research, an introduction, 8th Ed..
[6]. HadiBasirzadeh 2012, Applied Mathematical Sciences, 6(47), 2345-2355.
[7]. Singh S., Dubey G.C., Shrivastava R. (2012) IOSR Journal of Engineering, 2(8), 01-15.
[8]. Turkensteen M., Ghosh D., Boris Goldengorin, Gerard Sierksma, 2008 European Journal of Operational Research, 189, 775-788.
[9]. Zimmermman H.J. (1996) Fuzzy Set Theory and its Applications, 3rd ed., Kluwer Academic, Boston.
[10]. Pentico D.W. (2007) European Journal of Operation Research, 176, 774-793.
[11]. Singh S. (2011) International Journal of Operation Research and Information System, 3(3), 87-97.
[12]. AnshumanSahu, RudrajitTapador, 2007, Solving the assignment problem using Genetic Algorithm And Simulated annealing, IJAM.
[13]. Shayle R. Searle, 2006, Matrix algebra useful for statistics, John Willey.

Humayra Dil Afroz. "Divide Row and Subtract One Assignment Method for Solving Assignment Problem." *IOSR Journal of Mathematics (IOSR-JM)*, 17(5), (2021): pp. 01-07.