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Analytic Solution of Helix Curve Equation

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Abstract:
in the classical three dimoensional curve theroy,the geometry of a curve essentially characterized by two scalar functions,curvature κ and torsion τ,[1] which reperesent the rate of change of the tangent vector and the osculating plane respectively.given two continous function of one parmeter, there is space curve for which the two functions are its curvature and torsion ,which represent the rate of change of the tangent vector and the osculating plane ,respectively .Given two continous functions of one parmeter, there is a space curve for which the two functions are its curvature and torsion(parameterized by arc-length). The curve [2]

$$c(t) = (x(t), y(t), z(t)) \tag{0.0.1}$$

in 3-space, we define its velocity and acceleration by [2]

$$v(t) = \dot{c}(t) \tag{0.0.2}$$

and

$$a(t) = v(t) = \ddot{c}(t) \tag{0.0.3}$$

the speed citeCho81

$$\dot{s}(t) = ||\mathbf{v}(\mathbf{t})\ || \tag{0.0.4}$$

and arclength by

$$s = \int_0^{t_0} \dot{s}(t)\mathrm{d}x \tag{0.0.5}$$

unit tangent vector T(s):

: the unit tangent vector to the curve as [3]

$$T(t) = \frac{v(t)}{||\mathbf{v(t)}\ ||} \tag{0.0.6}$$

curvature κ : the curvatureκ of the curve and the normal $N(s)$ by [3]

$$\kappa(s) = ||\frac{\mathbf{dT}}{\mathbf{ds}}\ || \tag{0.0.7}$$

and

$$\frac{dT}{ds} = \kappa(s)N(s) \tag{0.0.8}$$

binormal B(s) the binormal B(s) by

$$B(s) = T(s) \times N(s) \tag{0.0.9}$$

and

$$T(s) = N(s) \times B(s) \tag{0.0.10}$$

the torsion τ : define the torsion by [?]

$$\frac{dB}{ds} = -\tau N(s) \tag{0.0.11}$$

with

$$N(s) = B(s) \times T(s) \tag{0.0.12}$$

and

$$\frac{dN}{ds} = \frac{dB}{ds} \times T + B \times \frac{dT}{ds} = \tau N \times T + \kappa B \times N \tag{0.0.13}$$

or

$$\frac{dN}{ds} = -\kappa(s)T(s) + \tau(s)B(s) \tag{0.0.14}$$

0.0.1 helix curve

$$c(t) = (a\cos t, a\sin t, bt) \tag{0.0.15}$$

then

$$v(t) = (-a\sin t, a\cos t, b) \tag{0.0.16}$$

and

$$||\mathbf{v(t)}\ || = \sqrt{a^2+b^2} \tag{0.0.17}$$

and

$$S = t\sqrt{a^2+b^2} \tag{0.0.18}$$

$$T(s) = \frac{1}{\sqrt{a^2+b^2}}(-a\sin t, a\cos t, b) \tag{0.0.19}$$

hence and [4]

$$\kappa(s) = \frac{a}{a^2+b^2} \tag{0.0.20}$$

,

$$N(s) = (-\cos t, -\sin t, 0) \tag{0.0.21}$$

'

$$B(s) = (\frac{1}{\sqrt{a^2+b^2}})(b\sin t, -b\cos t, a) \tag{0.0.22}$$

'

$$\frac{dB}{ds} = (\frac{1}{a^2+b^2})(b\cos t, b\sin t, 0) \tag{0.0.23}$$

'

$$\tau(s) = \frac{b}{a^2+b^2} \tag{0.0.24}$$

' let

$$F(s) = (T(s), N(s), B(s)) \tag{0.0.25}$$

and

$$\dot{F}(s) = F(s)A(s) \tag{0.0.26}$$

A(s)=

$$\begin{bmatrix} 0 & -\kappa(s) & 0 \\ \kappa(s) & 0 & -\tau(s) \\ 0 & \tau(s) & 0 \end{bmatrix}$$

with solution

$$F(s) = F_0 e^{(s-s_0)A} \tag{0.0.27}$$

the curvature of helix is

$$\kappa(s) = \frac{a}{a^2+b^2} \tag{0.0.28}$$

and the torsion of helix is

$$\tau(s) = \frac{b}{a^2+b^2} \tag{0.0.29}$$

and hence

$$a = \frac{\kappa}{\kappa^2+\tau^2} \tag{0.0.30}$$

$$b = \frac{\tau}{\kappa^2+\tau^2} \tag{0.0.31}$$

then the curve of helix

$$c(t) = (a\cos t, a\sin t, bt) \tag{0.0.32}$$

becomes

$$c(t) = (\frac{\kappa}{\kappa^2+\tau^2}\cos t, \frac{\kappa}{\kappa^2+\tau^2}\sin t, \frac{\tau}{\kappa^2+\tau^2}t) \tag{0.0.33}$$

Conculation

we investigate Bertrand curves corresponding to the spherical images of the tangent, binormal,[4] principal normal and Darboux indicatrices of a space curve in Euclidean 3-space. As a result, [1] in case of a space curve is a general helix, we show that the curves corresponding to the spherical images of its the tangent indicatrix and binormal indicatrix are both Bertrand curves and circular helices. [5]Similarly, in case of a space curve is a slant helix, we demonstrate that the curve corresponding to the spherical image of its the principal normal indicatrix is both a Bertrand curve and a circular helix.

Bibliography

[1]. Helix to helix packing in proteins Chothia, Cyrus and Levitt, Michael andRichardson, Douglas 1981, Elsevier

[2]. intuitive helix geometry analysis and abstraction Dahl, Anna Caroline E andChavent, Matthieu and Sansom, Mark SP 2012, Oxford University Press

[3]. Optimisation of variable helix tool geometry for regenerative chatter mitigation,

[4]. Yusoff, Ahmad R and Sims, Neil D, 2011, Elsevier intuitive helix geometry analysis and abstraction, Dahl, Anna Caroline E and Chavent, Matthieu and Sansom, Mark SP, year=2012, publisher=Oxford Uni-versity Press

[5]. Helix geometry in proteins, Barlow, DJ and Thornton, JM,year=1988, publisher=Elsevier