IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 17, Issue 6 Ser. II (Nov. – Dec. 2021), PP 16-34
www.iosrjournals.org

Heat and Mass Transfer in Flow of Copper Nanofluid Containing Different Shapes of Nanoparticles in a Porous Channel Part 1

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Abstract

An analytical study of energy transfer in flow of four different shapes of copper nanoparticles in ethylene glycol and water based fluids through a porous channel was carried out. The governing models which are Navier-Stokes, energy, concentration and continuity equations were non-dimensionalized and solutions obtained using perturbation method. Analysis of graphical and numerical results showed that increase in angular frequency, Schmidt term and chemical reaction, enhanced the concentration profile of the fluid while increase in radiation term, increases the temperature profile of the copper nanofluid but decreases the velocity profile of the fluid. Increase in the heat absorption term and chemical reaction term all depreciates the temperature and velocity profiles of the fluid.

Date of Submission: 10-12-2021 Date of Acceptance: 24-12-2021

I. Introduction

Energy transfer involves both heat transfer and mass transfer. Energy transfer in form of heat is the transport of heat from one point to another. Similarly, energy transfer in form of mass is the transport of mass from one point to another. To enhance the heat transfer of a fluid, changing the flow geometry, boundary conditions or thermal conductivity is essential. Various techniques have been proposed to enhance the heat transfer performance of fluids. Researchers have tried to increase the thermal conductivity of base fluids by suspending micro or larger sized solid particles in fluids. Fluids with nanoparticles suspended in them are nanofluids (Choi, 1995). This nanofluids according to him significantly increases the thermal conductivity of the base fluids as well as their convective heat transfer rate. Nanoparticles are mainly metals, oxides and carbides while the common base fluids are water, ethylene glycol propylene glycol, kerosene oil and many more. Owing to the improved thermal properties, nanofluids are needed for utilization in heat exchangers, thermal media and energy systems to improve the heat transfer rate and modify the thermal management of devices with high heat flux (Ramezanizadeh et al, 2019).

The structure of solid particles and fraction of nanostructures are some of the contributory factors of nanofluids in heat transfer modification. However, some of the main parameters with an impact on the features of nanofluids include but not limited to type of solid phase, pH, temperature, base fluids, synthesis procedure and size of nanostructures. Nanofluid has become a topic of attraction due to its environmentally friendly nature, and extraordinary heat transfer performance in various areas including cooling, power generation, defense nuclear, space, microelectronics and biomedical appliances. In several practical applications, mainly in industries, the primary requirement is heat transfer from source to sink. Improving heat transfer efficiency is vital in telecommunication systems. A rapid and sustained heat removal rate is required in electronic systems optical devices, x-rays and laser application (Das et al., 2008).

Several other definitions and explanations are abounded in literatures such as klemstrever and Fan (2011), that described nanofluids as a new class of heat transfer fluids by dispersing nanometer-size particles, with typical diameter scales of 1 to 100nm in traditional heat transfer fluids. Wang and Mujundar (2008), considered nanofluid to be the next generation heat transfer fluids because they offer exciting new possibilities to enhance heat transfer performance compared to pure liquids. Due to the poor heat transfer abilities of conventional base fluids and with global need for improvement, to develop advanced heat transfer fluids with significantly higher thermal conductivity than those presently available, there is need for nanofluid.

In today's science and technology, size does matter, therefore modern fabrication technology provides great opportunity to actively process materials at micro and nanometer scales (Ngiangia and Akaezue 2019). According to Mukherjee and Paria (2013), nanofluids possess some vital features for scientific and engineering applications than conventional base fluids. They include, rise in thermal conductivity beyond exception and much higher than theoretical prediction, ultrafast heat transfer ability, better stability than other colloids, reduction of corrosion and clogging in micro channels, reduction in pumping power, reduce friction coefficient and better lubrication. Nanofluids preparation methods require advanced and sophisticated equipments. This leads to higher production cost of nanofluids, but its characteristics outlined cannot be over emphasized. The choice of non-spherical shape nanoparticle in this research is predicated on the work of Aaiza et al (2015), where they mentioned desirable properties in cancer treatment. Also the choice of copper nanoparticle with ethylene glycol and water as based fluids to form copper nanofluid is because it possesses higher thermal conductivity and stability than other nanofluids.

The quality of nanofluid not only depends on the type of nanoparticles but also their shapes and temperature. To incorporate temperature into the models of thermal conductivity, require the Brownian motion of the nanoparticles but the works of keblinski et al. (2002) and Jang and Choi (2004), postulated that, the effect of Brownian motion can be ignored, since the contribution of thermal diffusion is much greater than Brownian diffusion. Although, Wang et al (1999), argued that the thermal conductivities of nanofluids should be dependent on the Brownian motion and inter-particle forces. As a result of technological development and improved methods of practice in almost every industry, the use of nanoscience and nano technology is fast gaining momentum. In 1974 Norio Tanigudi first used the term nanotechnology, since then, nanofluid has grasp the attention of many scholars around the world and by the year 2006, more than one thousand research papers where the term nanofluid was used has been published. In a study of nanofluids for magnetohydrodynamic (MHD),Mansur et al (2015), obtained results for embedded parameters and considered stretching and shrinking cases. Timofeera et al (2009), conducted theoretical and experimental study of alumina of various nanoparticles in a base fluid mixture of ethylene glycol and water of equal volumes. Loganathan et al (2013), carried out a study where they analyzed radiation effects and concluded that spherical shape nanofluids velocity is less than copper. Study of heat transfer of alumina water nanofluid in mixed convection flow inside a square cavity was done by Sebdani et al (2012). Xuan and Li (2000), published an article on heat transfer enhancement of nanofluids and introduced the theoretical study of the thermal conductivity of nanofluids. Kleinstreuer and Feng (2011), carried out an experimental and theoretical studies of nanofluid and catalogued several classical models for effective thermal conductivity of mixtures. They went further to compare dynamic models with experimental data. The integral transform technique was employed by Hajizadeh et al (2019) in the study of free convection flow of nanofluids between two vertical plates with damped thermal flux and used graphical illustrations to present their findings. Souayeh et al (2019), scrutinized the consequences of non-linear radiation on MHD Casson nanofluid along the thin needle and reported a comparison of fixed needle and moving needle was made and illustrated through graphs. According to the work of Makinde and Mutuku (2014), Runge-Kutta-Fehiberg method with shooting technique gives inspiring results on the interaction between the electrical conductivity of the conventional base fluid and that of the nanoparticles under the influence of magnetic field in a boundary layer flow with heat transfer. Rashidi et al (2014), examined the effect of buoyancy on MHD flow of nanofluid over a stretching sheet in the presence of thermal radiation and magnetic field. The influence of visions dissipation and chemical reaction on MHD flow of nanofluids through a porous medium was examined by Eshetu and Shankar (2014).

Abdul-Hakeem et al (2015), published the results of their work on the effect of magnetic field on second order slip flow of nanofluid with thermal radiation.

A very useful and interesting result was obtained by Sheremet et al (2015) on the investigation of unsteady free convection heat transfer characteristics of a nanofluid confined within a porous open wavy cavity. Gunnasegaran et al (2012), presented quantitative results of the heat transfer enhancement of compact heat exchanger with increasing volumetric concentrations of nanofluids at various Reynolds numbers regime. Alotaibi et al (2020), in modeling thermal conductivity of ethylene glycol-based nanofluids, they opined that augmenting the thermal conductivity of fluids make them more favourable for thermal applications. Owing to the vital role played by thermal conductivity in the heat transfer ability of nanofluid, some studies have focused on this property. In a work carried out by Izadkhah et al. (2019), they observed that the existence of the nanosheets in the base fluid at a given concentration, led to augmentation in the thermal conductivity. It has also been argued by Kleinstruer and Feng (2011) that of all the physical properties of nanofluids, the thermal conductivity is the most complex and for many applications the most important one. However, findings have been controversial and theories are yet to fully explain the mechanisms of elevated thermal conductivity. To be specific, several articles examined thermal conductivity enhancement of nanofluids. They include but not limited to Lee et al (1999), Hemmat et al(2014), Gao et al (2019), Li et al (2015), and Liu et al (2006). Others discussed thermo-physical properties of nanofluids. They are Michael et al (2019), Omrani et al (2019), Ramezanizadeh and Nazari (2019) and Ramezanizadeh et al (2019). The application of nanofluids in thermosyphons was also extensively discussed in the work of Ramezanizadeh et al (2018). Warrier and Teja (2011) and Zyla (2017) also studied the viscosity of nanofluids but as well added thermal conductivity. A study of the shapes effects of nanosize particles in copper nanofluid on entropy generation was reported by Ellahi et al (2015). The study critically examined different shapes of nanoparticles as it affects the viscosity of the nanofluid. A similar study by Ellahi et al (2014), examined the elliptic inner cylindrical geometry in a nanofluid filled enclosure. Sheikholeslami et al (2014), considered heat transfer in flow of nanofluids over a permeable stretching wall in a porous medium. A group of scientist led by Sheikholeslamic et al (2014) but different co authors from the former, carried out a simulation of copper(II) oxide nanofluid flow and convective heat transfer considering lorentz force. Ellahi (2013), examined temperature depended viscosity on flow of MHD non-Newtonian nanofluid in a pipe. In another study by Ellahi et al. (2013), non-Newtonian nanofluids flow through a porous medium between coaxial cylinders with heat transfer and variable viscosity was examined and the results shed light, the effect of permeability and variable viscosity. Three dimensional mesoscopic simulation study of the effect of magnetic field on natural convection of nanofluid was examined by Sheikholeslami and Ellahi (2015) and far reaching deductions were made. Ellahi et al (2015), studied MHD nanofluid by means of single and multi-walled carbon nanotubes suspended in a salt water solution and used graphs to explain their findings. James et al (2015), tackled nanofluid properties provoked by the effect of chemical reaction and thermal radiation in a porous medium and deduced that radiation is an important parameter in the description of temperature profile of nanofluid. The magnetic field effect on a steady two-dimensional laminar radiative flow of an incompressible viscous water based nanofluid over a stretching/shrinking sheet with second order slip boundary condition was investigated by Abdul Hakeem et al.(2015) using Lie symmetry group transformations and both analytical and numerical methods of solution. They concluded that unique exact solution exists for momentum equation in stretching sheet case and dual solutions are obtained for shrinking sheet case which has upper and lower branches.This work incorporate four different shapes of copper nanoparticles namely cylinder, platelet, blade and brick. These different shaped nanoparticles shall be investigated analytically through an even porous channel.

1.1 Mathematical Formulation of the Physical Problem

An unsteady two dimensional boundary layer flow of viscous, oscillatory, incompressible, radiating nanofluid along an infinite flat channel is considered. The $x' - axis$ is taken along the vertical infinite channel in the upward direction and the $y' - axis$ normal to the channel. Using the Boussineq's approximation, the partial differential equations of the nanofluid flow are given as

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \qquad \textbf{(1)}$$

$$\frac{\partial u'}{\partial t'} + \left(u' \frac{\partial u'}{\partial x'} + v' \frac{\partial u'}{\partial y'} \right) = - \frac{1}{\rho_{nf}} \left(\frac{\partial p}{\partial x'} + \frac{\partial p}{\partial y'} \right) + \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 u'}{\partial x'^2} + \frac{\partial^2 u'}{\partial y'^2} \right) - \frac{\vartheta}{\rho_{nf} k} u'$$
$$- \frac{\sigma B_0^2}{\rho_{nf}} u' + g \beta_{nf} (T - T_0) + g \beta'_{nf} (C' - C_0) \qquad \textbf{(2)}$$

$$\frac{\partial T}{\partial t'} + \left(u' \frac{\partial T}{\partial x'} + v' \frac{\partial T}{\partial y'} \right) = \frac{k_{nf}}{\left(\rho C_p\right)_{nf}} \left(\frac{\partial^2 T}{\partial x'^2} + \frac{\partial^2 T}{\partial y'^2} \right) - \frac{1}{\left(\rho C_p\right)_{nf}} \left(\frac{\partial q_r}{\partial x'} + \frac{\partial q_r}{\partial y'} \right) + \frac{Q'}{\left(\rho C_p\right)_{nf}} \left(T - T_0\right) \qquad \textbf{(3)}$$

$$\frac{\partial C'}{\partial t'} + \left(u' \frac{\partial C'}{\partial x'} + v' \frac{\partial C'}{\partial y'} \right) = \frac{D_{nf}}{\left(\rho C_p\right)_{nf}} \left(\frac{\partial^2 C'}{\partial x'^2} + \frac{\partial^2 C'}{\partial y'^2} \right) - \frac{k_r^2}{\left(\rho C_p\right)_{nf}} \left(C' - C_0\right) \qquad \textbf{(4)}$$

Subject to the boundary conditions to flow inside the channel with stationary walls, Aaiza (2015)

$$u'(0, t') = u'(1, t') = 0, T(0, t') = 0, T(1, t') = 1, C'(0, t') = 0, C'(1, t') = 1$$

Since the motion of the copper nanofluid is two dimensional and the length of the channel is very large compared to the width of the channel, all physical variables are independent of the coordinate (Aruna et al 2015). Therefore,

$$\frac{\partial v'}{\partial y'} = 0 \qquad \textbf{(5)}$$

$$\frac{\partial u'}{\partial t'}+\left(v'\frac{\partial u'}{\partial y'}\right)=-\frac{1}{\rho_{nf}}\left(\frac{\partial p}{\partial x'}\right)+\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u'}{\partial y'^2}\right)-\frac{\vartheta}{\rho_{nf}k}u'-\frac{\sigma B_0^2}{\rho_{nf}}u'+g\beta_{nf}\left(T-T_0\right)+g\beta*_{nf}\left(C'-C_0\right)$$

(6)

$$\frac{\partial T}{\partial t'}+\left(v'\frac{\partial T}{\partial y'}\right)=\frac{k_{nf}}{\left(\rho C_p\right)_{nf}}\left(\frac{\partial^2 T}{\partial y'^2}\right)-\frac{1}{\left(\rho C_p\right)_{nf}}\left(\frac{\partial q_r}{\partial y'}\right)+\frac{Q'}{\left(\rho C_p\right)_{nf}}\left(T-T_0\right) \quad \textbf{(7)}$$

$$\frac{\partial C'}{\partial t'}+\left(v'\frac{\partial C'}{\partial y'}\right)=\frac{D_{nf}}{\left(\rho C_p\right)_{nf}}\left(\frac{\partial^2 C'}{\partial y'^2}\right)-\frac{k_r^2}{\left(\rho C_p\right)_{nf}}\left(C'-C_0\right) \quad \textbf{(8)}$$

where u' and v' are velocities in x' and y' directions respectively, t' is time, C' is nanofluid concentration, p is pressure, ρ_{nf} is density of nanofluid, μ_{nf} is dynamic viscosity of nanofluid, σ is electrical conductivity of base fluid, B_0^2 is imposed magnetic induction, g is acceleration due to gravity, β_{nf} is thermal expansion due to temperature, β^*_{nf} is thermal expansion due to concentration, T is temperature of nanofluid, T_0 is free stream temperature, C_0 is free stream concentration, k_{nf} is thermal conductivity of nanofluid, $\left(C_p\right)_{nf}$ is specific heat at constant pressure, q_r is radiation term, k_r^2 is chemical reaction term, D is chemical molecular diffusivity of nanofluid, Q' is heat absorption term.

Several classical models for dynamic viscosity and effective thermal conductivity have been proposed in the work of Kleinstreuer and Feng (2011) for various spherical and non-spherical nanoparticles but that of Hamilton and Crosser (1962) is most suited due to its validity for both spherical and non-spherical shapes nanoparticles and is defined as

$$\mu_{nf}=\mu\left(1+a\phi+b\phi^2\right) \quad (9)$$

$$\frac{k_{nf}}{k_f}=\frac{k_s+(n-1)k_f+(n-1)\left(k_s-k_f\right)\phi}{k_s+(n-1)k_f-\left(k_s-k_f\right)\phi} \quad (10)$$

The **n** seen in equation (10) is the empirical shape factor given by $n=\frac{3}{\psi}$, where ψ is the sphericity. The values of ψ for different shape particles are shown on **Table 2**.

According to the work of Tiwari and Das (2007) and Asma et al (2015), density of nanofluid (ρ_{nf}), thermal expansion due to temperature of nanofluid (β_{nf}), thermal expansion due to concentration of nanofluid (β^*_{nf}), specific heat at constant pressure of nanofluid $\left(C_p\right)_{nf}$ are respectively

$$\begin{aligned}
\rho_{nf}&=(1-\phi)\rho_f+\phi\rho_s\\
\beta_{nf}&=(1-\phi)\beta_f+\phi\beta_s\\
\beta'_{nf}&=(1-\phi)\beta'_f+\phi\beta'_s\\
\left(C_p\right)_{nf}&=(1-\phi)\left(C_p\right)_f+\phi\left(C_p\right)_s
\end{aligned} \quad (8)$$

where ϕ is the nanoparticles volume fractions, ρ_f and ρ_s are the densities of the base fluid and solid nanoparticles, β_f and β_s are the thermal expansion due to temperature of base fluid and solid nanoparticles, β'_f and β'_s are the thermal expansion due to concentration of base fluid and solid nanoparticles and $\left(C_p\right)_f$ and $\left(C_p\right)_s$ are the specific heat at constant pressure due to base fluid and solid nanoparticles. **a** and **b** are constants that depend on the particle shape. The thermo physical properties of copper nanoparticles , Ethylene glycol and water H_2O as base fluids are presented in table 1.

Table 1: Thermo-physical properties of Ethylene glycol, Water and Copper nanoparticles **(**Aaiza 2015)

Property	Ethylene glycol	water	Copper
Specific heat (J/kgK)	0.58	4179	765
Density (kg/m^3)	1.115	997.1	3970
Thermal conductivity (w/mk)	0.149	0.6	40
Viscosity (m^2/s)	0.001095	0.00089	0.00046
Volumetric thermal expansion ($x10^{-5}k^{-1}$)	6.5	21	1.67

Table 2: Sphericity ψ and empirical shape factor for different shapes nanoparticles (Aaiza 2015)

Model	Platelet	Blade	Cylinder	Brick
ψ	0.52	0.36	0.62	0.81
n	5.76923	8.33333	4.83871	3.70370

Table 3: Constants **a** and **b** empirical shape factors (Aaiza 2015)

Model	Platelet	Blade	Cylinder	Brick
a	37.1	14.6	13.5	1.9
b	612.6	123.3	904.4	471.4

The integral of equation (5) is performed and the suction velocity takes the form

$$v' = -v_0\left(1 + \varepsilon A e^{n't'}\right) \quad \textbf{(15)}$$

where v_0 is characteristic velocity of the channel wall ε and A are constants.

To consider the effect of radiation on an optically thick model in which the thermal layer becomes very thick or highly absorbing as described by Rosseland approximation, Cogley et al (1968) stated it as

$$\frac{\partial q_r}{\partial y'} = -\frac{4\zeta}{3\alpha}\frac{\partial T^4}{\partial y'} \qquad (16)$$

where ζ is the Stefan-Boltzmann constant and α is the absorption coefficient. If temperature difference within the flow of the nanofluid is sufficiently small, we can approximate T^4 using Taylor series expansion about the point 0 and obtain

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 \qquad (17)$$

Makinde and Mhone (2005), also stated that both plates temperature of the porous channel be assumed high enough and produces the radiative heat transfer.

1.2 Dimensional Analysis

The Buckinham-π $-$ theorem is used and the various dimensionless parameters are stated as

$$u = \frac{u't'}{x'}, y = \frac{y'}{x'}, t = \frac{t'u'}{y'}, \mathrm{Re} = \frac{\mu_{nf} C'}{v_0 \rho_{nf}}, \theta = \frac{T - T_0}{T_0}, C = \frac{C' - C_0}{C_0}, \mathrm{Pr} = \frac{v'(\rho C_p)_{nf}}{k_{nf}}, \frac{\partial p}{\partial x'} = -pe^{i\omega t},$$

$$Sc = \frac{\mu_{nf}}{D}, k_0 = \frac{k_r^2 \mu_{nf}}{v'^2}, M = \frac{\sigma B_0^2 \mu_{nf}}{\rho_{nf} v'^2}, N = \frac{16 \zeta T_\infty^3}{3\alpha (C_p)_{nf}} Gt = \frac{g\beta_{nf} (T - T_0)\mu_{nf}}{v_0 u'^2}, Gc = \frac{g\beta_{nf}^* (C' - C_0)\mu_{nf}}{v_0 u'^2}$$

$$, Q = \frac{\mu_{nf}}{(\rho C_p)_{nf} u'^2}, \chi = \frac{\vartheta}{\rho_{nf} k u'^2}$$

Using the dimensionless variables and equation (15), equations (6) - (8) can be written as

$$\frac{\partial u}{\partial t} - (1 + \varepsilon A e^{nt})\frac{\partial u}{\partial y} = -pe^{i\omega t} + a_1 \mathrm{Re}^{-1} \frac{\partial^2 u}{\partial y^2} - u(\chi + M)a_4 + a_2 Gt\,\theta + a_3 GcC \tag{18}$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{nt})\frac{\partial \theta}{\partial y} = \left(\frac{a_5}{\mathrm{Pr}}\right)\frac{\partial^2 \theta}{\partial y^2} - a_6 \theta (N - Q) \tag{19}$$

$$\frac{\partial C}{\partial t} - (1 + \varepsilon A e^{nt})\frac{\partial C}{\partial y} = \frac{a_6}{Sc}\frac{\partial^2 C}{\partial y^2} - a_6 k_0 C \tag{20}$$

$$a_1 = \frac{1 + a\phi + b\phi^2}{1 - \phi + \phi \dfrac{\rho_s}{\rho_f}},\ a_2 = 1 - \phi + \phi \frac{\beta_s}{\beta_f},\ a_3 = 1 - \phi + \phi \frac{\beta_s'}{\beta_f'},\ a_4 = \left(1 - \phi + \phi \frac{\rho_s}{\rho_f}\right)^{-1}$$

$$a_5 = \left(\frac{k_s + (n-1)k_f + (n-1)(k_s - k_f)\phi}{k_s + (n-1)k_f - (k_s - k_f)\phi}\right) / \left(1 - \phi + \phi \frac{(\rho C_p)_s}{(\rho C_p)_f}\right) a_6 = \left(1 - \phi + \phi \frac{(\rho C_p)_s}{(\rho C_p)_f}\right)^{-1}$$

The boundary conditions also transform into

$$u(0,t) = u(1,t) = 0, \theta(0,t) = 0, \theta(1,t) = 1, C(0,t) = 0, C(1,t) = 1$$

where p is dimensionless pressure, χ is dimensionless porosity term, Gt is Grashof number, Gc is modified Grashof number, Re is Reynolds number, Pr is Prandtl number, N is dimensionless radiation term, Q is dimensionless absorption term, Sc is Schmidt term, k_0 is dimensionless chemical reaction term, u, θ, C,t are respectively dimensionless velocity, temperature, concentration of copper nanofluid and time.

1.3 Method of Solution

Following the methods adopted by Aaiza et al (2015), Ngiangia and Akaezue (2019) and Israel-Cookey et al (2003), a regular perturbation of the form

$$u(y,t) = u_0(y) + u_1(y)e^{i\omega t} \tag{21}$$

$$\theta(y,t) = \theta_0(y) + \theta_1(y)e^{i\omega t} \tag{22}$$

$$C(y,t) = C_0(y) + C_1(y)e^{i\omega t} \tag{23}$$

where ω is a dimensionless free stream frequency of oscillation. Using equations (21)-(23), equations (18)-(20), transform into

$$a_1 u_0''(y) + \mathrm{Re}\,\alpha^2 u_0'(y) - a_4 \mathrm{Re}\,(\chi + M)u_0(y) + a_2 Gt\,\theta_0(y) + a_3 GcC_0 = 0 \tag{24}$$

$$a_1 u_1''(y) + \mathrm{Re}\,\alpha^2 u_1'(y) - a_4 \mathrm{Re}\,(\chi + M + i\omega)u_1(y) + a_2 Gt\,\theta_1(y) + a_3 GcC_1 = p \tag{25}$$

$$a_5 \theta_0''(y) + \mathrm{Pr}\,\alpha^2 \theta_0'(y) - a_6 \mathrm{Pr}\,(N - Q)\theta_0(y) = 0 \tag{26}$$

$$a_5 \theta_1''(y) + \mathrm{Pr}\,\alpha^2 \theta_1'(y) - a_6 \mathrm{Pr}\,(i\omega + N - Q)\theta_1(y) = 0 \tag{27}$$

$$a_6 C_0''(y) + Sc\,\alpha^2 C_0'(y) - a_6 Sck_0 C_0(y) = 0 \tag{28}$$

$$a_6 C_1''(y) + Sc\,\alpha^2 C_1'(y) - a_6 Sc\,(k_0 + i\omega)C_1(y) = 0 \tag{29}$$

Also the boundary conditions take the form

$u_o(0) = u_0(1) = 0, u_1(0) = u_1(y) = 0$

$\theta_0(0) = 0, \theta_0(1) = 1, \theta_1(0) = 0, \theta_1(1) = 1$

$C_0(0) = 0, C_0(1) = 1, C_1(0) = 0, C_1(1) = 1$

where $\alpha^2 = (1 + \varepsilon A e^{nt})$ and prime indicates derivative with respect to y.
The solution of equations (26)-(29), using the method of undetermined coefficients and imposing the boundary conditions, the results are stated as

$$C_1(y) = \frac{e^{m_1 y} - e^{m_2 y}}{e^{m_1} - e^{m_2}} \tag{30}$$

$$C_0(y) = \frac{e^{m_3 y} - e^{m_4 y}}{e^{m_3} - e^{m_4}} \tag{31}$$

$$\theta_1(y) = \frac{e^{m_5 y} - e^{m_6 y}}{e^{m_5} - e^{m_6}} \tag{32}$$

$$\theta_0(y) = \frac{e^{m_7 y} - e^{m_8 y}}{e^{m_7} - e^{m_8}} \tag{33}$$

Similarly, equations (30)-(31) and equations (32)-(33) are respectively substituted into equations (25) and (24). The resulting solutions after imposing the boundary conditions are

$$u_1(y) = \beta_3 e^{m_9 y} + \beta_4 e^{m_{10} y} + A_1 e^{m_5 y} + A_2 e^{m_6 y} + A_3 e^{m_1 y} + A_4 e^{m_2 y} + A_5 \tag{34}$$

$$u_0(y) = \beta_1 e^{m_{11} y} + \beta_2 e^{m_{12} y} + A_6 e^{m_7 y} + A_7 e^{m_8 y} + A_8 e^{m_3 y} + A_9 e^{m_4 y} \tag{35}$$

Also, substitute equations (30)-(31), (32)-(33) and (34)-(35) into equations (23), (22) and (21) respectively, result in

$$C(y,t) = \frac{e^{m_3 y} - e^{m_4 y}}{e^{m_3} - e^{m_4}} + \frac{e^{m_1 y} - e^{m_2 y}}{e^{m_1} - e^{m_2}} e^{i\omega t} \tag{36}$$

$$\theta(y,t) = \frac{e^{m_7 y} - e^{m_8 y}}{e^{m_7} - e^{m_8}} + \frac{e^{m_5 y} - e^{m_6 y}}{e^{m_5} - e^{m_6}} e^{i\omega t} \tag{37}$$

$$u(y,t) = \beta_1 e^{m_{11} y} + \beta_2 e^{m_{12} y} + A_6 e^{m_7 y} + A_7 e^{m_8 y} + A_8 e^{m_3 y} + A_9 e^{m_4 y} + (\beta_3 e^{m_9 y} + \beta_4 e^{m_{10} y} + A_1 e^{m_5 y} + A_2 e^{m_6 y} + A_3 e^{m_1 y} + A_4 e^{m_2 y} + A_5) e^{i\omega t} \tag{38}$$

Special cases

Case 1: A situation where the upper wall of the channel is set into oscillatory motion while the lower wall is stationary, then the boundary conditions modifies into

$u(0,t) = 0, u(1,t) = H(t) e^{i\omega t}$ **t>0**

where H(t) is the Heaviside step function. Equation (38) takes the form

$$u(y,t) = \beta_{11} e^{m_{11} y} + \beta_{12} e^{m_{12} y} + A_6 e^{m_7 y} + A_7 e^{m_8 y} + A_8 e^{m_3 y} + A_9 e^{m_4 y} + (\beta_3 e^{m_9 y} + \beta_4 e^{m_{10} y} + A_1 e^{m_5 y} + A_2 e^{m_6 y} + A_3 e^{m_1 y} + A_4 e^{m_2 y} + A_5) e^{i\omega t} \tag{39}$$

Case 2: In this situation, the two channel walls are set into oscillatory motions and the boundary conditions modifies into

$u(0,t) = u(1,t) = H(t) e^{i\omega t}$ **t>0**

Equation (38) is therefore

$$u(y,t) = \beta_{13} e^{m_{11} y} + \beta_{14} e^{m_{12} y} + A_6 e^{m_7 y} + A_7 e^{m_8 y} + A_8 e^{m_3 y} + A_9 e^{m_4 y} +$$
$$(\beta_{15} e^{m_9 y} + \beta_{16} e^{m_{10} y} + A_1 e^{m_5 y} + A_2 e^{m_6 y} + A_3 e^{m_1 y} + A_4 e^{m_2 y} + A_5) e^{i\omega t} \quad (40)$$

where

$$A_1 = -\frac{\phi_1}{m_5^2 + \mathrm{Re}\ \alpha^2 m_5 - \mathrm{Re}\left(\chi + m + i\omega\right)}, \quad A_2 = \frac{\phi_1}{m_6^2 + \mathrm{Re}\ \alpha^2 m_6 - \mathrm{Re}\left(\chi + m + i\omega\right)}$$

$$A_3 = -\frac{\phi_2}{m_1^2 + \mathrm{Re}\ \alpha^2 m_1 - \mathrm{Re}\left(\chi + m + i\omega\right)}, \quad A_4 = \frac{\phi_2}{m_2^2 + \mathrm{Re}\ \alpha^2 m_2 - \mathrm{Re}\left(\chi + m + i\omega\right)}$$

$$A_5 = \frac{p}{-\mathrm{Re}\left(\chi + m + i\omega\right)}, \quad A_6 = -\frac{\phi_3}{m_7^2 + \mathrm{Re}\ \alpha^2 m_7 - a_1^{-1} a_4\ \mathrm{Re}\left(\chi + m\right)}$$

$$A_7 = \frac{\phi_3}{m_8^2 + \mathrm{Re}\ \alpha^2 m_8 - a_1^{-1} a_4\ \mathrm{Re}\left(\chi + m\right)}, \quad A_8 = -\frac{\phi_4}{m_3^2 + \mathrm{Re}\ \alpha^2 m_3 - a_1^{-1} a_4\ \mathrm{Re}\left(\chi + m\right)}$$

$$A_9 = \frac{\phi_4}{m_4^2 + a_1^{-1}\ \mathrm{Re}\ \alpha^2 m_4 - a_1^{-1} a_4\ \mathrm{Re}\left(\chi + m\right)}, \quad \phi_1 = \frac{a_1^{-1} a_2 Gt}{e^{m_5} - e^{m_6}}, \quad \phi_2 = \frac{a_1^{-1} a_3 Gc}{e^{m_1} - e^{m_2}}$$

$$\phi_3 = \frac{a_1^{-1} a_2 Gt}{e^{m_7} - e^{m_8}}, \quad \phi_4 = \frac{a_1^{-1} a_3 Gc}{e^{m_3} - e^{m_4}}, \quad \beta_1 = \left(\frac{\alpha_1 e^{m_{11}} - \alpha_2}{e^{m_{11}} - e^{m_{12}}}\right) - \alpha_1$$

$$\beta_2 = \left(\frac{\alpha_1 e^{m_{11}} - \alpha_2}{e^{m_{11}} - e^{m_{12}}}\right), \quad \beta_3 = \left(\frac{\alpha_4 e^{m_9} - \alpha_5}{e^{m_{10}} - e^{m_9}} + \alpha_4\right), \quad \beta_4 = \left(\frac{\alpha_4 e^{m_9} - \alpha_5}{e^{m_{10}} - e^{m_9}}\right)$$

$$\beta_{11} = \left(\frac{H(t) e^{i\omega t} + \alpha_1 e^{m_{11}} - \alpha_2}{e^{m_{11}} - e^{m_{12}}}\right) - \alpha_1, \quad \beta_{12} = \left(\frac{H(t) e^{i\omega t} + \alpha_1 e^{m_{11}} - \alpha_2}{e^{m_{11}} - e^{m_{12}}}\right)$$

$$\beta_{13} = \left(\frac{H(t) e^{i\omega t}\left(e^{m_{10}} - 1\right) - \alpha_5 - \alpha_4 e^{m_{10}}}{e^{m_9} - e^{m_{10}}}\right), \quad \beta_{14} = \left(\frac{\alpha_4 e^{m_9} - e^{m_9} H(t) e^{i\omega t} - H(t) e^{i\omega t} + \alpha_5}{e^{m_9} - e^{m_{10}}}\right)$$

$$\beta_{15} = H(t) e^{i\omega t} - \alpha_1 - \left(\frac{H(t) e^{i\omega t} - \alpha_2 - e^{m_{11}} H(t) e^{i\omega t} - \alpha_1 e^{m_{11}}}{e^{m_{12}} - e^{m_{11}}}\right)$$

$$\beta_{16} = \left(\frac{H(t) e^{i\omega t} - \alpha_2 - e^{m_{11}} H(t) e^{i\omega t} - \alpha_1 e^{m_{11}}}{e^{m_{12}} - e^{m_{11}}}\right)$$

$$\alpha_1 = A_6 + A_7 + A_8 + A_9, \quad \alpha_2 = A_6 e^{m_7} + A_7 e^{m_8} + A_8 e^{m_3} + A_9 e^{m_4}$$

$$\alpha_4 = A_1 + A_2 + A_3 + A_4 + A_5, \quad \alpha_5 = A_1 e^{m_5} + A_2 e^{m_6} + A_3 e^{m_1} + A_4 e^{m_2} + A_5$$

$$m_1 = \frac{-\left(a_6^{-1} Sc\ \alpha^2\right) + \sqrt{\left(a_6^{-1} Sc\ \alpha^2\right)^2 + 4 a_6^{-1} Sc\left(k_0 + i\omega\right)}}{2},$$

$$m_2 = \frac{-\left(a_6^{-1} Sc\ \alpha^2\right) - \sqrt{\left(a_6^{-1} Sc\ \alpha^2\right)^2 + 4 a_6^{-1} Sc\left(k_0 + i\omega\right)}}{2}$$

$$m_3 = \frac{-\left(a_6^{-1} Sc\,\alpha^2\right) + \sqrt{\left(a_6^{-1} Sc\,\alpha^2\right)^2 + 4a_6^{-1} Sc\left(k_0\right)}}{2},$$

$$m_4 = \frac{-\left(a_6^{-1} Sc\,\alpha^2\right) - \sqrt{\left(a_6^{-1} Sc\,\alpha^2\right)^2 + 4a_6^{-1} Sc\left(k_0\right)}}{2}$$

$$m_5 = \frac{-\left(a_5^{-1} \Pr\,\alpha^2\right) + \sqrt{\left(a_5^{-1} \Pr\,\alpha^2\right)^2 + 4a_5^{-1} \Pr\left(i\omega + N - Q\right)}}{2},$$

$$m_6 = \frac{-\left(a_5^{-1} \Pr\,\alpha^2\right) - \sqrt{\left(a_5^{-1} \Pr\,\alpha^2\right)^2 + 4a_5^{-1} \Pr\left(i\omega + N - Q\right)}}{2}$$

$$m_7 = \frac{-\left(a_5^{-1} \Pr\,\alpha^2\right) + \sqrt{\left(a_5^{-1} \Pr\,\alpha^2\right)^2 + 4a_5^{-1} \Pr\left(N - Q\right)}}{2},$$

$$m_8 = \frac{-\left(a_5^{-1} \Pr\,\alpha^2\right) - \sqrt{\left(a_5^{-1} \Pr\,\alpha^2\right)^2 + 4a_5^{-1} \Pr\left(N - Q\right)}}{2}$$

$$m_9 = \frac{-\left(a_1^{-1} \mathrm{Re}\,\alpha^2\right) + \sqrt{\left(a_1^{-1} \mathrm{Re}\,\alpha^2\right)^2 + 4a_4 a_1^{-1} \mathrm{Re}\left(i\omega + M + \chi\right)}}{2}$$

$$m_{10} = \frac{-\left(a_1^{-1} \mathrm{Re}\,\alpha^2\right) - \sqrt{\left(a_1^{-1} \mathrm{Re}\,\alpha^2\right)^2 + 4a_a a_1^{-1} \mathrm{Re}\left(i\omega + M + \chi\right)}}{2}$$

$$m_{11} = \frac{-\left(a_1^{-1} \mathrm{Re}\,\alpha^2\right) + \sqrt{\left(a_1^{-1} \mathrm{Re}\,\alpha^2\right)^2 + 4a_4 a_1^{-1} \mathrm{Re}\left(M + \chi\right)}}{2},$$

$$m_{12} = \frac{-\left(a_1^{-1} \mathrm{Re}\,\alpha^2\right) - \sqrt{\left(a_1^{-1} \mathrm{Re}\,\alpha^2\right)^2 + 4a_4 a_1^{-1} \mathrm{Re}\left(M + \chi\right)}}{2}$$

II. Results

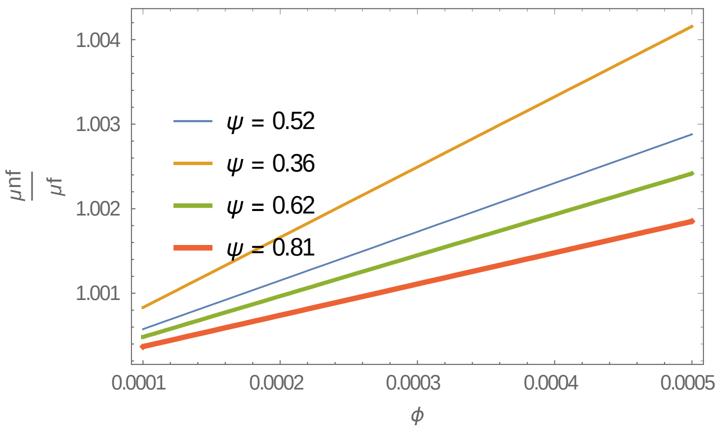

Figure 1 Dependence of effective viscosity on volume fraction of empirical shape factors of copper nanoparticles.

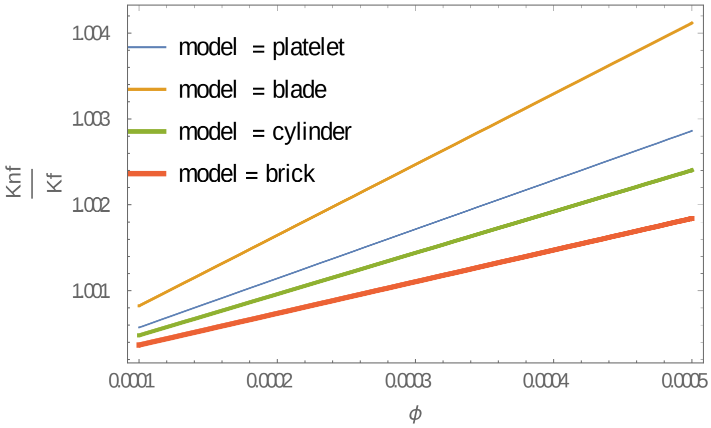

Figure 2 Dependence of effective thermal conductivity on volume fraction of copper nanoparticles in water as base fluid.

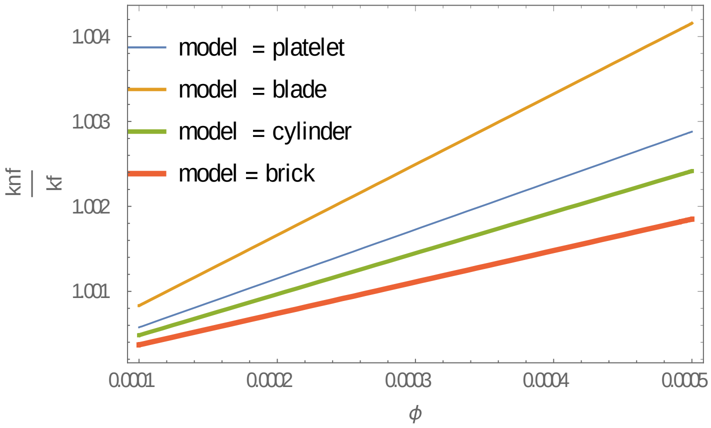

Figure 3 Dependence of effective thermal conductivity on volume fraction of copper nanoparticles in EG as base fluid.

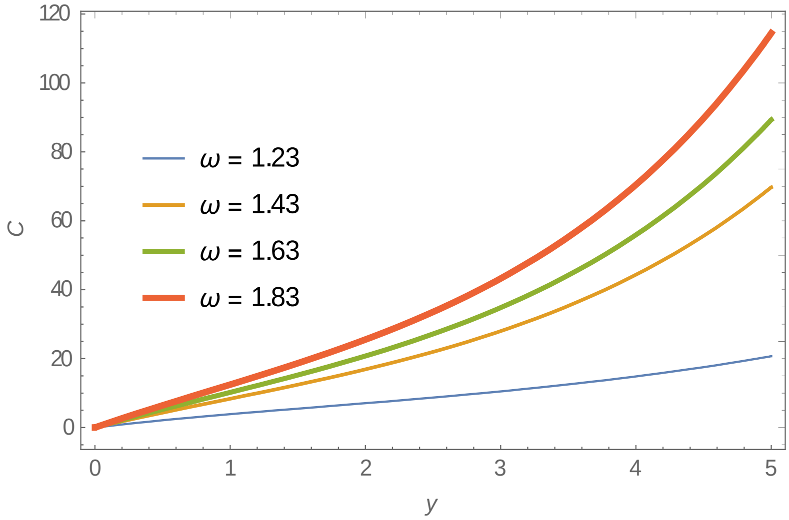

Figure 4: The dependence of concentration on Coordinate with frequency term varying

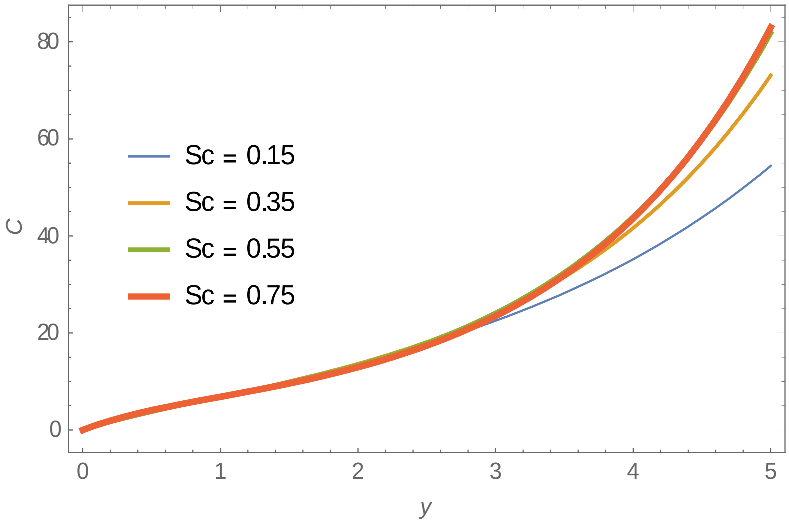

Figure 5: The dependence of concentration on Coordinate with Schmidt number varying

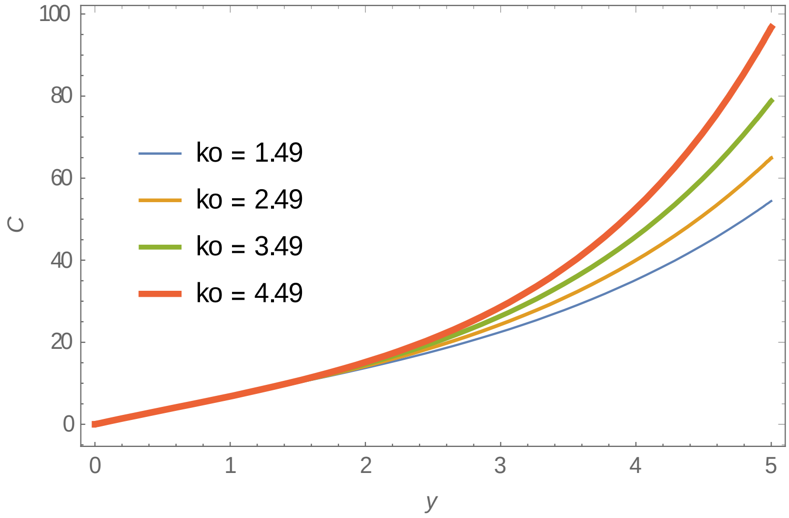

Figure 6: The dependence of concentration on Coordinate with chemical reaction term varying

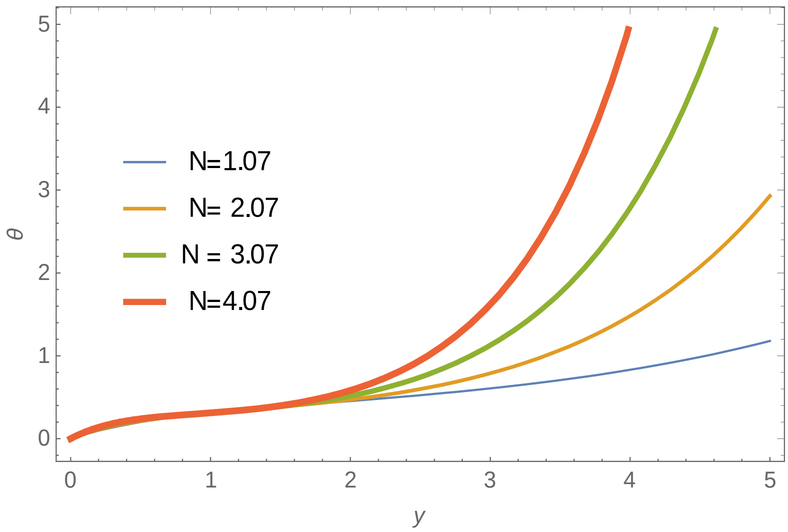

Figure 7: The dependence of Temperature on Coordinate with radiation term varying

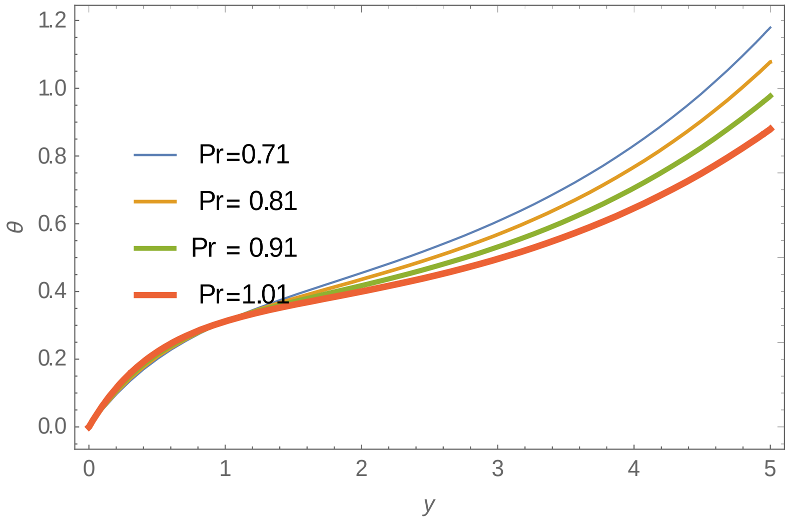

Figure 8: The dependence of Temperature on Coordinate with Prandtl number varying

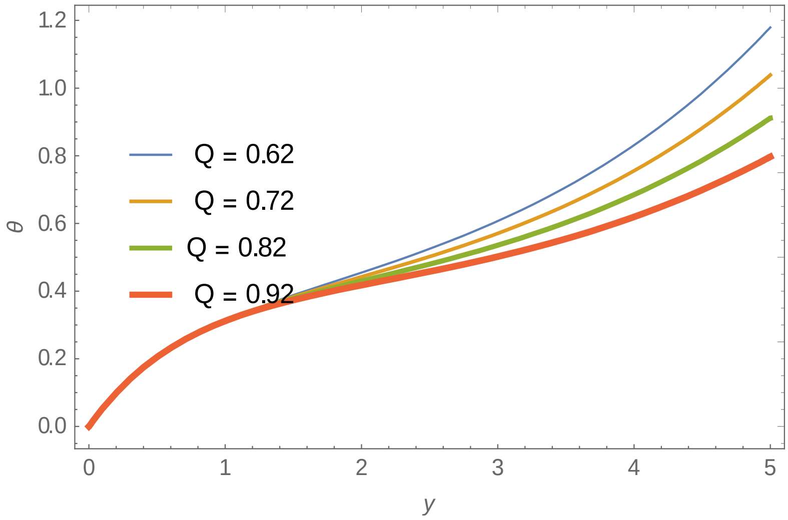

Figure 9: The dependence of Temperature on Coordinate with heat source term varying

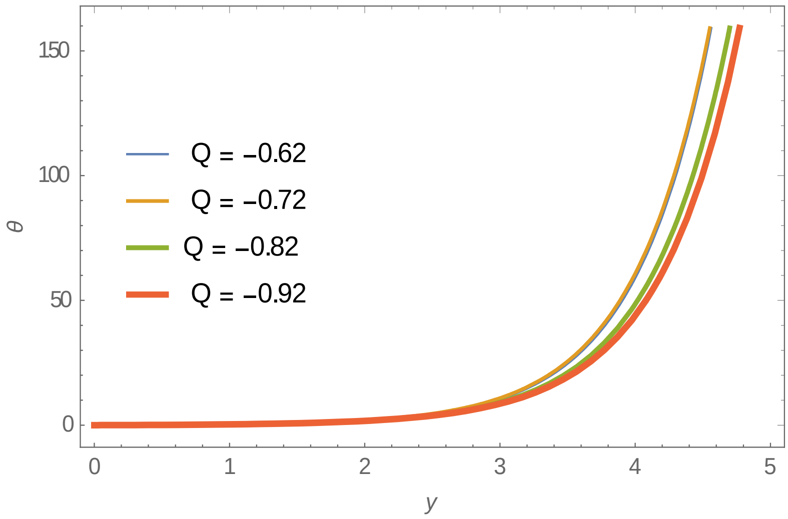

Figure 10: The dependence of Temperature on Coordinate with heat sink term varying

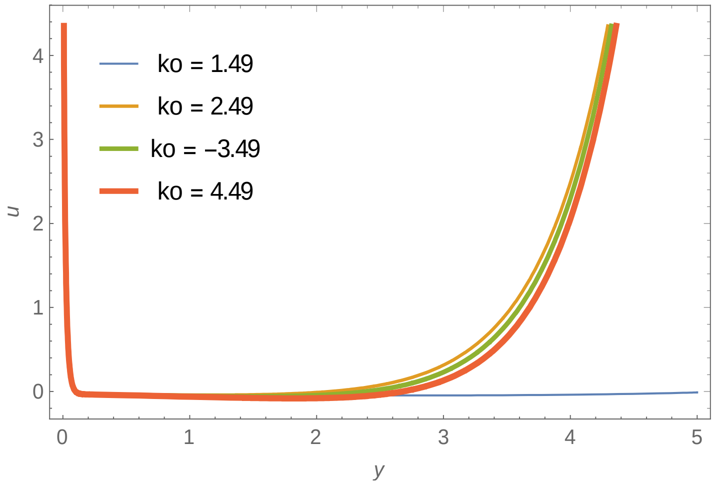

Figure 11: The dependence of velocity on Coordinate with chemical reaction term varying

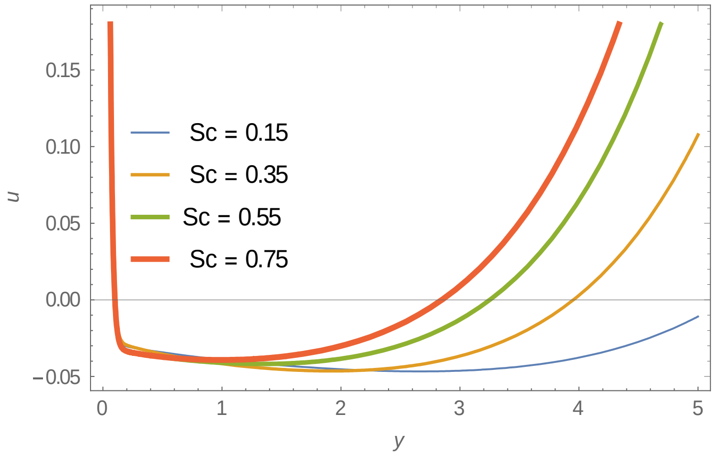

Figure 12: The dependence of velocity on Coordinate with Schmidt number varying

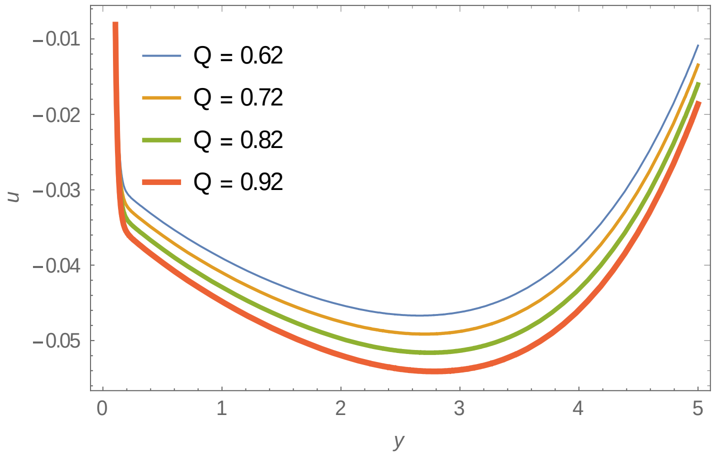

Figure 13: The dependence of velocity on Coordinate with heat source term varying

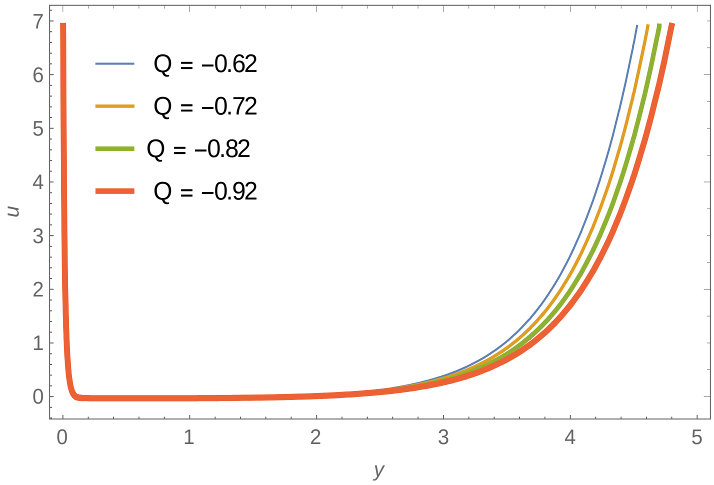

Figure 14: The dependence of velocity on Coordinate with heat sink term (Q<0) varying

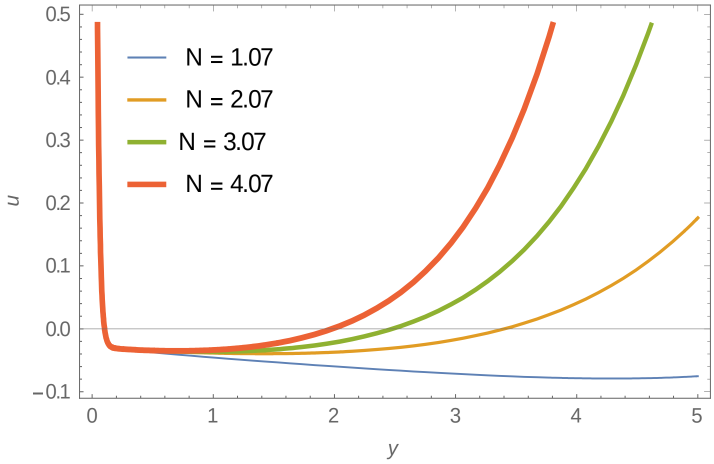

Figure 15: The dependence of velocity on Coordinate with radiation term varying

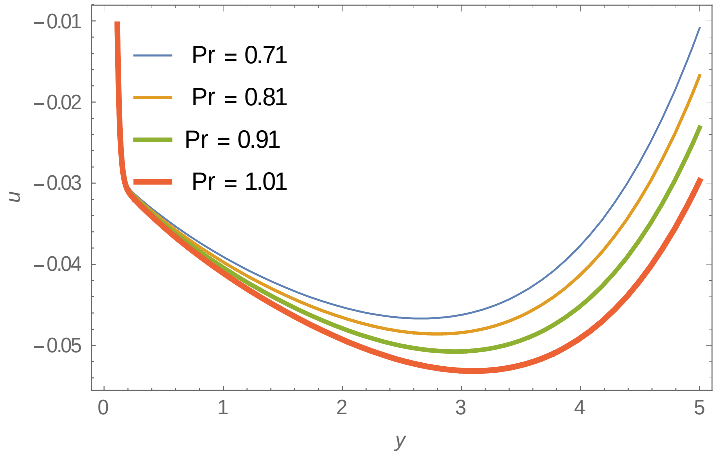

Figure 16: The dependence of velocity on Coordinate with Prandtl number varying

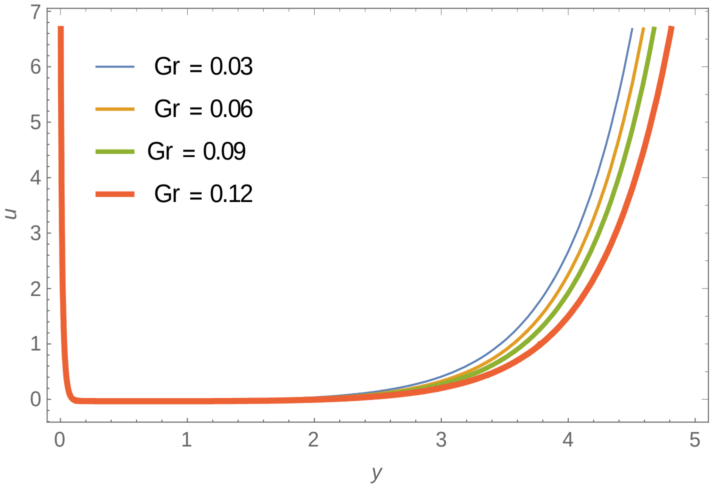

Figure 17: The dependence of velocity on Coordinate with Grashof number varying

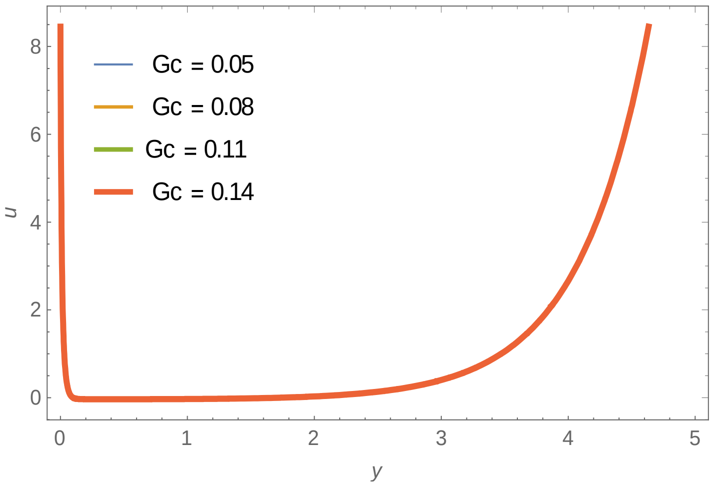

Figure 18: The dependence of velocity on Coordinate with modified Grashof number varying

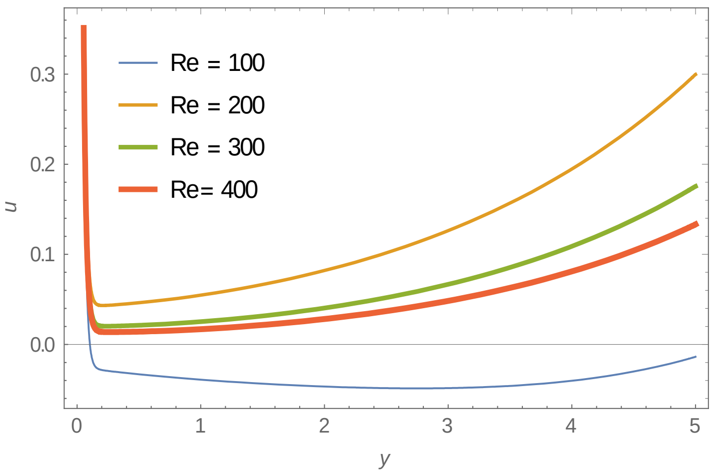

Figure 19: The dependence of velocity on Coordinate with Reynolds number varying

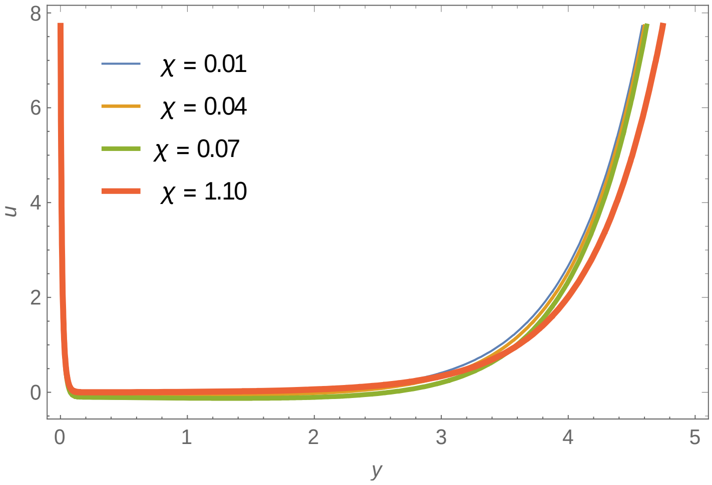

Figure 20: The dependence of velocity on Coordinate with porosity term number varying

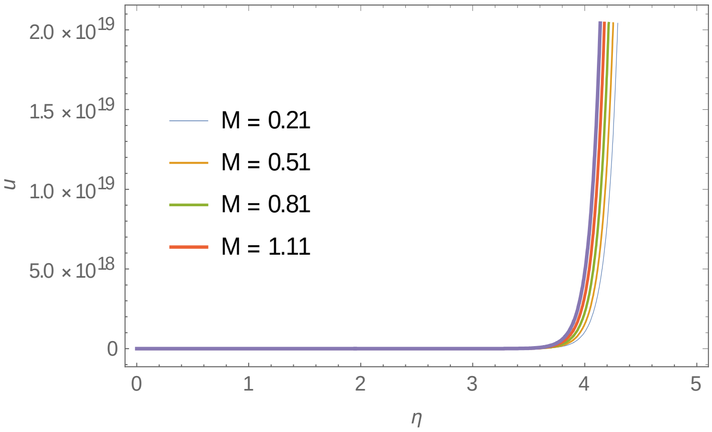

Figure 21: The dependence of velocity on Coordinate with Magnetic field parameter varying

III. Discussion

In order to get physical insight and numerical validation of the problem, an approximate values of $P = 1, \alpha^2 = H(t) = t = 1$ are chosen. The values of other parameters made use of are $\mathrm{Re} = 100, 200, 300, 400$, $Gr = 0.03, 0.06, 0.09, 0.12$, $M = 0.21, 0.51, 0.81, 1.11$, $\chi = 0.01, 0.04, 0.07, 0.10$, $\mathrm{Pr} = 0.71, 0.81, 0.91, 1.01$, $N = 1.07, 2.07, 3.07, 4.07$, $Gc = 0.05, 0.08, 0.91, 1.01$, $Sc = 0.15, 0.35, 0.55, 0.75$, $k_0 = 1.49, 2.49, 3.49, 4.49$, $\omega = 1.23, 1.43, 1.63, 1.83$, $Q = 0.62, 0.72, 0.82, 0.92$

Figure 1 and Table 3, showed the model of empirical shape factors for determining parameters of viscosities of different shapes of nanoparticles in either in base fluids of ethylene glycol or water, the viscosity of the nanoparticle shape followed the order, blade < brick < platelet < cylinder.
Figure 2 and Figure 3 showed that the thermal conductivity of platelet > blade > cylinder > brick in ethylene glycol and water based fluids but variation of the thermal conductivity is more distinct in water based fluid.

Concentration profile

Increasing the frequency of oscillation as depicted in Figure 4, correspond to increasing concentration of the copper nanoparticle in both the ethylene glycol and water based fluids but differ in magnitude.The Schmidt number is a ratio of momentum diffusivity to mass diffusivity and relates the relative thickness of the hydrodynamic boundary layer. It is illustrated in Figure 5 that an increase in concentration profile of the copper nanofluid is observed as result of increase in Schmidt number.The chemical reaction of the copper nanofluid lowers the concentration boundary layer and this result in an increase in the concentration profile of the fluid as the chemical reaction term increases. This relationship is shown in Figure 6.

Temperature profile

The thermal radiation transfer of copper nanofluid when increased, give rise to the temperature profile of the nanofluid as depicted in Figure 7. From astrophysical point of view, the Prandtl number of 0.71 at $25\ ^0C$, if increased further as shown in Figure 8, led to a decrease in the thermal boundary thickness hence a decrease in the temperature profile of the nanofluid. Increase in heat source Q > 0, decreases the temperature profile of the nanofluid as shown on Figure 9 and the reverse which is increase in heat sink Q < 0, increases the temperature profile of nanofluid as depicted in Figure 10. In the absence of the heat function term, the results are similar to that of Aaiza et al (2015) and Ngiangia and Akaezue (2019).

Velocity profile

The concentration boundary layer is lowered by chemical reaction and this effect also decreases the velocity profile of the nanofluid as shown in Figure 11. Figure 12 illustrates the effect of increasing the Schmidt number and observation showed that the velocity profile of the fluid is enhanced by the increase. Figure13 and Figure 14 demonstrated the effect of increasing heat source and heat sink term and the graphs clearly showed that increase in heat source term (Q > 0) as shown in Figure 13 corresponds to a decrease in the velocity profile of the copper nanofluid while a reverse which is increase in velocity profile is observed with heat sink term (Q < 0) as shown in Figure 14.The effect of radiation on the velocity profile of copper nanofluid flow as described in Figure 15, showed that increase in radiation results in an early increase in velocity and decreases the velocity profile of the fluid. Increasing Prandtl number as observed in Figure 16, showed that the velocity profile decreases steadily and converge to the free stream velocity. This observation agrees quantitatively with earlier results of Israel-Cookey et al (2003).

Increase in Grashof number (Gr, Gc) > 0 means cooling of the channel plates and increase cooling of the plates correspond to a decrease in the velocity profile of the nanofluid and these effect is demonstrated in Figures 17 and 18. The Reynolds number describes the behaviour of nanofluid flow from laminar to turbulence and its thermal conductivity. It is therefore verified from Figure 19 that its increase brings about a corresponding enhancement not only in the velocity profile of the nanofluid but also the thermal conductivity. The permeability term as shown in Figure 20, expresses the fact that its increasing pore spaces led to early decrease and later enhanced the velocity profile of the copper nanofluid in both the ethylene glycol and water based fluid. Finally, the magnetic Hartmann number is a resistive type of force and its increase in a region will lead to a decrease in the velocity profile of the fluid and that is the illustration in Figure 21.

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Amadi, Okechukwu. "Heat and Mass Transfer in Flow of Copper Nanofluid Containing Different Shapes of Nanoparticles in a Porous Channel Part 1." *IOSR Journal of Mathematics (IOSR-JM)*, 17(6), (2021): pp. 16-34.