IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 18, Issue 1 Ser. I (Jan. – Feb. 2022), PP 01-07
www.iosrjournals.org

Heat and Mass Transfer in Flow of Copper Nanofluid Containing Different Shapes of Nanoparticles in a Porous Channel Part 2

Amadi, Okechukwu [1] and Amos, Emeka [2]

[1] *Department of Mathematics,Ignatius Ajuru University of Education, Rumuolumeni*

[2] *Department of Mathematics,Rivers State University, Nkpolu Oroworukwo, Port Harcourt*

Abstract
An analytical study to determine the effect of certain material parameters on the rate of heat transfer and rate of mass transfer was made. The hydrodynamical equations of continuity, momentum, energy and concentration were non dimensionalized and solution obtained by perturbation method. Using numerical values, it was observed that increase in Prandtl number, radiation and heat absorption, depreciated the Nusselt number and skin friction of the nanofluid. Enhancement of the magnetic Hartmann number also enhanced the skin friction but increase in the frequency of oscillation depreciates skin friction and enhanced the rate of mass transfer. The Schmidt number depreciates both the skin friction and the Sherwood number as it is enhanced. The Grashof number and the modified Grashof number enhanced the skin friction while the Reynolds number depreciates it as they are enhanced. Increase in the porosity term, depreciates both the skin friction and the Sherwood number. Calculation of mass efflux, mean temperature and mean concentration was also determined.

Keywords*: Rate of Heat transfer; Rate of mass transfer; Pertubation method, Nanofluids, Skin friction.*

Date of Submission: 01-01-2022 Date of Acceptance: 12-01-2022

I. Introduction

As a result of technological development and improved methods of practice in almost every industry, the use of nanoscience and nano technology is fast gaining momentum. In 1974 Norio Tanigudi first used the term nanotechnology. The role of thermal radiation on the flow and heat transfer process is of major importance in the design of many advanced energy conversion system operating at higher temperature. Thermal radiation within this system is usually as a result of emission by hot walls and the working fluid, Seigel and Howell (1971). Preparation of nanofluids involves single-step and two-step preparation process. However, Lo et al (2005), developed vacuum based submerged arc nanoparticle synthesis to prepare copper (cu) based nanofluids with different dielectric liquids. The copper nanofluid used was not prepared since it is very expensive but relied on literatures of copper nanoparticles, ethylene glycol and water base fluids and their properties to determine the effect of heat function on the hydrodynamic properties of copper nanofluid. Eastman et al (1997) developed a modified vacuum evaporation onto a running oil substrate (VEROS) technique, in which copper (cu) vapour is directly condensed into nanoparticles by contact with flowing vapour – pressure ethylene glycol. Ngiangia and Orukari (2021) and Ngiangia et al (2022), investigated the heat transfer characteristics of copper nananofluid and made useful deductions. Soares et al,(2020), considered the Evaluation of Silica Nanofluids in Static and Dynamic conditions by an optical fiber sensor and concluded that the method offer better results than those obtained by traditional mathematical models. Nanofluids preparation methods require advanced and sophisticated equipments. This leads to higher production cost of nanofluids, but its characteristics outlined cannot be over emphasized. The choice of non-spherical shape nanoparticle in this research is predicated on the work of Aaiza et al (2015), where they mentioned desirable properties in cancer treatment. Also the choice of copper nanoparticle with ethylene glycol and water as based fluids to form copper nanofluid is because it possesses higher thermal conductivity and stability than other nanofluids. Enhanced thermal conductivity and improved heat transfer are potent factors in scientific and engineering applications. The usual known base fluids that are in use, needs not only modification but enhancement of the aforementioned properties. It is of necessity that the application, mainly in viscosity and thermal conductivity that the analytical study of copper nanofluid and its properties is proposed. The study will afford engineers and scientist the opportunity to select or choose among a host of base fluids which of them and the nanoparticle required for producing the needed mean thermal conductivity and effective viscosity for industrial application.

Mathematical Formulation of the Physical Problem
Following the formulation of part1 of Amadi and Amos (submitted) the dimensionless governing equations are rewritten as

$$\frac{\partial u}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)\frac{\partial u}{\partial y}=-pe^{i\omega t}+a_1\,\mathrm{Re}^{-1}\frac{\partial^2 u}{\partial y^2}-u\left(\chi+M\right)a_4+a_2 Gt\,\theta+a_3 GcC \quad (1)$$

$$\frac{\partial \theta}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)\frac{\partial \theta}{\partial y}=\left(\frac{a_5}{\mathrm{Pr}}\right)\frac{\partial^2 \theta}{\partial y^2}-a_6\theta\left(N-Q\right) \quad (2)$$

$$\frac{\partial C}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)\frac{\partial C}{\partial y}=\frac{a_6}{Sc}\frac{\partial^2 C}{\partial y^2}-a_6 k_0 C \quad (3)$$

The boundary conditions also transform into

$u(0,t)=u(1,t)=0,\theta(0,t)=0,\theta(1,t)=1,C(0,t)=0,C(1,t)=1$

And parameters defined.

Method of Solution
The problem is solved and the solutions as contained in Amadi and Amos (submitted) is restated as

$$C(y,t)=\frac{e^{m_3 y}-e^{m_4 y}}{e^{m_3}-e^{m_4}}+\frac{e^{m_1 y}-e^{m_2 y}}{e^{m_1}-e^{m_2}}e^{i\omega t} \quad (4)$$

$$\theta(y,t)=\frac{e^{m_7 y}-e^{m_8 y}}{e^{m_7}-e^{m_8}}+\frac{e^{m_5 y}-e^{m_6 y}}{e^{m_5}-e^{m_6}}e^{i\omega t} \quad (5)$$

$$u(y,t)=\beta_1 e^{m_{11}y}+\beta_2 e^{m_{12}y}+A_6 e^{m_7 y}+A_7 e^{m_8 y}+A_8 e^{m_3 y}+A_9 e^{m_4 y}+$$
$$(\beta_3 e^{m_9 y}+\beta_4 e^{m_{10}y}+A_1 e^{m_5 y}+A_2 e^{m_6 y}+A_3 e^{m_1 y}+A_4 e^{m_2 y}+A_5)e^{i\omega t} \quad (6)$$

And the parameters all defined.

Skin friction, Nusselt number and Sherwood number
The skin friction at the walls of the channel, the heat transfer coefficient and the mass transfer coefficient, give vital scientific and engineering information on some properties of the copper nanofluid.
The skin friction

$$\tau=\left(\frac{\partial u}{\partial y}\right)_{y=0}=\beta_1 m_{11}+\beta_2 m_{12}+A_6 m_7+A_7 m_8+A_8 m_3+A_9 m_4$$
$$+(\beta_3 m_9+\beta_4 m_{10}+A_1 m_5+A_2 m_6+A_3 m_1+A_4 m_2)e^{i\omega t} \quad (7)$$

Case 1

$$\tau=\left(\frac{\partial u}{\partial y}\right)_{y=0}=\beta_{11} m_{11}+\beta_{12} m_{12}+A_6 m_7+A_7 m_8+A_8 m_3+A_9 m_4$$
$$+(\beta_3 m_9+\beta_4 m_{10}+A_1 m_5+A_2 m_6+A_3 m_1+A_4 m_2)e^{i\omega t} \quad (8)$$

Case 2

$$\tau=\left(\frac{\partial u}{\partial y}\right)_{y=0}=\beta_{13} m_{11}+\beta_{14} m_{12}+A_6 m_7+A_7 m_8+A_8 m_3+A_9 m_4$$
$$+(\beta_{15} m_9+\beta_{16} m_{10}+A_1 m_5+A_2 m_6+A_3 m_1+A_4 m_2)e^{i\omega t} \quad (9)$$

The heat transfer coefficient is given as

$$Nu = -\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = -\left(\frac{m_7 - m_8}{e^{m_7} - e^{m_8}} + \frac{m_5 - m_6}{e^{m_5} - e^{m_6}} e^{i\omega t}\right) \tag{10}$$

The mass transfer coefficient is stated as

$$Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = -\left(\frac{m_3 - m_4}{e^{m_3} - e^{m_4}} + \frac{m_1 - m_2}{e^{m_1} - e^{m_2}} e^{i\omega t}\right) \tag{11}$$

Further, the mass flux (ϖ), the mean temperature (θ_m) and mean concentration C_m respectively are obtained by evaluating the integrals

$$\varpi = \int_0^1 u(y,t)dy = \tag{12}$$

$$\theta_m = \frac{\int_0^1 u(y,t)\theta(y,t)dy}{\int_0^1 u(y,t)dy} \tag{13}$$

$$C_m = \frac{\int_0^1 u(y,t)C(y,t)dy}{\int_0^1 u(y,t)dy} \tag{14}$$

II. Results And Discussion

RESULTS

Table 1: Computed values of the skin friction, Nusselt number and Sherwood number for various values of Prandtl number when

$N = 1.07, Q = 0.62, \omega = 1.23, \chi = 0.01, M = 0.21, \mathrm{Re} = 100,$

$Gr = 0.03, Gc = 0.05, Sc = 0.15, k_0 = 1.49, p = 1, \alpha^2 = 1$

Pr	τ	Nu	Sh
0.71	$-4.68180 \ x10^{22}$	-3.21613	--
0.81	$-4.72119 \ x10^{22}$	-3.54692	--
0.91	$-4.7598 \ x10^{22}$	-3.89158	--
1.01	$-4.79678 \ x10^{22}$	-4.25318	--

Table 2: Computed values of the skin friction, Nusselt number and Sherwood number for various values of Radiation term when

$\mathrm{Pr} = 0.71, Q = 0.62, \omega = 1.23, \chi = 0.01, M = 0.21, \mathrm{Re} = 100,$

$Gr = 0.03, Gc = 0.05, Sc = 0.15, k_0 = 1.49, p = 1, \alpha^2 = 1$

N	τ	Nu	Sh
1.07	$-4.68180 \ x10^{22}$	-3.21613	--
2.07	$-4.92317 \ x10^{22}$	-3.97385	--
3.07	$-5.07895 \ x10^{22}$	-4.80075	--

4.07	$-5.17379\ x10^{22}$	-5.68498	--

Table 3: Computed values of the skin friction, Nusselt number and Sherwood number for various values of Absorption term when

$N = 1.07, \Pr = 0.71, \omega = 1.23, \chi = 0.01, M = 0.21, \text{Re} = 100,$

$Gr = 0.03, Gc = 0.05, Sc = 0.15, k_0 = 1.49, p = 1, \alpha^2 = 1$

Q	τ	Nu	Sh
0.62	$-4.68180\ x10^{22}$	-3.21613	--
0.72	$-4.7272\ x10^{22}$	-3.23851	--
0.82	$-4.77408\ x10^{22}$	-3.25991	--
0.92	$-4.82322\ x10^{22}$	-3.28008	--

Table 4: Computed values of the skin friction, Nusselt number and Sherwood number for various values of Porosity term when

$N = 1.07, Q = 0.62, \omega = 1.23, \Pr = 0.71, M = 0.21, \text{Re} = 100,$

$Gr = 0.03, Gc = 0.05, Sc = 0.15, k_0 = 1.49, p = 1, \alpha^2 = 1$

χ	τ	Nu	Sh
0.01	$-4.03149\ x10^{22}$	--	--
0.04	$-6.2262\ x10^{22}$	--	--
0.07	$-1.08274\ x10^{23}$	--	--
0.10	$1.18991\ x10^{22}$	--	--

Table 5: Computed values of the skin friction, Nusselt number and Sherwood number for various values of Magnetic field when

$N = 1.07, Q = 0.62, \omega = 1.23, \chi = 0.01, \Pr = 0.71, \text{Re} = 100,$

$Gr = 0.03, Gc = 0.05, Sc = 0.15, k_0 = 1.49, p = 1, \alpha^2 = 1$

M	τ	Nu	Sh
0.21	$-4.03149\ x10^{22}$	--	--
0.51	$3.49064\ x10^{22}$	--	--
0.81	$1.23615\ x10^{22}$	--	--
1.11	$6.35315\ x10^{21}$	--	--

Table 6: Computed values of the skin friction, Nusselt number and Sherwood number for various values of Frequency of oscillation when

$N = 1.07, Q = 0.62, \Pr = 0.71, \chi = 0.01, M = 0.21, \text{Re} = 100,$

$Gr = 0.03, Gc = 0.05, Sc = 0.15, k_0 = 1.49, p = 1, \alpha^2 = 1$

ω	τ	Nu	Sh
1.23	$-4.03149\ x10^{22}$	-3.21613	0.0991857
1.43	$-4.08981\ x10^{22}$	-3.2118	0.11452
1.63	$-4.12406\ x10^{22}$	-3.20393	0.129572

1.83	$-4.1442 \ x10^{22}$	-3.05865	0.144344

Table 7: Computed values of the skin friction, Nusselt number and Sherwood number for various values of Schmidt number when

$N = 1.07, Q = 0.62, \omega = 1.23, \chi = 0.01, M = 0.21, \mathrm{Re} = 100,$

$Gr = 0.03, Gc = 0.05, \mathrm{Pr} = 0.71, k_0 = 1.49, p = 1, \alpha^2 = 1$

Sc	τ	Nu	Sh
0.15	$-4.03149 \ x10^{22}$	--	0.0991857
0.35	$-5.11531 \ x10^{22}$	--	0.0962341
0.55	$-5.161 \ x10^{22}$	--	0.0936341
0.75	$-5.03163 \ x10^{22}$	--	0.0470213

Table 8: Computed values of the skin friction, Nusselt number and Sherwood number for various values of Grashof number due to temperature when

$N = 1.07, Q = 0.62, \omega = 1.23, \chi = 0.01, M = 0.21, \mathrm{Re} = 100,$

$\mathrm{Pr} = 0.71, Gc = 0.05, Sc = 0.15, k_0 = 1.49, p = 1, \alpha^2 = 1$

Gr	τ	Nu	Sh
0.03	$-4.03149 \ x10^{22}$	--	--
0.06	$-3.94369 \ x10^{22}$	--	--
0.09	$-3.23663 \ x10^{22}$	--	--
0.12	$-3.6954 \ x10^{22}$	--	--

Table 9: Computed values of the skin friction, Nusselt number and Sherwood number for various values of Grashof number due to concentration when

$N = 1.07, Q = 0.62, \omega = 1.23, \chi = 0.01, M = 0.21, \mathrm{Re} = 100,$

$Gt = 0.03, \mathrm{Pr} = 0.71, Sc = 0.15, k_0 = 1.49, p = 1, \alpha^2 = 1$

Gc	τ	Nu	Sh
0.05	$-4.03149 \ x10^{22}$	--	--
0.08	$-4.08967 \ x10^{22}$	--	--
0.11	$-4.33908 \ x10^{22}$	--	--
0.14	$-4.58848 \ x10^{22}$	--	--

Table 10: Computed values of the skin friction, Nusselt number and Sherwood number for various values of Reynolds number when

$N = 1.07, Q = 0.62, \omega = 1.23, \chi = 0.01, M = 0.21, \mathrm{Re} = 100,$

$Gr = 0.03, Gc = 0.05, Sc = 0.15, k_0 = 1.49, p = 1, \alpha^2 = 1$

Re	τ	Nu	Sh
100	$-4.03149 \ x10^{22}$	--	--
200	$-2.08562 \ x10^{44}$	--	--

300	$-1.17991\ x10^{66}$	--	--
400	$-5.62616\ x10^{87}$	--	--

Table 11: Computed values of the skin friction, Nusselt number and Sherwood number for various values of Porosity term when

$N = 1.07, Q = 0.62, \omega = 1.23, \chi = 0.01, M = 0.21, \mathrm{Re} = 100,$

$Gr = 0.03, Gc = 0.05, Sc = 0.15, \mathrm{Pr} = 0.71, p = 1, \alpha^2 = 1$

k_0	τ	Nu	Sh
1.49	$-4.03149\ x10^{22}$	--	0.0991857
2.49	$-5.54165\ x10^{22}$	--	0.0918653
3.49	$-6.32473\ x10^{22}$	--	0.0838305
4.49	$-7.07076\ x10^{22}$	--	0.0770789

III. Discussion

Table 1 shows the variation of the skin friction and the heat transfer coefficient. Taking the astrophysical mean Prandtl number of 0.71at mean temperature of $25\,^0C$, further increase, reveals that both the skin friction and the Nusselt number are depreciated.

Radiation is brought about by conduction heat transfer as shown in **Table 2**, increase in radiation, caused a decrease in the skin friction and Nusselt number. This observation is consistent with the study of Gao et al (2018). **Table 3**, shows that, increase in the absorption term corresponds to a decrease in both the skin friction and Nusselt number of the channel walls through which the copper nanofluid flows. As depicted in **Table 4**, increase in porosity showed that the skin friction decreases but later showed marginal enhancement. **Table 5** displayed the variation between the skin friction and the magnetic Hartmann number. Increase in magnetic field term shows a corresponding increase in skin friction early but later showed a decrease as the Hartmann number is further increased. The relationship between the frequency of oscillation and the skin friction as well as the rate of heat transfer coefficient and the rate of mass transfer coefficient depicted in **Table 6**, showed that, as the frequency of oscillation increases, a decrease is observed in the skin friction. Increasing the Schmidt number as shown in **Table 7**, brings about a decrease in both the skin friction and the rate of mass transfer coefficient. Grashof number approximates the ratio of the buoyancy to viscous force acting on a fluid in boundary layer. Therefore, the skin friction enhances as a result of increase in Grashof number due to temperature as illustrated in **Table 8** and same result is observed in the skin friction as the Grashof number due to concentration is increased as shown on **Table 9**. Increase in Reynolds number means more collision of the copper nanofluid with the channel walls but increase viscosity hence reduces the skin friction as observed on **Table 10**. This result is the same with the result of Ngiangia and Nwabuzor (2019). **Table 11** illustrates that an increase in porosity term results to a corresponding depreciation in the skin friction as well as the rate of mass transfer coefficient and the result is in agreement the work of Ngiangia and Nwabuzor (2022).

References

[1]. Seigel R, and Howell J R.(1971) Thermal Radiation Heat Transfer. Student ed. Macgraw-Hill.

[2]. Lo, C.H, Tsung, T.T, Chen, L.C, Su, C.H and Lin, H.M (2005). Fabrication of Copper Oxide Nanofluid Using Submerged Arc Nanoparticle Synthesis System. Journal of Nanoparticle Research, 7: 313-320

[3]. Eastman, J.A, Choi, U.S, Li, S, Thompson, L.J and Lee, S (1997). Enhanced Thermal Conductivity Through the Development of Nanofluids. Materials Research Society. 3-11

[4]. Ngiangia, Alalibo.T and Orukari M.A. (2021); Heat Transfer Coefficient and Skin Friction Determination of Thermal Radiation Effects on MHD Convective Flow of Alumina Nanofluid Through a Non-Darcian Porous Plate. International Journal of Scientific and Engineering Research 12(5): 355-378

[5]. Ngiangia, Alalibo.T, Orukari M.A. Amadi, O and Nwabuzor, P. (2021); Onset of Transition to Non-Newtonian MHD Chemically Reacting Couette Copper Nanofluid Flow in a Radiative Porous Medium. International Journal of Mathematics Trends and Technology 67(5): 126 – 149 Doi: 10.14445/22315373/IJMTT-V6715P515

[6]. Soares, M. C. P, , Rodrigues, M. S, Schenkel, E. A, Perli, G, Silva, W. H. A, Gomes, M. K, Fujiwara,E and Suzuki, C.K (2020). Evaluation of Silica Nanofluids in Static and Dynamic Conditions by an Optical Fiber Sensor. Sensors, 20, 707-722: doi:10.3390/s20030707

[7]. Aaiza, G; Khan I and Shafie S. 2015. Energy Transfer in Mixed Convection MHD Flow of Nanofluid Containing Different Shapes of Nanoparticles in a Channel Filled with Saturated Porous Medium. Nanoscale Research Letters. 10(490): 1-14

[8]. Gao, Y., Wang, H., Sasmito, A. P. and Mujumdar, A. S. (2018). Measurement and modeling of thermal conductivity of graphene Nanoplatelet water and ethylene glycol

[9]. base nanofluids. International Journal of Heat and Mass Transfer, 123, 97–109. doi:10.1016/J.IJHEATMASSTRANSFER.2018.02.089

[10]. Ngiangia, A.T, and Akaezue,N.N. (2019); Heat Transfer of Mixed Convection Electroconductivity Flow of Copper Nanofluid with Different Shapes in a Porous Micro Channel Provoked by Radiation and First Order Chemical Reaction. Asian Journal of Physical and Chemical Sciences 7(1): 1-14.

[11]. Ngiangia, A T and Nwabuzor, P. O. (2019); Double Diffusive MHD Flow of a Chemically Reacting Alumina Nanofluid Past a Semi-Infinite Plate Part 2. Science and Technology 5: 106-113.

[12]. Ngiangia, Alalibo.T and Nwabuzor, P. (2022); Investigation of Heat Transfer charateristics of Spherical copper and Alumina Nanoparticles in Water and Ethylene Glycol Based Fluids. World Scientific News 163: 78-98.

Amadi, Okechukwu. "Heat and Mass Transfer in Flow of Copper Nanofluid Containing Different Shapes of Nanoparticles in a Porous Channel Part 2." *IOSR Journal of Mathematics (IOSR-JM)*, 18(1), (2022): pp. 01-07.