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Saturation in the 3·n+1 problem and a conjecture

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Abstract
We construct and analyse the orbits of the 3·n+1i.e. the (3·n+1)/2 problem in a finite set of the integer n, and we observe the presence of a "saturation point" for the 3·n+1 at n=118 (notice l(97)=118) and for the (3·n+1)/2 formulation at l(73)=73. The point is a value n_0 for which $l(n) \le n$, $\forall n \ge n_0$ where l(n) is the length of the orbit of the integer n to reach the unit i.e. 1, in the cycle 4→2→1 or 2→1.
Alternatively, we then pose the conjecture that, above the saturation point, for the tree of the inverse orbits starting at 1 and of depth k, the number of integers not exceeding k present on the tree is equal to k for $k \ge k_0$where $k_{0.}$ is the depth of the chalice at the saturation point, i.e. k_0=118 respectively k_0 =73 in the second formulation.
We then check the truth of the conjecture in the domain of n in the ranges of k∈[118..250] and k∈ [73..250] respectively.
Key words: *Collatz problem in the two formulation (3n+1) and (3n+1/2), inverse orbits, total stopping time, saturation point, conjecture, stochastic like Fibonacci Sequences, numerical experiment.*

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I. Introduction

The 3·n+1or (3·n+1)/2 problem is characterized by having "only" a very small cycle (probably the arrival of the orbits of all the integers n) given respectively by 4→2→1→4 and 2→1→2.Infact there is still the possibility that an infinite number of integers do not fall into the cycle and have an infinite trajectory diverging to infinity or that a set of integer belongs to a big possible cycle: very very "large", containing many odd.

See the extensive work of Lagarias for many important contributions, explanations and also results for sequences related to the 3n+1 [1,2].

A point of interest is that all similar problems i.e. 3·n+a, a odd, have the elementary cycle (multiple of the above of the 3·n+1 problem), i.e. a→4·a→2·a→a, arrivals of "all" multiple of 3, (a=3), of "all" multiple of 5(a=5), of "all" multiple of7, and so on, in addition to other possible more large cycles.

In fact, if we look at cycles containing just one odd in the 3·n+a, sequence, where a is an odd integer, we have to solve the Equation (let α be an integer):

$$\frac{(3\cdot n+a)}{2^{\alpha}} = n \qquad (1)$$

$$n \cdot (2^{\alpha} - 3) = a \qquad (2)$$

with the solution n=a and α = 2, i.e. the cycle a→4a→2a→a. For a=1, a=3, a=5, a=7,....
For a=1, if the conjecture is true one obtains all multiple of 3, of 5, of 7, ..., then all even numbers i.e. all integers falling into the cycle 1→4→2→1.
Numerical studies are very important in few of the fact that it is partly believed (in the scientific community) that the problem is presently very difficult for a complete solution (it may be for a long time). Keeping this in mind, additional experiments may still be interesting also for finite sets of integers not necessarily large [3], reduced - as an example - to a set of a thousand of integers (See Section3).
In fact as for special models of statistical mechanics connected with integers, numerical experiments with very small number of terms, i.e. N small may suggest interesting additional information about the system under investigation in the "thermodynamic" limit [4].
Now for the 3n+1, Tables of the lengths of the orbits calculated are given explicitly only up to n=250 in Appendix 1.

An analysis of the orbits reveals the emergence of a point which we call “saturation point” in such a finite domain; it is located for the 3·n+1 formulation at n=118 and for the (3·n+1)/2 at n=73.
These saturation points are defined to be such that the length l(n) of the orbit of an integer n reaching1 is smaller or equal to itself, i.e. n, thus $l(n) \le n\ \forall n \ge 118$ and $n \ge 73$ (Section 3).

Equivalently, the tree of the inverse orbits of depth k is expected to contain all numbers from 1 to k giving rise to a conjecture (of course equivalent to the truth of the Collatz conjecture; to the best of our knowledge this point is new or it was not analysed along our lines given below).
In a more extended analysis [11] we then present the experiment we have performed up to n=1000 to check the correctness of the conjecture i.e., (but) only for the finite domain above (up to n=1000).
(We have nevertheless controlled that as the intervals of n grows, i.e. from [250..500], [500..750] to [750..1000], the ratio between the length of the longest orbits over n, i.e. l(n)/n, decreases as a function of the “center” of the intervals - asymptote - that the conjecture may continue to be true as n increases (See Section 4 for the relative plots of l(n) as a function of n for some n with the largest l(n) values in the corresponding interval and given here only for the first one [1-250]).
We then close our note, setting the conjecture and present the leaves of the original chalice (tree of the inverse orbits in the 3·n+1 formulation) of height k=15 [5].

II. Construction of the orbits of the 3·n+1 and of the (3·n+1)/2 in the range n=2-250. (See Appendix1)

In our studies, we calculatedthe orbits for n comprise between 2 and 250 for 3·n+1 and (3·n+1)/2, respectively.
The tables (in Appendix 1) are created using different ad hoc C and C++ programs.
An example of source code is in the Table 1.

```
#INCLUDE <IOSTREAM>
#INCLUDE <CSTDLIB>
INT MAIN(INT ARGC, CHAR** ARGV) {
{
INT N, R, C;
PRINTF("INPUT AN INTEGER \N");
SCANF("%D", &N);
C=0;
WHILE (N > 1)
{ IF(R == 0)
{ N = N/2;
}
ELSE {
N = N * 3 + 1;}
C=C+1;
PRINTF("\T %D", N);
}
PRINTF("\N ORBITS: %D\N", C);
RETURN 0;
}
}
```

Table 1. A program (C language) to generate the orbits in the (3n+1) problem.

III. Observation, Saturation of the orbits in the two “cases” (3·n+1 and (3·n+1)/2).

Following the numerical results given in the Appendix1 we give the pointplot of l(n) in the above range where l(n) is the length of the orbits of n to reach 1 in the cycle $1 \rightarrow 4 \rightarrow 2 \rightarrow 1$ (3·n+1). The point (118,118) on the red line is our saturation point for the (3·n+1) case.

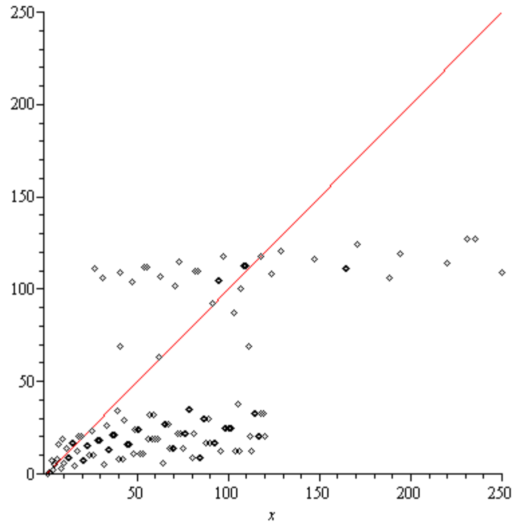


Fig.1. Pointplot of (n, l(n)) for the (3·n+1) formulation. From n=118 we have plotted points only for arguments n with the highest l(n); (118,118) is our saturation point. Above n=118, all points up to n=250 are below the line of Equation y = f(n) = n (in red).

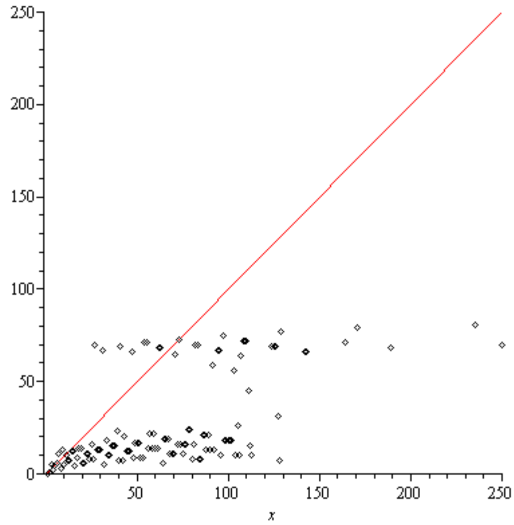


Fig.2. Pointplot of (n, l(n)) for the (3·n+1)/2 formulation. From n=73 we have plotted only some points with the highest l(n); (73,73) is our saturation point. Above n=73, all points up to n=250 are below the line of Equation y=f(n)= n (in red).

Remark 1

The two Figures are of course similar. We notice now that in the case of the 3·n+1, the number of the odd in the orbit of n=115 is 42 and that of the even is 73; the same as in the case (3·n+1)/2 where the number of the even is 31 (42+31=73, 73+42=115, (l(73) =115 for the 3·n+1 and l(73) = 73 for the (3·n+1)/2), 115-73 = 42 is equal to the number of the odds in both the formulations).

Remark2

The possible saturation in both cases 3·n+1 and (3·n+1)/2 are of course related: for n=118, l(97)=118 in the 3·n+1 while for n=73, l(73)=73. Here in the orbit of n=73, there are 42 odd, n_o =42 and 31 even, 31+42=73= l(73). In the 3·n+1, the corresponding orbit is that of n=115 where there are 42 more even then in that of the (3·n+1)/2, i.e. n_e = 73 and 73+42=115=l(73), but following the above strategy, the number of integers for n=115 are only 114 (since l(97) = 118 in the 3·n+1). With k=118 we have f(118) = 118 and n=97 is included. Notice that for n=97, we have l(97)=118 resp. l(97) = 75; 118-75 = n_o= 43 and n_e = 75 – 43=32, i.e. 75+43= 118. Saturation point at: k=118.

Let now N(k) be the number of the integers not exceeding k present on a chalice of the inverse orbits of depth k for the 3·n+1.

IV. Some numerical computations

We are here aware that in number theory n~ 250 or n ~ 1000 are "very Small Numbers". We also agree that ("as pointed out by some experts in the field"), $n=2^{68}$ is still a Small Number even if it is not (we say) "a very Small Number". We nevertheless know (from international Tables on the 3n+1 or on the (3n+1)/2 formulation on the Collatz problem) - up to now- (in a numerical context within stochastic models), that the maximum of the length of a trajectory of an integer n to reach the cycle 1,4,2,1 or 1,2,1, is expected to have as upper bound the Lagarias-Weiss Bound given by $l(n) < 41{\cdot}7{\cdot}\log(n)$; (notice that if this bound if translated into the 3n+1 formulation, the bound becomes $l(n)< 61{\cdot}\log(n)$, as explained in [5]).
We think that since $l(n) < n$ is a much weaker proposed bound, it will be very difficult to obtain a counterexample too. In fact, the last number of the Table 4 of Ref [6] (even if not so big) has a low total stopping time given by: $l(n=13371194527) < 2000$, and $n/\log(n) \leq 61$ in the (3n+1) formulation.
Notice here that $l(n)<61{\cdot}\log(n)< n$ for n~358 (n=226 in the (3·n+1)/2 formulation.
It is our opinion that in this context, the problem is very different from that concerning the fluctuations of the function Li(n) around Pi(n) (with a change of the signum of the difference at very very big arguments {n}.
We also think that the analysis of a new kind of inverse orbits in both the formulations and possibly related to other systems may be of interest [11].

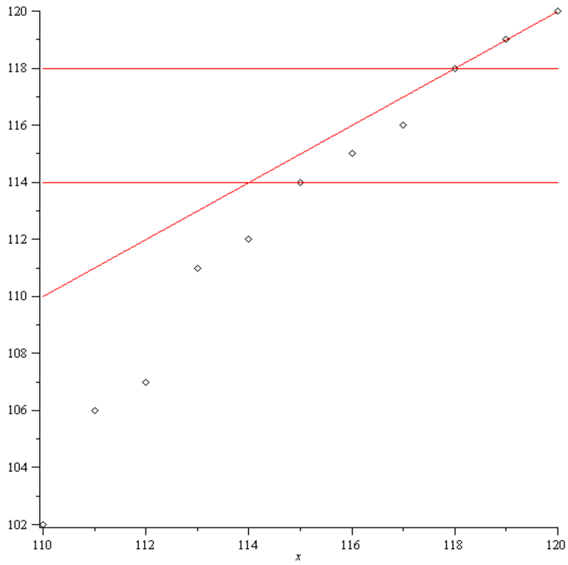


Fig.3. N(k) in the range k= 110-120 in the case of the 3·n+1. Pointplot in black, in red the function y= g(k)=k and the constant functions y=114 and y=118 (in red).

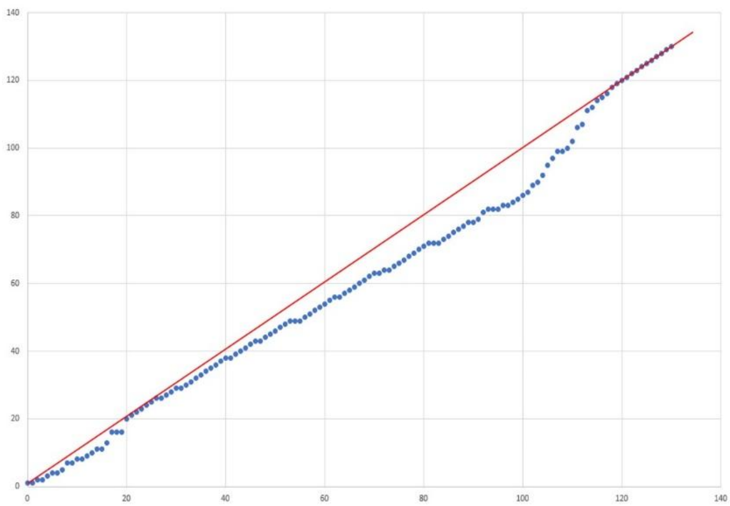


Fig.4. Pointplot of N(k) i.e. the number of integers not exceeding k appearing in the tree of the inverse orbit of the (3·n+1, n/2), as a function of the depth of the tree, in the range k ϵ [0..130].At k=115, N(115)=114 (Notice that (l(97) = 118!).

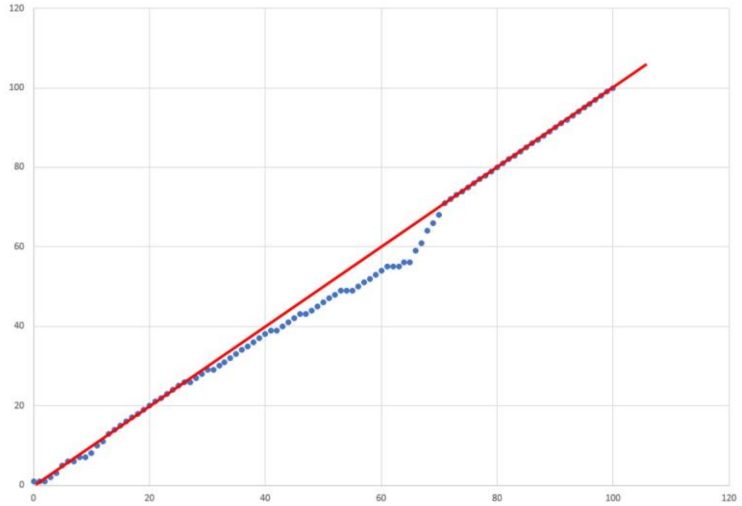


Fig.5. Pointplot of N(k) i.e. the number of integers not exceeding k appearing in the tree of the inverse orbit of the ((3·n+1)/2, n/2), as a function of the depth of the tree, in the range kϵ[0..100]). At k=73, N(73)=73.

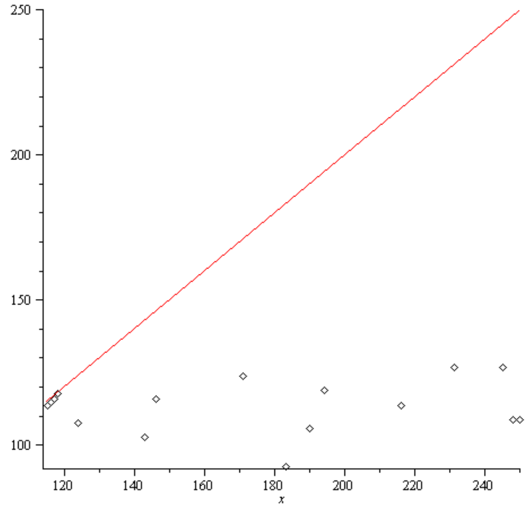


Fig.6. The length l(n) of some longest orbits in the 3·n+1as a function of n in the range n=[115..250]. In red the function y=n.

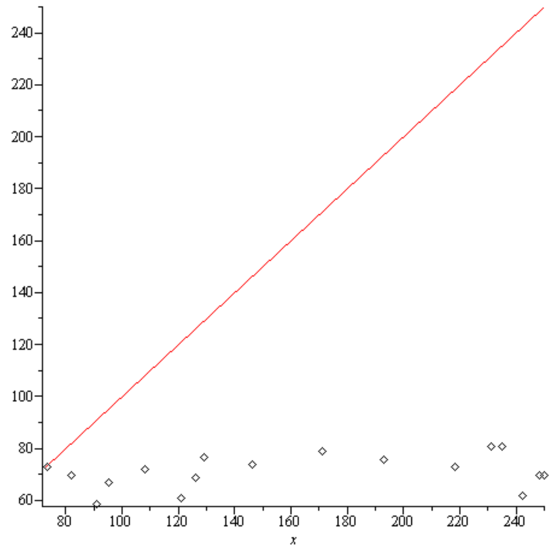


Fig. 7. The length l(n)of some longest orbits in the (3·n+1)/2 as a function of n in the range n=[73..250]. In red the function y=n.

Remark

We have observed that the largest values of l(n) in the subsequent intervals decrease i.e. l(n)/n is decreasing-let say- as a function of the "center" of the intervals we have considered i.e. in the range i.e. [1..250],[250..500],[500..750], [750..1000]; for the 3n+1, we have l(871)/871 = 178/871~ 0.2< 1 and for the (3n+1)/2 we have l(871)/871=113/871~ 0.13.

The plots have been given here only for the first interval, i.e. n ϵ [0..250]for both the formulations. To make contact with important models for the (3·n+1)/2 case we add below the plot of l(n) in the range n=[500.. 1000] and the bound l(n) =41·7·log(n) of Lagarias-Weiss in their stochastic models [6] (in red).In red also the function y = n. For some large values of n, l(n) ~36·log(n), see the remark of Applegate and Lagarias about Vyssotsky [12].

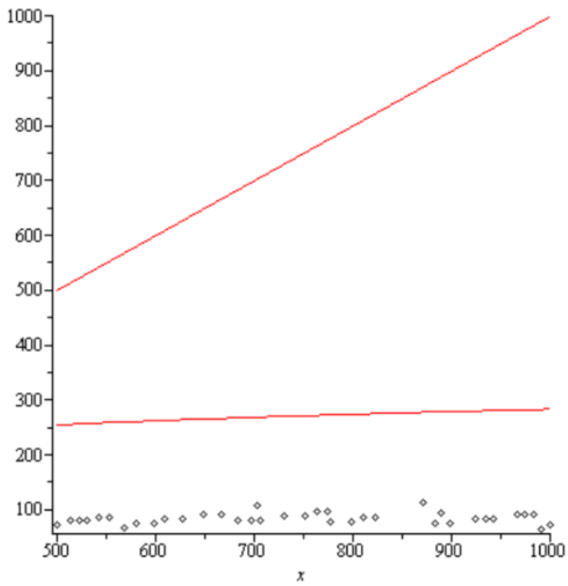
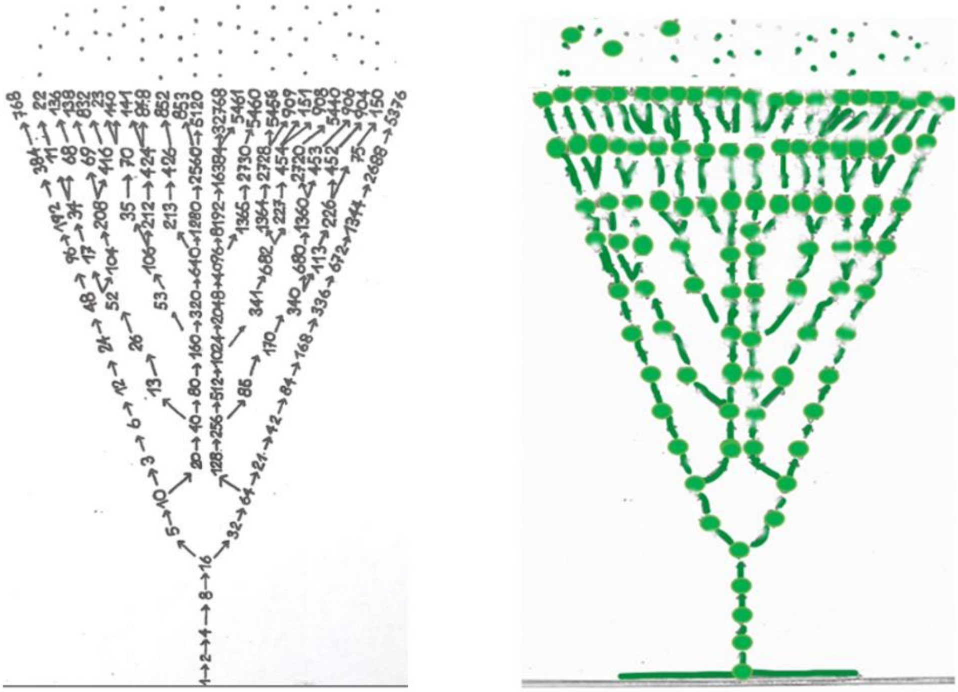


Fig.8. Some largest values of l(n) in the interval [500-1000]. In red the Lagarias-Weiss bound 41·7·log(n) in their stochastic models and the function y=n·(in red).

V. Conjecture

There are at least the first k integers 1,2,3...k on the chalice of depth k,for k≥118 in the 3·n+1 and for k ≥73 in the (3·n+1)/2formulation.The numbers k=118 resp. k=73 have been called here "saturation points". An experiment in the range of n=[119..1000] for the 3·n+1 and in the range n=[74..1000] for the (3·n+1)/2 confirms 100% our conjecture in such a finite domain [11].

Below, we present on the Figure 9 our original chalice [5]of the inverse orbits in the 3·n+1 case of depth k=15 where N(k=15)=11(i.e. 11 integers≤15) (figure 9a) and the chalice in green without the numbers on it (figure 9b), illustrating the equality of leaves at the top of the chalice with the number of bifurcations inside the chalice, i.e. the number of all odd on the full chalice (24 leaves, i.e. 24 bifurcations) and the number of the evens at the level k=15 (18) equal to the number of odds up to the level k-1 = 14, i.e. the cardinality of the numbers at the level k-1=14 (18). The cardinality of the chalice of depth 15 is equal to 103.



a) b)

Fig. 9 a) Chalice of the inverse orbits in the 3·n+1 case of depth k=15 where N(k=15)=11 [5]. b) Chalice of the inverse orbit for the 3·n+1 of depth k=15 with the 24 leaves in green.

Concluding remark

This work represents an attempt to understand more the truthfulness of Collatz's hypothesis, in agreement to other some recent studies [7, 8, 9, 10].

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Appendix 1

The next two tables present the orbits calculated for n comprise between 2 and 250 for 3·n+1 and (3·n+1)/2, respectively. The tables are calculated using different ad hoc C and C++ programs.

n	Orbits	n	Orbits	n	Orbits	n	Orbits	n	Orbits
1		51	24	101	25	151	15	201	18
2	1	52	11	102	25	152	23	202	26
3	7	53	11	103	87	153	36	203	39
4	2	54	112	104	12	154	23	204	26
5	5	55	112	105	38	155	85	205	26
6	8	56	19	106	12	156	36	206	88
7	16	57	32	107	100	157	36	207	88
8	3	58	19	108	113	158	36	208	13
9	19	59	32	109	113	159	54	209	39
10	6	60	19	110	113	160	10	210	39
11	14	61	19	111	69	161	98	211	39
12	9	62	107	112	20	162	23	212	13
13	9	63	107	113	12	163	23	213	13
14	17	64	6	114	33	164	111	214	101
15	17	65	27	115	33	165	111	215	101
16	4	66	27	116	20	166	111	216	114
17	12	67	27	117	20	167	67	217	26
18	20	68	14	118	33	168	10	218	114
19	20	69	14	119	33	169	49	219	52
20	7	70	14	120	20	170	10	220	114
21	7	71	102	121	95	171	124	221	114
22	15	72	22	122	20	172	31	222	70
23	15	73	115	123	46	173	31	223	70
24	10	74	22	124	108	174	31	224	21
25	23	75	14	125	108	175	80	225	52
26	10	76	22	126	108	176	18	226	13
27	111	77	22	127	46	177	31	227	13
28	18	78	35	128	7	178	31	228	34
29	18	79	35	129	121	179	31	229	34
30	18	80	9	130	38	180	18	230	34
31	106	81	22	131	28	182	18	232	127
32	5	82	110	132	28	182	93	232	21
33	26	83	110	133	28	183	93	233	83
34	13	84	9	134	28	184	18	234	21
35	13	85	9	135	41	185	44	235	127
36	21	86	30	136	15	186	18	236	34
37	21	87	30	137	90	187	44	237	34
38	21	88	17	138	15	188	106	238	34
39	34	89	30	139	41	189	106	239	52
40	8	90	17	140	15	190	106	240	21
41	109	91	92	141	15	191	44	241	21
42	8	92	17	142	103	192	13	242	96
43	29	93	17	143	103	193	119	243	96
44	16	94	105	144	23	194	119	244	21
45	16	95	105	145	116	195	119	245	21
46	16	96	12	146	116	196	26	246	47
47	104	97	118	147	116	197	26	247	47
48	11	98	25	148	23	198	26	248	109
49	24	99	25	149	23	199	119	249	47
50	24	100	25	150	15	200	26	250	109

Table 2. The orbits of the 3·n+1, n =[2..250]

n	Orbits	n	Orbits	n	Orbits	n	Orbits	n	Orbits
1		51	17	101	18	151	12	201	14
2	1	52	9	102	18	152	17	202	19
3	5	53	9	103	56	153	25	203	27
4	2	54	71	104	10	154	17	204	19
5	4	55	71	105	26	155	55	205	19
6	6	56	14	106	10	156	25	206	57
7	11	57	22	107	64	157	25	207	57
8	3	58	14	108	72	158	25	208	11
9	13	59	22	109	72	159	36	209	27
10	5	60	14	110	72	160	9	210	27
11	10	61	14	111	45	161	63	211	27
12	7	62	68	112	15	162	17	212	11
13	7	63	68	113	10	163	17	213	11
14	12	64	6	114	23	164	71	214	65
15	12	65	19	115	23	165	71	215	65
16	4	66	19	116	15	166	71	216	73
17	9	67	19	117	15	167	44	217	19
18	14	68	11	118	23	168	9	218	73
19	14	69	11	119	23	169	33	219	35
20	6	70	11	120	15	170	9	220	73
21	6	71	65	121	61	171	79	221	73
22	11	72	16	122	15	172	22	222	46
23	11	73	73	123	31	173	22	223	46
24	8	74	16	124	69	174	22	224	16
25	16	75	11	125	69	175	52	225	35
26	8	76	16	126	69	176	14	226	11
27	70	77	16	127	31	177	22	227	11
28	13	78	24	128	7	178	22	228	24
29	13	79	24	129	77	179	22	229	24
30	13	80	8	130	20	180	14	230	24
31	67	81	16	131	20	182	14	232	81
32	5	82	70	132	20	182	60	232	16
33	18	83	70	133	20	183	60	233	54
34	10	84	8	134	20	184	14	234	16
35	10	85	8	135	28	185	30	235	81
36	15	86	21	136	12	186	14	236	24
37	15	87	21	137	58	187	30	237	24
38	15	88	13	138	12	188	68	238	24
39	23	89	21	139	28	189	68	239	35
40	7	90	13	140	12	190	68	240	26
41	69	91	59	141	12	191	30	241	16
42	7	92	13	142	66	192	11	242	62
43	20	93	13	143	66	193	76	243	62
44	12	94	67	144	17	194	76	244	16
45	12	95	67	145	74	195	76	245	16
46	12	96	10	146	74	196	19	246	32
47	66	97	75	147	74	197	19	247	32
48	9	98	18	148	17	198	19	248	70
49	17	99	18	149	17	199	76	249	32
50	17	100	18	150	12	200	19	250	70

Table 3. The orbits for the (3·n+1)/2, n=2-250.

Appendix 2: Table of the formation of the integers from 1 to n in the tree of the inverse orbits of the 3·n+1
In the Table 4, we write in the horizontal lines from the left to the right the ordered natural numbers appeared in the chalice as a function of the height or depth k starting with k=0.

k	
0	1
1	1, 2
2	1,2, 4,
3	1,2, 4, , ,8,
4	1,2, 4, ,8, , , , , , 16
5	1,2, 4, 5, ,8, 16
6	1,2, 4, 5, ,8 , ,10, 16
7	1,2, 3,4, 5, ,8, ,10, 16 , 20 ,21
8	1,2, 3,4, 5,6 , ,8 , ,10, 16 ,20,21
9	1,2,3, 4 , 5,6 , ,8 , ,10, ,12,13 16 ,20,21
10	1,2,3, 4 , 5,6, ,8, ,10, ,12,13 16 ,20,21, ,24
11	1,2,3, 4 , 5,6, ,8, ,10 , ,12,13 16 ,20,21 ,24
12	1,2,3,4 , 5,6, ,8 , ,10 , ,12,13, , 16,17 ,20,21 ,24
13	1,2,3,4 , 5,6 ,8 , ,10 , ,12,13, , 16,17 ,20,21 ,24
14	1,2,3,4, 5,6 ,8 , ,10, 11 ,12,13 , 16,17 ,20,21 ,24
15	**1,2,3,4, 5,6 ,8 , ,10, 11 ,12,13** , 16,17 ,20,21,22,23,24
16	1,2,3,4, 5,6 ,7 ,8 , ,10, 11 ,12,13 , 16,17 ,20,21,22,23,24
17	1,2,3,4, 5,6 ,7 ,8 , ,10, 11 ,12,13,14,15, 16,17 ,20,21,22,23,24
18	1,2,3,4, 5,6 ,7 ,8 , ,10, 11 ,12,13,14,15, 16,17 ,20,21,22,23,24
19	1,2,3,4, 5,6 ,7 ,8 , 9 ,10, 11 ,12,13,14,15,16,17 ,20,21,22,23,24
20 .	1,2,3,4, 5,6 ,7 ,8 , 9 ,10, 11 ,12,13,14,15,16,17,18,19,20,21,22,23,24
21.	24, ,26 *,28 ,29,30, *,32, ,34,35,36, 37,38, , 40,* ,42,43, 44,45,46 ,48
22.	24, ,26 * ,28 ,29,30, *,32, ,34,35,36, 37,38, , 40,* ,42,43, 44,45,46 ,48
23.	24,25,26 *, 28 ,29,30, *,32, ,34,35,36, 37 ,38, , 40, * ,42,43, 44,45,46, 48 51
24	24,25,26 *, 28 ,29,30, *32, ,34,35,36, 37, 38, ,40,* ,42, 43, 44,45,46,48,49,50,51
25	24,25,26, *, 28 ,29,30,*32, ,34,35,36, 37, 38, , 40,* ,42, 43, 44,45,46,48,49,50
26	24,25, 26, *, 28 ,29,30,*32, 33 ,34,35,36, 37, 38, , 40,* ,42, 43, 44,45,46,48,49,50
27	24,25, 26, *, 28 ,29,30,*32, 33 ,34,35,36, 37, 38, * 40,* ,42, 43, 44,45,46,48,49,50
28	24,25, 26, *, 28 ,29,30,*32, 33 ,34,35,36, 37,38,39,40,* ,42, 43, 44,45,46,48,49,50

Notice:
27, 31, 41 long orbit: l(27)=111, l(31)=106,l(41)=109, ….and l(97)= 118.

Table 4. Inverse Orbits starting from the 0.

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