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On Totally $\mathcal{N}g^{\#}-$ Continuous Functions in Neutrosophic Topological Space

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Abstract:
In this article, we introduce a new concept of Neutrosophic continuous functions called totally $Ng^{\#}-$ continuous functions and study their properties in Neutrosophic topological spaces.

***Key Word**: $\mathcal{N}g^{\#}-$ closed set, $\mathcal{N}g^{\#}-$ continuous function, totally $\mathcal{N}g^{\#}-$continuous function.*

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I. Introduction

Smarandache [4] introduced the idea of Neutrosophic set, and in 2014 Salama et.al. [12] initiated further studies into Neutrosophic closed sets and Neutrosophic continuous functions. Recently Pious Missier et.al.[7],[8], introduced the concept of $\mathcal{N}g^{\#}-$ closed sets, continuous and irresolute mappings, in Neutrosophic Topological Spaces. In this paper, we introduce a new type of continuity in the consept of Neutrosophic topology called totally $\mathcal{N}g^{\#}-$ continuous functions and investigate their properties with necessary examples.

II. Preliminaries

Definition 2.1 [4] A Neutrosophic set $(\mathcal{NS})\mathcal{A}_{\mathcal{N}}$ is an object having the form
$\mathcal{A}_{\mathcal{N}}=\{\langle x,\mu_{\mathcal{A}_{\mathcal{N}}}(x),\sigma_{\mathcal{A}_{\mathcal{N}}}(x),\gamma_{\mathcal{A}_{\mathcal{N}}}(x)\rangle : x\in\mathcal{X}\}$ where $\mu_{\mathcal{A}_{\mathcal{N}}}(x),\sigma_{\mathcal{A}_{\mathcal{N}}}(x)$ and $\gamma_{\mathcal{A}_{\mathcal{N}}}(x)$ represent the degree of membership, degree of indeterminacy and the degree of non- membership respectively of each element $x\in\mathcal{X}$ to the set $\mathcal{A}_{\mathcal{N}}$. A Neutrosophic set $\mathcal{A}_{\mathcal{N}}=\{\langle x,\mu_{\mathcal{A}_{\mathcal{N}}}(x),\sigma_{\mathcal{A}_{\mathcal{N}}}(x),\gamma_{\mathcal{A}_{\mathcal{N}}}(x)\rangle : x\in\mathcal{X}\}$ can be identified as an ordered triple $\langle\mu_{\mathcal{A}_{\mathcal{N}}}(x),\sigma_{\mathcal{A}_{\mathcal{N}}}(x),\gamma_{\mathcal{A}_{\mathcal{N}}}(x)\rangle$ in $]-0,1+[$ on $\mathcal{X}$.

Definition 2.2 [12] For any two Neutrosophic sets $\mathcal{A}_{\mathcal{N}}=\{\langle x,\mu_{\mathcal{A}_{\mathcal{N}}}(x),\sigma_{\mathcal{A}_{\mathcal{N}}}(x),\gamma_{\mathcal{A}_{\mathcal{N}}}(x)\rangle : x\in\mathcal{X}\}$ and $\mathcal{B}_{\mathcal{N}}=\{\langle x,\mu_{\mathcal{B}_{\mathcal{N}}}(x),\sigma_{\mathcal{B}_{\mathcal{N}}}(x),\gamma_{\mathcal{B}_{\mathcal{N}}}(x)\rangle : x\in\mathcal{X}\}$ we have

- $\mathcal{A}_{\mathcal{N}}\subseteq\mathcal{B}_{\mathcal{N}}\Leftrightarrow\mu_{\mathcal{A}_{\mathcal{N}}}(x)\le\mu_{\mathcal{B}_{\mathcal{N}}}(x),\sigma_{\mathcal{A}_{\mathcal{N}}}(x)\le\sigma_{\mathcal{B}_{\mathcal{N}}}(x)$ and $\gamma_{\mathcal{A}_{\mathcal{N}}}(x)\ge\gamma_{\mathcal{B}_{\mathcal{N}}}(x)$.
- $\mathcal{A}_{\mathcal{N}}\cap\mathcal{B}_{\mathcal{N}}=\langle x,\mu_{\mathcal{A}_{\mathcal{N}}}(x)\wedge\mu_{\mathcal{B}_{\mathcal{N}}}(x),\sigma_{\mathcal{A}_{\mathcal{N}}}(x)\wedge\sigma_{\mathcal{B}_{\mathcal{N}}}(x)$ and $\gamma_{\mathcal{A}_{\mathcal{N}}}(x)\vee\gamma_{\mathcal{B}_{\mathcal{N}}}(x)\rangle$
- $\mathcal{A}_{\mathcal{N}}\cup\mathcal{B}_{\mathcal{N}}=\langle x,\mu_{\mathcal{A}_{\mathcal{N}}}(x)\vee\mu_{\mathcal{B}_{\mathcal{N}}}(x),\sigma_{\mathcal{A}_{\mathcal{N}}}(x)\vee\sigma_{\mathcal{B}_{\mathcal{N}}}(x)$ and $\gamma_{\mathcal{A}_{\mathcal{N}}}(x)\wedge\gamma_{\mathcal{B}_{\mathcal{N}}}(x)\rangle$

Definition 2.3 [12] Let $\mathcal{A}_{\mathcal{N}}=\langle\mu_{\mathcal{A}_{\mathcal{N}}}(x),\sigma_{\mathcal{A}_{\mathcal{N}}}(x),\gamma_{\mathcal{A}_{\mathcal{N}}}(x)\rangle$ be a $\mathcal{N}$S on $\mathcal{X}$, then the complement ${\mathcal{A}_{\mathcal{N}}}^{c}$ defined as

- ${\mathcal{A}_{\mathcal{N}}}^{c}=\{\langle x,\gamma_{\mathcal{A}_{\mathcal{N}}}(x),1-\sigma_{\mathcal{A}_{\mathcal{N}}}(x),\mu_{\mathcal{A}_{\mathcal{N}}}(x)\rangle : x\in\mathcal{X}\}$

Note that for any two Neutrosophic sets $\mathcal{A}_{\mathcal{N}}$ and $\mathcal{B}_{\mathcal{N}}$,

- $(\mathcal{A}_{\mathcal{N}} \cup \mathcal{B}_{\mathcal{N}})^{c} = {\mathcal{A}_{\mathcal{N}}}^{c} \cap {\mathcal{B}_{\mathcal{N}}}^{c}$
- $(\mathcal{A}_{\mathcal{N}} \cap \mathcal{B}_{\mathcal{N}})^{c} = {\mathcal{A}_{\mathcal{N}}}^{c} \cup {\mathcal{B}_{\mathcal{N}}}^{c}$.

Definition 2.4 [12] A Neutrosophic topology $(\mathcal{NT})$ on a non-empty set $\mathcal{X}$ is a family τ of Neutrosophic subsets in $\mathcal{X}$ satisfies the following axioms:

1. $\mathbf{0}_{\mathcal{N}}, \mathbf{1}_{\mathcal{N}} \in \tau$
2. $R_{N_1} \cap R_{N_2} \in \tau$ for any $R_{N_1}, R_{N_2} \in \tau$
3. $\cup R_{N_i} \in \tau \quad \forall \quad R_{N_i} : i \in I \subseteq \tau$

Here the empty set $\mathbf{0}_{\mathcal{N}}$ and the whole set $\mathbf{1}_{\mathcal{N}}$ may be defined as follows:

1. $\mathbf{0}_{\mathcal{N}} = \{\langle x, 0,0,1\rangle : x \in \mathcal{X}\}$
2. $\mathbf{1}_{\mathcal{N}} = \{\langle x, 1,1,0\rangle : x \in \mathcal{X}\}$

Definition 2.5 [12] Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{N}$S in $\mathcal{NTS}X_{\mathcal{N}}$. Then

1. $\mathcal{N}\text{int}(\mathcal{A}_{\mathcal{N}}) = \cup \{G : G$ isa $\mathcal{NOS}$ in $X_{\mathcal{N}}$ and $G \subseteq \mathcal{A}_{\mathcal{N}}\}$ is called a Neutrosophic interior of $\mathcal{A}_{\mathcal{N}}$.
2. $\mathcal{N}\text{cl}(\mathcal{A}_{\mathcal{N}}) = \cap \{K : K$ is a $\mathcal{NCS}$ in $X_{\mathcal{N}}$ and $\mathcal{A}_{\mathcal{N}} \subseteq K\}$ is called Neutrosophic closure of $\mathcal{A}_{\mathcal{N}}$.

Definition 2.6 [5] A Neutrosophic set $\mathcal{A}_{\mathcal{N}}$ of a $\mathcal{NTS}$ $(\mathcal{X}, \tau)$ is called a neutrosophic $\mathcal{N}\alpha$gCS if $\mathcal{N}\alpha\text{cl}(\mathcal{A}_{\mathcal{N}}) \subseteq \mathcal{U}_{\mathcal{N}}$, whenever $\mathcal{A}_{\mathcal{N}} \subseteq \mathcal{U}_{\mathcal{N}}$ and $\mathcal{U}_{\mathcal{N}}$ is a $\mathcal{NOS}$ in $\mathcal{X}$. The complement of $\mathcal{N}\alpha$gCS is $\mathcal{N}\alpha$gOS.

Definition 2.7 [7] A Neutrosophic set $\mathcal{A}_{\mathcal{N}}$ of a $\mathcal{NTS}$ $(\mathcal{X}, \tau)$ is called a Neutrosophic $g^{\#}-$closed ($\mathcal{N}g^{\#}$CS) if $\mathcal{N}\text{cl}(\mathcal{A}_{\mathcal{N}}) \subseteq \mathcal{Q}_{\mathcal{N}}$ whenever $\mathcal{A}_{\mathcal{N}} \subseteq \mathcal{Q}_{\mathcal{N}}$ and $\mathcal{Q}_{\mathcal{N}}$ is $\mathcal{N}\alpha$gOS in $\mathcal{X}$. The complement of $\mathcal{N}g^{\#}$CS is $\mathcal{N}g^{\#}$OS.

Definition 2.8 [11] Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{N}$S in $\mathcal{NTS}$ $\mathcal{X}$. Then

1. $\mathcal{N}g^{\#}\text{int}(\mathcal{A}_{\mathcal{N}}) = \cup \{G : G$ is a $\mathcal{N}g^{\#}$OS in $\mathcal{X}$ and $G \subseteq \mathcal{A}_{\mathcal{N}}\}$ is called a Neutrosophic $g^{\#}-$ interior of $\mathcal{A}_{\mathcal{N}}$.
2. $\mathcal{N}g^{\#}\text{cl}(\mathcal{A}_{\mathcal{N}}) = \cap \{K : K$ is a $\mathcal{N}g^{\#}$CS in $\mathcal{X}$ and $\mathcal{A}_{\mathcal{N}} \subseteq K\}$ is called Neutrosophic $g^{\#}-$ closure of $\mathcal{A}_{\mathcal{N}}$.

Definition 2.9 [8] A function $f_{\mathcal{N}} : (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ is said to be $\mathcal{N}g^{\#}-$ continuous function if $f_{\mathcal{N}}^{-1}(\mathcal{V}_{\mathcal{N}})$ is a $\mathcal{N}g^{\#}-$ closed set of $(\mathcal{X}, \tau)$ for every neutrosophic closed set $\mathcal{V}_{\mathcal{N}}$ of $(\mathcal{Y}, \zeta)$.

Definition 2.10 [8] A function $f_{\mathcal{N}} : (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ is said to be Neutrosophic $g^{\#}-$ irresolute function if $f_{\mathcal{N}}^{-1}(\mathcal{V}_{\mathcal{N}})$ is a $\mathcal{N}g^{\#}$CS of $(\mathcal{X}, \tau)$ for every $\mathcal{N}g^{\#}$CS $\mathcal{V}_{\mathcal{N}}$ of $(\mathcal{Y}, \zeta)$.

Definition 2.11 [11] A Neutrosophic Topological space $(\mathcal{X}, \tau)$ is called a $T_{\mathcal{N}}g^{\#}-$ space if every $\mathcal{N}g^{\#}$CS in $(\mathcal{X}, \tau)$ is $\mathcal{NCS}$ in $(\mathcal{X}, \tau)$.

Definition 2.14 [9] A function $f_{\mathcal{N}} : (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ is said to be $\mathcal{N}g^{\#}-$ contra continuous if $f_{\mathcal{N}}^{-1}(\mathcal{V}_{\mathcal{N}})$ is a $\mathcal{N}g^{\#}-$ closed set of $(\mathcal{X}, \tau)$ for every neutrosophic open set $(\mathcal{Y}, \zeta)$.

Definition 2.15 [9] A function $f_{\mathcal{N}} : (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ is said to be $\mathcal{N}-$ contra continuous if $f_{\mathcal{N}}^{-1}(\mathcal{V}_{\mathcal{N}})$ is a $\mathcal{N}-$ closed set of $(\mathcal{X}, \tau)$ for every neutrosophic open set $(\mathcal{Y}, \zeta)$.

Definition 2.16 [10] A function $f_{\mathcal{N}} : (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ is said to be perfectly $\mathcal{N}g^{\#}-$ continuous if the inverse image of every $\mathcal{N}g^{\#}-$ closed set in $(\mathcal{Y}, \zeta)$is Neutrosophic clopen set in $(\mathcal{X}, \tau)$.

III. Totally $\mathcal{N}g^{\#}-$ Continuous Functions

In this section, we introduce totally $\mathcal{N}g^{\#}-$ continuous functions and discuss some of their interesting properties.

Definition 3.1 A function $f_{\mathcal{N}} : (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ is said to be totally $\mathcal{N}g^{\#}-$ continuous if the inverse image of every Neutrosophic closed set in $(\mathcal{Y}, \zeta)$ is both $\mathcal{N}g^{\#}$CS and $\mathcal{N}g^{\#}$OS (ie, $\mathcal{N}g^{\#}-$ clopen set) in $(\mathcal{X}, \tau)$.

Example 3.2 Let $\mathcal{X} = \{l, m\} = \mathcal{Y}$ Consider the Neutrosophic sets

$\mathcal{M}_{\mathcal{N}_1} = \{\langle l, (0.4,0.3,0.6)\rangle, \langle m, (0.4,0.4,0.6)\rangle\}$,

$\mathcal{M}_{\mathcal{N}_2} = \{\langle p, (0.6,0.7,0.4)\rangle, \langle q, (0.6,0.7,0.4)\rangle\}$.

Now Then $\tau = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathcal{M}_{\mathcal{N}_2}, \mathbf{1}_{\mathcal{N}}\}$ and $\zeta = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_2}, \mathbf{1}_{\mathcal{N}}\}$ are $\mathcal{NT}$s on $\mathcal{X}$ and $\mathcal{Y}$ respectively. Define $f_{\mathcal{N}}: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ by $f_{\mathcal{N}}(l) = l$ and $f_{\mathcal{N}}(m) = m$. Here $\mathcal{N}g^{\#}COS(X) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathcal{M}_{\mathcal{N}_2}, \mathbf{1}_{\mathcal{N}}\}$. Now $\mathcal{M}_{\mathcal{N}_2}$ is $\mathcal{NCS}$ in $\mathcal{Y}$ and $f_{\mathcal{N}}{}^{-1}(\mathcal{M}_{\mathcal{N}_2})$ is $\mathcal{N}g^{\#}$ − clopen set in $(\mathcal{X}, \tau)$. Therefore, $f_{\mathcal{N}}$ is totally $\mathcal{N}g^{\#}$ − continuous.

Theorem 3.3 Every perfectly $\mathcal{N}g^{\#}$ − continuous function is totally $\mathcal{N}g^{\#}$ − continuous function but not conversely.

Proof. Let $f_{\mathcal{N}}: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ be any neutrosophic function. Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{NCS}$ in $(\mathcal{Y}, \zeta)$. Then $\mathcal{A}_{\mathcal{N}}$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{Y}, \zeta)$. Since $f_{\mathcal{N}}$ is a perfectly $\mathcal{N}g^{\#}$ − continuous function, $f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}})$ is both $\mathcal{NCS}$ and $\mathcal{NOS}$ in $(\mathcal{X}, \tau)$. Which implies $f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}})$ is both $\mathcal{N}g^{\#}CS$ and $\mathcal{N}g^{\#}OS$ in $(\mathcal{X}, \tau)$. Hence, $f_{\mathcal{N}}$ is totally $\mathcal{N}g^{\#}$ − continuous function.

Example 3.4 Let $\mathcal{X} = \{l, m\} = \mathcal{Y}$ Consider the Neutrosophic sets

$\mathcal{M}_{\mathcal{N}_1} = \{\langle l, (0.4,0.3,0.6)\rangle, \langle m, (0.4,0.4,0.6)\rangle\}$,

$\mathcal{M}_{\mathcal{N}_2} = \{\langle p, (0.6,0.7,0.4)\rangle, \langle q, (0.6,0.7,0.4)\rangle\}$,

$\mathcal{M}_{\mathcal{N}_3} = \{\langle l, (0.7,0.8,0.3)\rangle, \langle m, (0.8,0.8,0.3)\rangle\}$,

$\mathcal{M}_{\mathcal{N}_4} = \{\langle l, (0.3,0.2,0.7)\rangle, \langle m, (0.3,0.2,0.8)\rangle\}$.

Now Then $\tau = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathcal{M}_{\mathcal{N}_2}, \mathbf{1}_{\mathcal{N}}\}$ and $\zeta = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathbf{1}_{\mathcal{N}}\}$ are $\mathcal{NT}$s on $\mathcal{X}$ and $\mathcal{Y}$ respectively. Define $f_{\mathcal{N}}: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ by $f_{\mathcal{N}}(l) = l$ and $f_{\mathcal{N}}(m) = m$. Here $\mathcal{N}g^{\#}COS(X) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathcal{M}_{\mathcal{N}_2}, \mathbf{1}_{\mathcal{N}}\} = \mathcal{NCOS}(\mathcal{X})$, $\mathcal{N}g^{\#}CS(Y) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_2}, \mathcal{M}_{\mathcal{N}_3}, \mathbf{1}_{\mathcal{N}}\}$. Now $\mathcal{M}_{\mathcal{N}_2}$ is $\mathcal{NCS}$ in $\mathcal{Y}$ and $f_{\mathcal{N}}{}^{-1}(\mathcal{M}_{\mathcal{N}_2})$ is $\mathcal{N}g^{\#}$ − clopen set in $(\mathcal{X}, \tau)$. Therefore, $f_{\mathcal{N}}$ is totally $\mathcal{N}g^{\#}$ − continuous. But $\mathcal{M}_{\mathcal{N}_3}$ is $\mathcal{N}g^{\#}CS$ in $\mathcal{Y}$ and $f_{\mathcal{N}}{}^{-1}(\mathcal{M}_{\mathcal{N}_3})$ is not $\mathcal{N}$ − clopen set in $(\mathcal{X}, \tau)$. Therefore, $f_{\mathcal{N}}$ is not perfectly $\mathcal{N}g^{\#}$ − continuous.

Theorem 3.5 Every totally $\mathcal{N}g^{\#}$ − continuous function is $\mathcal{N}g^{\#}$ − continuous function.

Proof. Let $f_{\mathcal{N}}: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ be any neutrosophic function. Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{NCS}$ in $(\mathcal{Y}, \zeta)$. Since $f_{\mathcal{N}}$ is a totally $\mathcal{N}g^{\#}$ − continuous function, $f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}})$ is both $\mathcal{N}g^{\#}CS$ and $\mathcal{N}g^{\#}OS$ in $(\mathcal{X}, \tau)$. Which implies $f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{N}g^{\#}CS$ and in $(\mathcal{X}, \tau)$. Therefore, $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}$ − continuous function

Example 3.6 Let $\mathcal{X} = \{l, m\} = \mathcal{Y}$ Consider the Neutrosophic sets

$\mathcal{M}_{\mathcal{N}_1} = \{\langle l, (0.4,0.3,0.6)\rangle, \langle m, (0.4,0.4,0.6)\rangle\}$,

$\mathcal{M}_{\mathcal{N}_2} = \{\langle p, (0.6,0.7,0.4)\rangle, \langle q, (0.6,0.7,0.4)\rangle\}$,

$\mathcal{M}_{\mathcal{N}_3} = \{\langle l, (0.7,0.8,0.3)\rangle, \langle m, (0.8,0.8,0.3)\rangle\}$,

$\mathcal{M}_{\mathcal{N}_4} = \{\langle l, (0.3,0.2,0.7)\rangle, \langle m, (0.3,0.2,0.8)\rangle\}$.

Now Then $\tau = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathcal{M}_{\mathcal{N}_4}, \mathbf{1}_{\mathcal{N}}\}$ and $\zeta = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathbf{1}_{\mathcal{N}}\}$ are $\mathcal{NT}$s on $\mathcal{X}$ and $\mathcal{Y}$ respectively. Define $f_{\mathcal{N}}: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ by $f_{\mathcal{N}}(l) = l$ and $f_{\mathcal{N}}(m) = m$. Here $\mathcal{N}g^{\#}COS(X) = \{\mathbf{0}_{\mathcal{N}}, \mathbf{1}_{\mathcal{N}}\}$, $\mathcal{N}g^{\#}CS(X) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_2}, \mathcal{M}_{\mathcal{N}_3}, \mathbf{1}_{\mathcal{N}}\}$. Now $\mathcal{M}_{\mathcal{N}_2}$ is $\mathcal{NCS}$ in $\mathcal{Y}$ and $f_{\mathcal{N}}{}^{-1}(\mathcal{M}_{\mathcal{N}_2})$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}, \tau)$. Therefore, $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}$ − continuous. But $\mathcal{M}_{\mathcal{N}_2}$ is $\mathcal{NCS}$ in $\mathcal{Y}$ and $f_{\mathcal{N}}{}^{-1}(\mathcal{M}_{\mathcal{N}_2})$ is not $\mathcal{N}g^{\#}$ − clopen set in $(\mathcal{X}, \tau)$. Therefore, $f_{\mathcal{N}}$ is not totally $\mathcal{N}g^{\#}$ − continuous.

Theorem 3.7 Let $f_{\mathcal{N}}: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ be totally $\mathcal{N}g^{\#}$ − continuous function and $(\mathcal{Y}, \zeta)$ be $T_{\mathcal{N}}g^{\#}$ − spaces. Then $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}$ − irresolute function.

Proof. Let $\mathcal{A}_{\mathcal{N}}$ be any $\mathcal{N}g^{\#}CS$ in $(\mathcal{Y}, \zeta)$. Since $(\mathcal{Y}, \zeta)$ is $T_{\mathcal{N}}g^{\#}$ − space, $\mathcal{A}_{\mathcal{N}}$ is $\mathcal{NCS}$ in $(\mathcal{Y}, \zeta)$. Since $f_{\mathcal{N}}$ is totally $\mathcal{N}g^{\#}$ − continuous, $f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}})$ is both $\mathcal{N}g^{\#}CS$ and $\mathcal{N}g^{\#}OS$ in $(\mathcal{X}, \tau)$. $f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}, \tau)$. Therefore, $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}$ − irresolute.

Remark 3.8 Composition of two totally $\mathcal{N}g^{\#}$ − continuous functions need not be a totally $\mathcal{N}g^{\#}$ − continuous.

Example 3.9 Let $\mathcal{X} = \{p, q\} = \mathcal{Y} = \mathcal{Z}$. Consider the Neutrosophic sets
$\mathcal{M}_{\mathcal{N}_1} = \{\langle p, (0.4,0.5,0.6)\rangle, \langle q, (0.3,0.4,0.7)\rangle\}$,
$\mathcal{M}_{\mathcal{N}_2} = \{\langle p, (0.6,0.5,0.4)\rangle, \langle q, (0.7,0.6,0.3)\rangle\}$,
$\mathcal{M}_{\mathcal{N}_3} = \{\langle p, (0.3,0.4,0.7)\rangle, \langle q, (0.4,0.5,0.6)\rangle\}$,
$\mathcal{M}_{\mathcal{N}_4} = \{\langle p, (0.7,0.6,0.3)\rangle, \langle q, (0.6,0.5,0.4)\rangle\}$.

Now $(\mathcal{X}, \tau) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathcal{M}_{\mathcal{N}_2}, \mathbf{1}_{\mathcal{N}}\}, (\mathcal{Y}, \zeta) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_3}, \mathcal{M}_{\mathcal{N}_4}, \mathbf{1}_{\mathcal{N}}\} = (\mathcal{Z}, \eta)$ are Neutrosophic topological spaces. Then $\tau = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathcal{M}_{\mathcal{N}_2}, \mathbf{1}_{\mathcal{N}}\}, \zeta = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_3}, \mathcal{M}_{\mathcal{N}_4}, \mathbf{1}_{\mathcal{N}}\}$ and $\eta = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_3}, \mathbf{1}_{\mathcal{N}}\}$ are Neutrosophic topologies on $\mathcal{X}, \mathcal{Y}$ and $\mathcal{Z}$ respectively. Define a function $f_{\mathcal{N}}: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ by $f_{\mathcal{N}}(p) = q$ and $f_{\mathcal{N}}(q) = p$ and define a function $g_{\mathcal{N}}: (\mathcal{Y}, \zeta) \longrightarrow (\mathcal{Z}, \eta)$ by $g_{\mathcal{N}}(p) = p$ and $g_{\mathcal{N}}(q) = q$. Then $f_{\mathcal{N}}$ and $g_{\mathcal{N}}$ are $\mathcal{N}g^{\#}$ − contra continuos functions. Now define a function $g_{\mathcal{N}} \circ f_{\mathcal{N}}: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Z}, \eta)$ by $g_{\mathcal{N}} \circ f_{\mathcal{N}}(p) = p$ and $g_{\mathcal{N}} \circ f_{\mathcal{N}}(q) = q$. Here $\mathcal{M}_{\mathcal{N}_3}$ is a $\mathcal{NCS}$ in $(\mathcal{Z}, \eta)$. But $(g_{\mathcal{N}} \circ f_{\mathcal{N}})^{-1}(\mathcal{M}_{\mathcal{N}_4})$ is not a $\mathcal{N}g^{\#}$COS in $(\mathcal{X}, \tau)$. Hence $(g_{\mathcal{N}} \circ f_{\mathcal{N}})$ is not totally $\mathcal{N}g^{\#}$ − continuous function.

Theorem 3.10 Let $f_{\mathcal{N}}: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ be a $\mathcal{N}g^{\#}$ − irresolute function. Let $g_{\mathcal{N}}: (\mathcal{Y}, \zeta) \longrightarrow (\mathcal{Z}, \eta)$ be a totally $\mathcal{N}g^{\#}$ − continuous function. Then $(g_{\mathcal{N}} \circ f_{\mathcal{N}}): (\mathcal{X}, \tau) \longrightarrow (\mathcal{Z}, \eta)$ is totally $\mathcal{N}g^{\#}$ − continuous function.

Proof. Let $\mathcal{W}_{\mathcal{N}}$ be a $\mathcal{NCS}$ in $(\mathcal{Z}, \eta)$. Since $g_{\mathcal{N}}$ is totally $\mathcal{N}g^{\#}$ − continuous, ${g_{\mathcal{N}}}^{-1}(\mathcal{W}_{\mathcal{N}})$ is $\mathcal{N}g^{\#}$CS and $\mathcal{N}g^{\#}$OS in $(\mathcal{Y}, \zeta)$. Since $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}$ − irresolute, $f_{\mathcal{N}}^{-1}(g_{\mathcal{N}}^{-1}(\mathcal{W}_{\mathcal{N}}))$ is $\mathcal{N}g^{\#}$CS and $\mathcal{N}g^{\#}$OS in $(\mathcal{X}, \tau)$. Hence $g_{\mathcal{N}} \circ f_{\mathcal{N}}(\mathcal{W}_{\mathcal{N}})$ is totally $\mathcal{N}g^{\#}$ − continuous function.

Theorem 3.11 Let $f_{\mathcal{N}}: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ be a totally $\mathcal{N}g^{\#}$ − continuous function. Let $g_{\mathcal{N}}: (\mathcal{Y}, \zeta) \longrightarrow (\mathcal{Z}, \eta)$ be a $\mathcal{N}$ − continuous function. Then $(g_{\mathcal{N}} \circ f_{\mathcal{N}}): (\mathcal{X}, \tau) \longrightarrow (\mathcal{Z}, \eta)$ is totally $\mathcal{N}g^{\#}$ − continuous function.

Proof. Let $\mathcal{W}_{\mathcal{N}}$ be a $\mathcal{NCS}$ in $(\mathcal{Z}, \eta)$. By hypothesis, ${g_{\mathcal{N}}}^{-1}(\mathcal{W}_{\mathcal{N}})$ is a $\mathcal{NCS}$ in $(\mathcal{Y}, \zeta)$. Since $f_{\mathcal{N}}$ is totally $\mathcal{N}g^{\#}$ − continuous, $f_{\mathcal{N}}^{-1}(g_{\mathcal{N}}^{-1}(\mathcal{W}_{\mathcal{N}}))$ is a $\mathcal{N}g^{\#}$CS and $\mathcal{N}g^{\#}$OS in $(\mathcal{X}, \tau)$. Hence $g_{\mathcal{N}} \circ f_{\mathcal{N}}(\mathcal{W}_{\mathcal{N}})$ is totally $\mathcal{N}g^{\#}$ − continuous function.

Theorem 3.12 Let $f_{\mathcal{N}}: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ and $g_{\mathcal{N}}: (\mathcal{Y}, \zeta) \longrightarrow (\mathcal{Z}, \eta)$ be totally $\mathcal{N}g^{\#}$ − continuous function and $(\mathcal{Y}, \zeta)$ be $T_{\mathcal{N}}g^{\#}$ − spaces. Then $g_{\mathcal{N}} \circ f_{\mathcal{N}}: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Z}, \eta)$ is totally $\mathcal{N}g^{\#}$ − continuous function.

Proof. Let $\mathcal{A}_{\mathcal{N}}$ be any $\mathcal{NCS}$ in $(\mathcal{Z}, \eta)$. Since $g_{\mathcal{N}}$ is totally $\mathcal{N}g^{\#}$ − continuous, $g_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}})$ is both $\mathcal{N}g^{\#}$CS and $\mathcal{N}g^{\#}$OS in $(\mathcal{Y}, \zeta)$. Since $(\mathcal{Y}, \zeta)$ is $T_{\mathcal{N}}g^{\#}$ − spaces, $g_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{NCS}$ in $(\mathcal{Y}, \zeta)$. Since $f_{\mathcal{N}}$ is totally $\mathcal{N}g^{\#}$ − continuous, $f_{\mathcal{N}}^{-1}(g_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}})) = (g_{\mathcal{N}} \circ f_{\mathcal{N}})^{-1}(\mathcal{A}_{\mathcal{N}})$ is both $\mathcal{N}g^{\#}$CS and $\mathcal{N}g^{\#}$OS in $(\mathcal{X}, \tau)$. Therefore, $g_{\mathcal{N}} \circ f_{\mathcal{N}}$ is totally $\mathcal{N}g^{\#}$ − continuous function.

IV. Conclusion

In this article we introduced a new class of continuous function in Neutrosophic Topological space called totally $Ng^{\#}$ − continuous functions. Moreover, characterizations of totally $Ng^{\#}$ − continuous functions are analyzed and studied their properties.

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