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A Study of the Beam Fixed At Ends and Loaded In the Middle and Cantilever

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Abstract: *A cantilever is a thin unvaried cross-sectional beam unchanging horizontally at one end and loaded at the additional end. In this paper, we talk about the theory of the cantilever and the beam supported at its ends and loaded in the centervia the application of residue theorem to findtotal depression at a given point on them and it comes out to be very effective tool for analyzing the theory of the cantilever and the beam supported at its ends and loaded in the middle for finding thetotal depression at a given point on their length.*

Keywords: *Residue Approach, Cantilever, Beam, Mahgoub Transformation*

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I. Introduction:

The Residue Theorem approach has been apply for analyzing most of the problems in science and engineering [1, 2, 3, 4,] like analysis of electric circuits, analysis of quantum theory of free electron gas, integral equations of convolution type, etc. It also comes out to be very powerful mathematical tool for analyzing the theory of cantilever and the beam supported at its ends and loaded in the middle to find their total depression at a given point on them. A structure of uniform cross-section, whose length is large in comparison to its breadth and thickness, is called as beam. For such a structure, the shearing stress over any cross-section is negligibly small. A cantilever is a thin uniform beam fixed horizontally at one end and loaded at the other end [5]. Beams are used in the construction of bridges or for supporting the heavy loads, and are commonly used in the structure of multistoried buildings [6,7,8,9,10,11,12]. The Mahgoub Transformhas been applied in solving boundary value problems in most of the science and engineering disciplines. Theelectrical circuits are usually analyzed by adopting different integral transforms or methods [13,1415,16,17,18,19,20].In this paper, we will study a Mahgoub integral analyze the impulsive response of network circuits and reveals that it can also be obtained easily by the application of MahgoubTransform.

II. Definitions

2.1 Mahgoub Transform

If the function $f(\mathrm{y})$, $\mathrm{y} \geq 0$ is having an exponential order and is a piecewise continuous function on any interval, then the Mahgoub transform of$\hbar(\mathrm{y})$ is given by

$$\mathrm{M}\{\hbar(\mathrm{y})\} = \overline{\hbar}(p) = \mathrm{p}\int_0^\infty e^{-py}\,\hbar(\mathrm{y})dy.$$

The Mahgoub Transform [1, 2, 3] of some of the functions are given by

- $M\{y^n\} = \frac{n!}{p^n}, where n = 0,1,2,..$
- $M\{e^{ay}\} = \frac{p}{p-a},$
- $M\{sinay\} = \frac{ap}{p^2+a^2},$
- $M\{cosay\} = \frac{p^2}{p^2+a^2},$
- $M\{sinhay\} = \frac{ap}{p^2-a^2},$
- $M\{coshay\} = \frac{p^2}{p^2-a^2}.$

2.2 Inverse Mahgoub Transform

The Inverse Mahgoub Transform of some of the functions are given by

- $\mathrm{M}^{-1}\{\frac{1}{p^n}\} = \frac{y^n}{n!}$ $, n = 2, 3, 4 ...$

- $M^{-1}\{\frac{p}{p-a}\} = e^{ay}$
- $M^{-1}\{\frac{p}{p^2+a^2}\}= \frac{1}{a}\sin ay$
- $M^{-1}\{\frac{p^2}{p^2+a^2}\} =\cos ay$
- $M^{-1}\{\frac{ap}{p^2-a^2}\}= \frac{1}{a}\sin hay$
- $M^{-1}\{\frac{p^2}{p^2-a^2}\} =\frac{1}{a}\cos hay$

2.3 Mahgoub Transform of Derivatives

The Elzaki Transform [1, 2, 3] of some of the Derivatives of h(y) are given by

- $M\{h'(y)\} = pM\{h(y)\} - \mathrm{p}\, h(0)$
- $M\{\hbar''(y)\} = p^2\bar{\hbar}(p) - p^2\hbar(0) - \mathrm{p}\hbar'(0),$

$andsoon.$

Residue theorem

It states [1, 2] that $y(\mathrm{x})$ = sum of residues of $e^{px}\bar{y}(\mathrm{p})$ at the poles of$\bar{y}(\mathrm{p})$, where $\bar{y}(\mathrm{p})$ is the Laplace transform of$y(\mathrm{x})$.

Methods

The important methods [1, 4, 8,] of finding residues are

(i) If $y(\mathrm{x})$ has a simple pole at x = b, then residue of $y(\mathrm{x})$ at x = b i.e. Res $y(\mathrm{b}) =\lim_{x\to b}(x-b)y(x)$.

(ii) $Residue\ (for\ pole\ b\ of\ order\ n) = \lim_{x\to b}\frac{1}{n-1!}\frac{d^{n-1}}{dx^{n-1}}[x^n \quad y(x)]$

III. Methodology

CANTILEVER

Assume a horizontal beam CDof length L, fixed at the end C and loaded with load W vertically downwards at the free end D. The bending moment [5, 6] at the section X is given by the differential equation

$\ddot{y}(\mathrm{x}) + \frac{\mathrm{W}}{\mathrm{YI}}(\mathrm{L} - \mathrm{x}) = 0\ldots$ (a), where I is the geometrical moment of inertia and Y is the Young's modulus.Here y is the depression of the beam at the section X at the distance x from fixed end A.

On taking the Mahgoub Transform [9, 10] of (1), we get

$p^2\bar{y}(\mathrm{p}) - p^2y(0) - p\dot{y}(0)+\frac{\mathrm{W}}{\mathrm{YI}}(L-\frac{1}{p^2}) = 0$... (b)

Here $\bar{y}(\mathrm{p})$ is the mahgoub transform of $y(\mathrm{x})$.

$Put\ y(0) = 0, \dot{y}(0) = 0$ therefore, (2) becomes,

$p^2\bar{y}(\mathrm{p}) + \frac{\mathrm{W}}{\mathrm{YI}}(\mathrm{L}-\frac{1}{p}) = 0$

Or

$$\bar{y}(\mathrm{p}) = -\frac{\mathrm{W}}{\mathrm{YI}}(Lp-1)\frac{1}{p^3}$$

The pole of $\bar{\mathrm{y}}(\mathrm{p})$ is 0 and it is of order four. Residue of $e^{px}\,\bar{\mathrm{y}}(\mathrm{p})$ at p = 0 is equal to

$$\lim_{p\to 0}\frac{1}{2!}\frac{d^2}{dp^2}[e^{px}\ p^3\bar{\mathrm{y}}(\mathrm{p})]$$

$$= \lim_{p\to 0}\frac{1}{2!}\frac{d^2}{dp^2}\left[e^{px}\ p^3\frac{-\frac{\mathrm{W}}{\mathrm{YI}}(Lp-1)}{p^3}\right]$$

On solving, we get

Residue of $e^{px}\,\bar{\mathrm{y}}(\mathrm{p})$ at p = 0 is$\frac{1}{2!}\frac{\mathrm{W}}{\mathrm{YI}}(2Lx - x^2)$.

Applying Residue Theorem, we have

y (x) = Sum of residues of $e^{py}\bar{\mathrm{y}}(\mathrm{p})$ at its poles i.e.

$\mathrm{y(x)} = \frac{1}{2!}\frac{\mathrm{W}}{\mathrm{YI}}(2Lx - x^2)$….. (c)

At x = L i.e. at the free end, the total depression is given by

$\mathrm{y(L)} = \frac{1}{2!}\frac{\mathrm{W}}{\mathrm{YI}}(2L^2 - L^2)$

Or

$\mathrm{y(L)} = \frac{1}{2}\frac{\mathrm{W}L^2}{\mathrm{YI}}$….. (d)

For a beam of circular cross-section [12, 13]

$\mathrm{I} = \pi r^4/4$

Hence from (4), we have

$$y(L) = \frac{2WL^2}{Y\pi r^4}$$

BEAM SUPPORTED AT ENDS AND LOADED IN THE MIDDLE

Consider a beam supported on the two knife edges C and D and loaded in the middle with a load W vertically downwards. Assume L be the length of the beam between the points A and B. The bending moment [5, 6] at the section X is given by the differential equation

$\ddot{y}(x) + \frac{W}{2YI}x = 0$… (e), where I is the geometrical moment of inertia and Y is the Young's modulus. Here y is the depression of the beam at the section X at the distance x from the end A.

$$\cdot \equiv \frac{d}{dx}$$

On taking the Laplace Transform [10, 11] of (5), we get

$p^2\bar{y}(p) - p^2y(0) - p\dot{y}(0) + \frac{W}{2YI}\frac{1}{p^2} = 0$... (f)

Here $\bar{y}(p)$ is the Laplace transform of $y(x)$.

$Put\ y(0) = 0, \dot{y}(0) = K$ therefore, equation (6) becomes,

$p^2\bar{y}(p) - K + \frac{W}{2YI}\frac{1}{p} = 0$

$\bar{y}(p) = \frac{Kp - \frac{W}{2YI}}{p^3}$

The pole of $\bar{y}(p)$ is 0 and it is of order four. Residue of $e^{px}\,\bar{y}(p)$ at p = 0 is equal to

$$\lim_{p\to 0}\frac{1}{2!}\frac{d^2}{dp^2}[e^{px}\ p^3\bar{y}(p)]$$

$$= \lim_{p\to 0}\frac{1}{2!}\frac{d^2}{dp^2}\left[e^{px}\ p^3\frac{Kp - \frac{W}{2YI}}{p^3}\right]$$

On solving, we get

Residue of $e^{px}\,\bar{y}(p)$ at p = 0 is $\frac{1}{2!}(4Kx - \frac{W}{2YI}x^2)$.

Applying Residue Theorem, we have

y (x) = Sum of residues of $e^{py}\bar{y}(p)$ at its poles i.e.

$y(x) = \frac{1}{2!}(4Kx - \frac{W}{2YI}x^2)$…… (g)

At $x = \frac{L}{2}, \dot{y}\left(\frac{L}{2}\right) = 0$. Therefore, using (g) and solving for K, we get

$C = \frac{Wl}{4YI}$ …. (i)

Use (g) in (i), we get

$y(x) = \frac{1}{2!}[\frac{WLx}{2YI} - \frac{Wx^2}{2YI}]$………(j)

At the middle, the total depression is given by

$y(L/2) = \frac{1}{2!}[\frac{WLx}{2YI} - \frac{Wx^2}{2YI}]$

Or

$y(L/2) = \frac{WL^2}{16YI}$ ……. (k)

For a beam of circular cross-section [12, 13]

$I = \pi r^4/4$

Hence from (10), we have

$$y(L/2) = \frac{WL^2}{Y\pi r^4}$$

IV. Conclusion:

We have Analysis the theory ofthe cantilever and the beam supported at its ends and loaded in the middle and found the total depression at the free end of the cantilever, and at the middle of the beam supported at its ends and loaded in the middle via the application of residue theorem. It is found that the depression increases as the lengths of the cantilever andthe beam supported at its ends and loaded in the middle increases.

References:

[1]. Mahgoub, M.A.M. (2016) The new integral transform "Mahgoub Transform", Advances in theoretical and applied mathematics, 11(4), 391- 398.
[2]. Fadhil, R.A. (2017) Convolution for Kamal and Mahgoub transforms, Bulletin of mathematics and statistics research, 5(4), 11-16.
[3]. M.M. AbdelrahimMahgoub, "The NewIntegral Transform "Mahgoub Transform",Advances in Theoretical and Applied Mathematics,vol. 11, pp. 391-398, 2016.
[4]. Mohamed and A. E. Elsayed, "Elzaki Transformation for Linear Fractional and Differential Equations",Journal of Computational Theoretical Nanoscience,vol. 12, pp. 2303-2305, 2015. application of new transform "Mahgoub Transform" to partial differential equations,
[5]. Rohit Gupta , Rahul Gupta, Residue approach to mathematical analysis of the moving coil galvanometer, International Journal of Advanced Trends in Engineering and Technology, Volume 4, Issue 1, 2019.
[6]. Rohit Gupta, Anamika Singh, Rahul Gupta, Quantum Theory Of One Dimensional Free Electron Gas By Means Of Residue Theorem, Compliance Engineering Journal, Volume 10, Issue 12, 2019.
[7]. Harold Cohen, Complex Analysis with Applications in Science and Engineering, Second Edition.
[8]. Dinesh Verma, Rohit Gupta, Amit Pal Singh, Analysis of integral Equations of convolution type via ResidueTheorem Approach,TheInternational journal of analytical and experimental modal analysis, Volume XII, Issue I, January/2020.
[9]. Theory of Structures by S. Ramamrutham and R. Narayan, Danpat Rai Publishing Company.
[10]. Theory of structures by R.S. Khurmi, S. Chand And Company LTD.
[11]. Theory and analysis of structures by A.S. Arya and O.P. Jain.
[12]. N.P. Bali and Dr. Manish Goyal, A Text Book of Engineering Mathematics, 9th edition, 2014.
[13]. Rohit Gupta, Tarun Singhal, Dinesh Verma, Quantum mechanical reflection and transmission coefficients for a particle through a one-dimensional vertical step potential, International Journal of Innovative Technology and Exploring Engineering, Volume-8 Issue-11, September 2019.
[14]. Rahul Gupta, Rohit Gupta, Dinesh Verma, Application ofConvolution Method to the Impulsive Response of A Lightly Damped Harmonic Oscillator, International Journal of Scientific Research in Physics and Applied Sciences, Vol.7, Issue.3, pp.173-175, June (2019).
[15]. Rohit Gupta, Rahul Gupta, Dinesh Verma, Laplace Transform Approach for the Heat Dissipation from an Infinite Fin Surface, Global Journal of Engineering Science and Researches, 6(2), February 2019, pp. 96-101. DOI- 10.5281/zenodo.2565939
[16]. Dr. K. K. Sharma, Mechanics, Oscillations and Relativity, Sharma Publications, Revised Edition, 2019.
[17]. Dinesh Verma , Elzaki –Laplace Transform of some significant Functions, Academia Arena, Volume-12, Issue-4, April 2020..
[18]. Dinesh Verma, Aftab Alam, Analysis of Simultaneous Differential Equations By Elzaki Transform Approach, Science, Technology And Development Volume Ix Issue I January 2020.
[19]. Dinesh Verma Analytical Solutuion of Differential Equations by Dinesh Verma Tranforms (DVT), ASIO Journal of Chemistry, Physics, Mathematics & Applied Sciences (ASIO-JCPMAS), Volume -4, Issue-1, 2020, PP:24-27.
[20]. Dinesh Verma, Empirical Study of Higher Order Diffeential Equations with Variable Coefficient by Dinesh Verma Transformation (DVT), ASIO Journal of Engineering & Technological Perspective Research (ASIO-JETPR), Volume -5, Issue-1, 2020, pp:04-07.

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