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Investigating the Power of Heteroscedasticity in Constant Elasticity of Substitution Production Function

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Abstract
This paper presents the power of some tests for detecting heteroscedasticity in Constant Elasticity of Substitution (CES) nonlinear model. The tests used include Breusch-Pagan, Glejser, White, Park and Goldfeld-Quandt. The CES production function was transformed to intrinsically linear model through Kmenta linearization approach. Using the parameter estimates of the model, the residual was computed and used as the dependent for the auxiliary regression. The error structure data was drawn from a normal distribution with mean, zero and variance, σ^2. The sample sizes for the simulation were 10 and 30, 50 and 100, 150 and 200 for small, medium and large sample sizes, respectively with 10,000 replications. The levels of heteroscedasticity introduced were 0.1, 0.5 and 0.9 for mild, moderate and severe heteroscedasticity, respectively. The result indicates that the power of the test for Glejser and Park tests increases as the sample size increases at every level of heteroscedasticity both at 1% and 5% levels of significance.

***Keywords**: Heteroscedasticity, CES, Power of the test, Production Function*

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I. Introduction

Regression disturbances whose variances are not constant across observations are heteroscedastic. Heteroscedasticity arises in numerous applications, in both cross-sectional and time series data. One of the assumptions of classical linear regression model (CLRM) is that the disturbances u_i entering the population regression function (PRF) are homoscedastic; that is, they all have the same variance, σ^2. If the errors are heteroscedastic, the ordinary least square (OLS) estimator remains unbiased but becomes inefficient [Draper and Smith (1981), Neter et al. (1985)]. More importantly, estimates of the standard errors are inconsistent. The estimated standard errors can either be too large or small, in either case resulting in incorrect inferences. There are certain circumstances in which the assumption of constant error variance, homoscedasticity, in the linear model is not tenable; researchers have observed that heteroscedasticity is usually found in cross sectional data. Such as income and expenditure of individual families. Here, the assumption of homoscedasticity is not very plausible on a prior ground since we would expect less variation in consumption for low income families' than for high income families. At low levels of income the average level of consumption is low, and variation around this level is restricted. This constraint is likely to be less binding at higher income levels [Paris and Houthakker (1955), Jan Kmenta (1971)].

Fasoranbaku (2005) in course of the analysis of the power of tests for homoscedasticity in a single equation econometric model analysed the power of the tests using aggregated and disaggregated approach. He showed that the three tests, Goldfeld-Quandt, Glejser and Breush-Pagan are among the most powerful while Park, White and Cook-Weisberg tests are the least powerful. Marie and Pegiun-Feissolle (2007) proposed two tests for homoscedasticity that require little knowledge of the functional relationship determining the variance of the error term. The idea of the first test is to approximate the true relationship by Tailor's series expansion, which is essentially linearizing the function in a neighbourhood. Gianluigi, Timo and Rolf (1999) earlier applied this idea to non-linear variable selection, while Pegiun-Feissolle (2008) focused on causality testing in a non-linear framework. Peguin-Feissolle (1999) also compared the power in small samples of different tests for conditional heteroscedasticity in which two new tests based on neural networks are proposed: the main interest in them arises from the fact that they do not require the exact specification of the conditional variance under the alternative.

Timo (2011) presented a brief survey of nonlinear models of autoregressive conditional heteroscedasticity. The models in question are parametric nonlinear extensions of the original model by Engle (1982). After presenting the individual models, linearity testing and parameter estimation are discussed, forecasting volatility with nonlinear models is considered. Finally, parametric nonlinear models based on multiplicative decomposition of the variance received attention. Muhammed (2012) in his work titled, a study on the violation of homoscedasticity assumption in linear regression models, after using different methods of detecting the presence of heteroscedasticity, found out that, Goldfeld-Quandt , Glejser and Park are most powerful while White, Breusch Pagan and Levene tests are the least powerful.

II. Material and Methods

A. Nonlinear Regression Model

A nonlinear regression model is one for which the first order conditions for least squares estimation of the parameters are nonlinear functions of the parameters.
Suppose the postulated model is of the form

$$Y = f\left(X_1, X_2, \ldots, X_k, \theta_1, \theta_2, \ldots, \theta_j\right) + u \qquad 2.1$$

and that (2.1) is assumed to be intrinsically nonlinear given a sample of n observation on Y and $X's$, then we can write

$$Y_i = f\left(X_{1i}, X_{2i}, \ldots, X_{ki}, \theta_1, \theta_2, \ldots, \theta_j\right) + u \qquad 2.2$$

$$Y = f\left(X, \theta\right) + u \qquad 2.3$$

$$Y = \begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{pmatrix} \quad X = \begin{pmatrix} X_{11} & X_{21} & \cdots & X_{k1} \\ X_{12} & X_{22} & \cdots & X_{k2} \\ \vdots & \vdots & \vdots & \vdots \\ X_{1n} & X_{2n} & \cdots & X_{kn} \end{pmatrix} \quad \theta = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_j \end{pmatrix} \quad u = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix}$$

B. Production Function Nonlinear Model

A production function is a heuristic device that describes the maximum output that can be produced from different combinations of inputs using a given technology. This can be expressed mathematically as a mapping $f : \mathbf{R}_+^N \rightarrow \mathbf{R}_+$ such that $Y = f(X)$, where X is a vector of factor inputs $(X_1, X_2, \cdots, X_n)'$ and $f(X)$ is the maximum output that can be produced for a given set of inputs $X_i \in \mathbf{R}_+$. This can be applied at both microeconomic (individual firm) and macroeconomic (overall economy) levels, [Eric (2008), Barro and Sala (2004)].

Constant Elasticity of Substitution (CES) Production Function

Solow, Minhas, Arrow and Chenery (1961) developed the Constant Elasticity of Substitution function. It can be expressed in the form:

$$Y_i = \theta_1 \left[\theta_2 K^{-\theta_3} + (1-\theta_2) L^{-\theta_3}\right]^{-\frac{1}{\theta_3}} e^{u_i} \qquad 2.4$$

Where

Y_i is a vector of the dependent variables,

θ_1 is the intercept ,

θ_2 and θ_3 are the regression coefficients,

K is the Capital

L is the Labour

u_i is the random error.

By applying the natural logarithms to the two sides of (3.4), gives

$$\ln(Y_i) = \ln(\theta_1) - \frac{1}{\theta_3}\ln\left[\theta_2 K^{-\theta_3} + (1-\theta_2) L^{-\theta_3}\right] + u_i \qquad 2.5$$

The result in (2.5) is intrinsically nonlinear since taking logarithms will not make the nonlinear function linear in parameters.
The model in (2.4) is linearised using Kmenta (1967) linearisation approach. Besides, a linear Taylor Series expansion around $\theta_3 = 0$ produced an intrinsically linear model.

$$\ln(Y) = \ln\theta_1 - \frac{1}{\theta_3} f(\theta_3) + u_i \qquad 2.6$$

By Taylor's Series expansion,

$$f(\theta_3) = \frac{\theta_3^0}{0!} f(0) + \frac{\theta_3^1}{1!} f'(0) + \frac{\theta_3^2}{2!} f''(0)$$

$$= f(0) + \theta_3 f'(0) + \frac{1}{2}\theta_3^2 f''(0)$$

2.7

where,

$$f(\theta_3) = \ln\left[\theta_2 K^{-\theta_3} + (1-\theta_2)L^{-\theta_3}\right]$$

$$f(0) = \ln\left[\theta_2 K^0 + (1-\theta_2)L^0\right]$$

$$= \ln\left[\theta_2 + (1-\theta_2)\right]$$

$$= \ln 1$$

$$= 0 \qquad 2.8$$

$$f'(\theta_3) = \frac{\frac{d}{d\theta_3}\left[\theta_2 K^{-\theta_3} + (1-\theta_2)L^{-\theta_3}\right]}{\left[\theta_2 K^{-\theta_3} + (1-\theta_2)L^{-\theta_3}\right]}$$

$$= \frac{-\theta_2 K^{-\theta_3}\ln K + (1-\theta_2)(-L^{-\theta_3}\ln L)}{\left[\theta_2 K^{-\theta_3} + (1-\theta_2)L^{-\theta_3}\right]} \qquad 2.9$$

$$f'(0) = \frac{-\theta_2 K^0 \ln K - (1-\theta_2)(-L^0 \ln L)}{\theta_2 K^0 + (1-\theta_2)L^0}$$

$$= \frac{-\theta_2 \ln K - (1-\theta_2)\ln L}{\theta_2 + (1-\theta_2)}$$

$$= -\left[\theta_2 \ln K + (1-\theta_2)\ln L\right] \qquad 2.10$$

$$f''(\theta_3) = \frac{\left[\theta_2 K^{-\theta_3} + (1-\theta_2)L^{-\theta_3}\right]\left[-\theta_2(-K^{-\theta_3}\ln K)\ln K - (1-\theta_2)\ln L(-L^{-\theta_3}\ln L)\right] - \left[-\theta_2 K^{-\theta_3}\ln K - (1-\theta_2)L^{-\theta_3}\ln L\right]\left[-\theta_2 K^{-\theta_3}\ln K - (1-\theta_2)L^{-\theta_3}\ln L\right]}{\left[\theta_2 K^{-\theta_3} + (1-\theta_2)L^{-\theta_3}\right]^2}$$

$$= \frac{\left[\theta_2 K^{-\theta_3} + (1-\theta_2)L^{-\theta_3}\right]\left[\theta_2 K^{-\theta_3}(\ln K)^2 + (1-\theta_2)L^{-\theta_3}(\ln L)^2\right] - \left[\theta_2 K^{-\theta_3}\ln K + (1-\theta_2)L^{-\theta_3}\ln L\right]^2}{\left[\theta_2 K^{-\theta_3} + (1-\theta_2)L^{-\theta_3}\right]^2} \qquad 2.11$$

$$f''(0) = \frac{\left[\theta_2 K^0 + (1-\theta_2)L^0\right]\left[\theta_2 K^0(\ln K)^2 + (1-\theta_2)L^0(\ln L)^2\right] - \left[\theta_2 K^0 \ln K + (1-\theta_2)L^0 \ln L\right]^2}{\left[\theta_2 K^0 + (1-\theta_2)L^0\right]^2}$$

$$= \frac{\left[\theta_2 + (1-\theta_2)\right]\left[\theta_2(\ln K)^2 + (1-\theta_2)(\ln L)^2\right] - \left[\theta_2 \ln K + (1-\theta_2)\ln L\right]^2}{\left[\theta_2 + (1-\theta_2)\right]^2}$$

$$= \frac{\left[\theta_2 + (1-\theta_2)\right]\left[\theta_2(\ln K)^2 + (1-\theta_2)(\ln L)^2\right] - \left[\theta_2 \ln K + (1-\theta_2)\ln L\right]^2}{\left[\theta_2 + (1-\theta_2)\right]^2}$$

$$= \frac{\left[\theta_2(\ln K)^2 + (1-\theta_2)(\ln L)^2\right] - \left[\theta_2 \ln K + (1-\theta_2)\ln L\right]^2}{\left[\theta_2 + (1-\theta_2)\right]^2}$$

$$= \left[\theta_2(\ln K)^2 + (1-\theta_2)(\ln L)^2\right] - \left[\theta_2 \log K + (1-\theta_2)\ln L\right]^2 \qquad 2.12$$

Putting (3.8), (3.10) and (3.12) into (3.7), that is,

$$f(\theta_3) = f(0) + \theta_3 f'(0) + \frac{1}{2!}\theta_3^2 f''(0)$$

$$f(\theta_3) = 0 + \theta_3\left[-(\theta_2 \ln K + (1-\theta_2)\ln L\right] + \frac{1}{2}\theta_3^2\left[\theta_2(\ln K)^2 + (1-\theta_2)(\ln L)^2\right] - \left[\theta_2 \ln K + (1-\theta_2)\ln L\right]^2$$

gives,

$$= -\theta_3\left[\theta_2 \ln K + (1-\theta_2)\ln L\right] + \frac{1}{2}\theta_3^2\left[\theta_2(\ln K)^2 + (1-\theta_2)(\ln L)^2\right] - \left[\theta_2 \ln K + (1-\theta_2)\ln L\right]^2 \qquad 2.13$$

Putting (3.13) into (3.6)

$$\ln(Y_i) = \ln\theta_1 - \frac{1}{\theta_3}\left\langle -\theta_3\left\{\theta_2 \ln K + (1-\theta_2)\ln L\right\} + \frac{1}{2}\theta_3^2\left\{\left[\theta_2(\ln K)^2 + (1-\theta_2)(\ln L)^2\right] - \left[\theta_2 \ln K + (1-\theta_2)\ln L\right]^2\right\}\right\rangle + u_i$$

$$= \ln\theta_1 - \left\langle -\left\{\theta_2 \ln K + (1-\theta_2)\ln L\right\} + \frac{1}{2}\theta_3\left\{\left[\theta_2(\ln K)^2 + (1-\theta_2)(\ln L)^2\right] - \left[\theta_2 \ln K + (1-\theta_2)\ln L\right]^2\right\}\right\rangle + u_i$$

$$= \ln\theta_1 + \theta_2 \ln K + (1-\theta_2)\ln L - \frac{1}{2}\theta_3\left\{\left[\theta_2(\ln K)^2 + (1-\theta_2)(\ln L)^2\right] - \left[\theta_2 \ln K + (1-\theta_2)\ln L\right]^2\right\} + u_i$$

2.14

Since,

$$\left[\theta_2(\ln K)^2 + (1-\theta_2)(\ln L)^2\right] - \left[\theta_2 \ln K + (1-\theta_2)\ln L\right]^2$$

$$= \theta_2(\ln K)^2 + (1-\theta_2)(\ln L)^2 - \theta_2^2(\ln K)^2 - 2\theta_2(1-\theta_2)(\ln K)(\ln L) - (1-\theta)^2(\ln L)^2$$

$$= \theta_2(\ln K)^2 - \theta_2^2(\ln K)^2 + (1-\theta_2)(\ln L)^2 - (1-\theta)^2(\ln L)^2 - 2\theta_2(1-\theta_2)(\ln K)(\ln L)$$

$$= \theta_2(1-\theta_2)(\ln K)^2 + (1-\theta_2)\left[1-(1-\theta_2)\right](\ln L)^2 - 2\theta_2(1-\theta_2)(\ln K)(\ln L)$$

$$= \theta_2(1-\theta_2)(\ln K)^2 + \theta_2(1-\theta_2)(\ln L)^2 - 2\theta_2(1-\theta_2)(\ln K)(\ln L)$$

$$= \theta_2(1-\theta_2)\left[(\ln K)^2 + (\ln L)^2 - 2(\ln K)(\ln L)\right]$$

$$= \theta_2(1-\theta_2)\left[\ln K - \ln L\right]^2 \qquad 2.15$$

Putting (2.15) into (2.14)

$$\ln(Y_i) = \ln\theta_1 + \theta_2 \ln K + (1-\theta_2)\ln L - \frac{1}{2}\theta_3\left[\theta_2(1-\theta_2)(\ln K - \ln L)^2\right] + u_i$$

$$= \ln\theta_1 + \theta_2 \ln K + (1-\theta_2)\ln L + \theta_3\theta_2(1-\theta_2)\left[-\frac{1}{2}(\ln K - \ln L)^2\right] + u_i$$

2.16

The intrinsically linear model in 2.16 is expressed as

$$Y_i^* = \theta_1^* + \theta_2 K^* + (1-\theta_2)L^* + (\theta_3\theta_2)(1-\theta_2)\left[-\frac{1}{2}(K^* - L^*)^2\right] + u_i$$

2.17

where,

$$\ln(Y_i) = Y_i^*, \ln(\theta_1) = \theta^*, \ln(K) = K^*, \ln(L) = L^*$$

C. Error Variance Structures and Tests for Heteroscedasticity

We considered, the multiplicative heteroscedasticity error structure model discussed by Harvey (1976). This can be shown as follows:

$$y_i = x_i\beta + u_i \qquad 2.18$$

Where,

$$u_i \sim N(0, \sigma_i^2)$$

$$\sigma_i^2 = \sigma^2 E(y_i)^2$$

$$= \sigma^2(\beta_1 x_{i1} + \cdots + \beta_k x_{ik})^2$$

$$= \sigma^2 \exp(q_i, \lambda) \qquad 2.19$$

For $i = 1, 2, \cdots, n$ where y_i is the i^{th} observation, x_i and q_i are the i^{th} $I \times k$ and $I \times (J - I)$ vectors of explanatory variables, respectively. β and λ are the vectors of unknown parameters.

Let $Z_i = (I, q_i)$ and $\delta = (\log\sigma^2, \lambda')'$ where Z_i and δ denote $I \times J$ and $J \times I$ vectors.

Then we can rewrite (2.19) as

$$\sigma_i^2 = \exp(Z_i, \delta) \qquad 2.20$$

$$= \sigma^2 (Z_i)^\lambda$$

Where σ^2 and λ are both unknown real constants, which determines the levels or degree of heteroscedasticity.

The following five tests were considered in this paper namely, Breusch-Pagan (1979), White (1980), Glejser (1969), Park (1966) and Goldfeld-Quandt (1965).

III. Simulation

An infected heteroscedasticity sample using uniform distribution was used to generate data for Capital (K), Labour (L) and Output (Y). The study used an arbitrary initial values for θ_1 =0.2, θ_2 =0.5 and θ_3 =0.3 for the model. The set of parameter estimates obtained were used to compute the residuals which represented the dependent variable for the auxiliary regression. The error structure data were drawn from a normal distribution with mean, zero and variance, σ^2.

The sample sizes for the simulation were specified as 10 and 30, 50 and 100, 150 and 200 for small, medium and large sample sizes respectively. Each sample size was replicated in 10,000 times. The levels of heteroscedasticity, λ , introduced were 0.1, 0.5 and 0.9 for mild, moderate and severe heteroscedasticity, respectively. The statistical package used for the analysis was STATA 12.0 version.

IV. Result

Table 1: Power of the Tests for CES Model at $\alpha = 0.01$

TEST	λ	SAMPLE SIZE					
		10	**30**	**50**	**100**	**150**	**200**
BREUSCH-PAGAN	**0.1**	-	0.5011	0.3651	0.5625	0.4562	0.5941
WHITE		-	0.2873	0.1368	0.0146	0.1022	0.3057
GLEJSER		-	0.9133	0.9999	1.0000	1.0000	1.0000
PARK		-	0.1888	0.9830	0.9999	0.9999	1.0000
GOLDFELD QUANDT		-	-	0.0421	0.3982	0.1912	0.1646
BREUSCH-PAGAN	**0.5**	-	0.2916	0.1981	0.3074	0.0026	0.9159
WHITE		-	0.3062	0.4889	0.2451	0.2500	0.4280
GLEJSER		-	0.9972	0.9999	1.0000	1.0000	1.0000
PARK		-	0.8127	0.9975	0.9999	1.0000	1.0000
GOLDFELD QUANDT		-	-	0.0002	0.1226	0.0000	0.0001
BREUSCH-PAGAN	**0.9**	-	0.9103	0.5769	0.3794	0.4787	0.1879
WHITE		-	0.2873	0.3234	0.5208	0.3492	0.0000
GLEJSER		-	0.8822	0.9999	1.0000	1.0000	1.0000
PARK		-	0.5123	0.8896	0.9999	1.0000	1.0000
GOLDFELD QUANDT		-	-	0.2483	0.0022	0.0362	0.0000

Table 2: Power of the Tests for CES Model at $\alpha = 0.05$

TEST	λ	SAMPLE SIZE					
		10	**30**	**50**	**100**	**150**	**200**
BREUSCH-PAGAN	**0.1**	-	0.5011	0.1604	0.3744	0.9103	0.2233
WHITE		0.2231	0.2873	0.0726	0.5204	0.2255	0.0797
GLEJSER		-	0.9133	0.9989	1.0000	1.0000	1.0000
PARK		-	0.1888	0.5755	0.9999	1.0000	1.0000
GOLDFELD QUANDT		-	-	0.0114	0.0022	0.0595	0.1054
BREUSCH-PAGAN	**0.5**	-	0.6873	0.5766	0.3769	0.4809	0.1877
WHITE		0.2231	0.2873	0.3227	0.5206	0.3492	0.0000
GLEJSER		1.0000	0.8851	0.9999	1.0000	1.0000	1.0000
PARK		-	0.1856	0.8898	0.9999	1.0000	1.0000
GOLDFELD QUANDT		-	-	0.1903	0.0022	0.0364	0.0004
BREUSCH-PAGAN	**0.9**	-	0.9103	0.5769	0.3794	0.4787	0.0067
WHITE		0.2231	0.2873	0.3234	0.5208	0.3492	0.0000
GLEJSER		-	0.8822	0.9999	1.0000	1.0000	1.0000
PARK		-	0.5123	0.8896	0.9999	1.0000	1.0000
GOLDFELD QUANDT		-	-	0.1907	0.0022	0.0362	0.0000

V. Discussion

A test is considered to have high power, if it has a value of 0.8 and above .Table 1 shows the power of the tests for CES Model at $\alpha = 0.01$. There is no result for sample size 10 at all levels of heteroscedasticity (λ) for all the tests due to insufficient sample size. The result shows that at every level of heteroscedasticity, as the sample size increases, the power of the test for Glejser is very high [Machhado and Silva (2000)] while the power of Park test also improves as the sample size increases at every levels of heteroscedasticity. White test has low power [Ayoola and Olubusoye (2012)] and Goldfeld-Quandt test loses power of the test at every level of heteroscedasticity as sample size increases [Carmelo and Subhash (2008)]

Table 2 shows the power of the tests for CES Model at $\alpha = 0.05$. The result showed that at every level of heteroscedasticity, as the sample size increases, the power of the test for Glejser test is high while the power of test for Park improves as the sample size increases at all levels of heteroscedasticity. Goldfeld-Quandt test has low power at every level of heteroscedasticity as sample size increases.

VI. Conclusion

It is evident that Glejser and Park tests performed better than other tests such as Breusch-Pagan, White and Goldfeld-Quandt both at 1% and 5% levels of significance. Based on these facts, one can suggest the two tests for detecting heteroscedasticity in nonlinear models.

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