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Buoyancy Driven Unsteady Magnetohydrodynamic Turbulent Dissipative Fluid Flow Past An Infinite Vertical Plate

Joyce C. Rono[1], Wilys O. Mukuna[1*], and John K. Rotich[1]
[1]*Department of Mathematics, Actuarial and Physical Sciences, University of Kabianga,*
P.O. Box 2030-20200, KERICHO, KENYA
[*]*Corresponding author*

Abstract
An analysis of unsteady turbulent magnetohydrodynamic (MHD) buoyancy driven fluid flow past an infinite vertical plate was developed. The study of magnetohydrodynamics is important due to its industrial and non-industrial applications. Incompressible viscous dissipative fluid was impulsively started to flow in y-axis direction along the infinite vertical plate and flow problem analysed thereafter. A strong transverse uniform magnetic field is applied along horizontal z-axis normal to the infinite vertical plate in x-y plane. The fluid flow was modelled using laws of conservation of mass, momentum and energy. The model together with its boundary and initial conditions are then transformed to non-dimensional form to obtain non-linear partial dimensionless differential equations with non-dimensional flow parameters. The resultant model, together with boundary and initial conditions was discretized by a fully explicit finite difference method scheme to obtain nonlinear algebraic equations. The resulting systems of non-linear algebraic equations are then solved numerically using the MATLAB that is mathematical computer software. The Reynolds stress terms were solved using Prandtl mixing length hypothesis. The impacts of the non- dimensional flow parameters such as magnetic parameter(M), Hall parameter(m), Prandtl number(Pr), Grashof number(Gr) and heat parameter (S) on fluid velocity and temperature profiles were investigated. It is evident from the results that when $Gr > 0$, the primary velocity (V) increase with an increase in Grashof number(Gr) and Hall parameter (m) with a decrease in heat parameter(S)but decreases with an increase in magnetic parameter(M). The secondary velocity increases with the increase in both magnetic parameter(M) and Grashof number(Gr) with a decrease in heat parameter(S), but decreases with the increase in Hall parameter(m) and prandtl number(Pr). There is no effect on temperature profile, during cooling of the plate with increase of the magnetic parameter(M), Hall parameter (m) and Grashof number (Gr) but it decreases as Prandtl number (Pr) is increased while heat parameter(S) is decreased.
Keywords: *Buoyancy, Unsteady, Turbulent, Vertical plate, Finite difference and Magneto hydrodynamics*

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Nomenclature

g	Acceleration due to gravity [ms^{-2}]
L	Characteristic lengths
x, y, z	Coordinates in x, y, z directions, $[m]$
H_0	Constant Magnetic field intensity (Wb/m^2)
Q_0	Constant heat generation
J	Current density vector, $[Am^{-2}]$
X , Y	Dimensionless horizontal and vertical coordinates
θ	Dimensionless temperature
U , V	Dimensionless velocity components in X and Y directions
Ec	Eckert number
E	Electric field $[Vm^{-1}]$
D	Electric flux density, $[C_{m^{-2}}]$
P	Pressure of the fluid, $[N_{m^{-2}}]$
Gr	Grash of number

$\Delta\tau$	Grid size in time [s]
ΔY	Grid size in vertical direction [m]
m	Hall parameter
S	Heat source/sink parameter
B_0	Magnetic field strength [kgs^{-2}A^{-1}]
M	Magnetic parameter
H	Magnetic field intensity $[Wbm^{-2}]$
Pr	Prandtl number
Ra	Rayleign number
U_0	Reference velocity
c_p	Specific heat at constant pressure of the fluid [J/kg/K]
c_V	Specific heat at constant volume of the fluid [J/kg/K]
T	Temperature [K]
T_∞	Temperature at free stream
T_w	Temperature at the wall
k	Thermal conductivity [W/m/K]
t	Time [s]
Pr_t	Turbulent Prandtl number
v, u	Velocity component in y and x directions [ms^{-1}]
$\bar{q}$	Velocity vector of velocity components u, v, w in the x, y, z direction respectively
W	Work, $[J]$
τ_e	Collision time of electron [s]
ρ	Density of the fluid [kgm^{-3}]
τ	Dimensionless time
σ	Electrical conductivity of the fluid $[\Omega^{-1}m^{-1}s]$
ω_e	Electron cyclotron frequency [Hz]
ρ_e	Electron density, $[kgm^{-2}]$
μ_e	Electron permeability, Hm^{-1}
$\omega_e\tau_e$	Hall parameter
$\omega_i\tau_i$	Ion slip parameter
ω_i	Ionic cyclotron frequency, H_z
τ_i	Ionic cyclotron period, s
ν	Kinematic viscosity [m^{2}s^{-1}]
μ_0	Magnetic permeability $[Hm^{-1}]$
σ_{ij}	Normal stress in i and j directions
τ_{ij}	Shear stress in i and j directions
α	Thermal diffusivity [m^{2}s^{-1}]
β	Volumetric coefficient of thermal expansion [K^{-1}]
κ	Von Karman Constant
i	Designates grid point along x direction
j	Designates grid point along y direction
∞	Free stream condition
w	Wall condition

I. Introduction

Magnetohydrodynamics (MHD) is the study of flow of electrically conducting fluid in presence of magnetic field.The MHD problem is of great interest in current mathematical modelling due to its applications in engineering problems such as MHD generators, plasma studies, nuclear reactors, oil exploration, geothermal extraction and boundary layer control in the field of aerodynamics. The analysis of systems involving fluid flow and the associated phenomena by using computer-based simulations referred to as computational fluid dynamics (CFD). CFD is powerful technique useful in both industrial and non-industrial applications. Modelling and simulation of turbulence is the key in numerical solution of fluid flow since most practical fluid flows are turbulent Zhang *et al*. [8]. Fluid motion generally and particularly within the boundary layer can either be laminar or turbulent. A transition from a laminar to turbulence flow depends on the relative magnitude of the buoyancy forces and viscous forces in the fluid.

Chebos *et al*. [1] presented an investigation on an unsteady MHD free convective flow past an oscillating vertical porous plate with oscillatory heat flux and they noted that the velocity increase with decrease in suction parameter and magnetic parameter and increase with an increase in Darcy number. They also found that the temperature increase with decrease in Prandtl number and increase with increase in radiation parameter and suction parameter.

Falodun and Fadugba [3] investigated the effects of heat transfer on unsteady magneto hydrodynamics (MHD) boundary layer flow of an incompressible fluid on a moving vertical plate. They employed Roseland model and the results showed that an increase in thermal Grashof number increases the velocity profile and an increase in Prandtl number leads to decrease in both the velocity and the temperature profile. Increasing thermal radiation intensifies the velocity and temperature profile and also speeds up the convections flow and thermal boundary layer of the fluid. Increase in magnetic parameter reduces the velocity profile as a result of the applied transverse magnetic field.

Vijayalakshmi *et al.* [7] studied the unsteady electrically transmitting fluid past an oscillating semi-infinite vertical plate with uniform temperature and mass diffusion under chemical reactions. They found out that heat transfer progress enhanced with the oscillating frequency, Prandtl number and thermal Grashof numbers.

Mukuna *et al.* [6] developed a mathematical model of buoyancy driven hydromagnetic turbulent fluid flow over a vertical infinite plate using turbulent Prandtl number. They established that the primary velocity increases with decreasing magnetic parameter while it increases with increase in Hall parameter and Grashoff number.

Mukuna [5] on studying an MHD turbulent free convection fluid flow over an infinite vertical heat-generating cylinder noted that the effect of Hall current on primary velocity was not observed due to turbulence and a decrease in secondary velocity profile when Hall current was increased. He also observed that primary velocity profiles increased with an increase in Grashoff number while temperature profiles decreased with an increased Prandtl number.

Chepkemoi and Mukuna [2] carried out an analysis of hydromagnetic free convection turbulent fluid flow over an infinite heat absorbing vertical plate. From this study, they found that primary velocity increases with an increase in both Hall parameter and Grashoff number but decreases with an increase in magnetic parameter. Secondary velocity increases with decreasing in both Hall current and magnetic parameter. The temperature profile increases with increasing magnetic parameter and decrease in Prandtl number but decreases with increasing Hall parameter.

Dimensionless physical-mathematical modeling of turbulent natural convection on an isothermal plate in an"infinite" open environment, with the application of non-dimensionalization techniques of transport equations and the$\kappa - \varepsilon$, $\kappa - \omega$ turbulence models was consolidated by Júnior, *et al.* [4]. They found out that there are significant similarities in constitution and presentation between the dimensional and non-dimensional forms of the physical transport equations with mathematical turbulence models.

In this paper investigation is done of MHD turbulent buoyancy driven fluid flow with viscous and electrical dissipation past an infinite vertical plate in the presence of transverse magnetic field. The present problem takes into account unsteady fluid flow, thermal diffusivity terms and heat source in both energy and momentum equations.

II. Mathematical Formulation

A two dimensional turbulent boundary layer flow is considered in this study. The fluid flow is in the positive y-axis direction along a vertical plate lying in the x_y plane and perpendicular to the x_z plane of a Cartesian plane. The desired geometry of the current problem on Cartesian plane is as shown in figure 1 below.

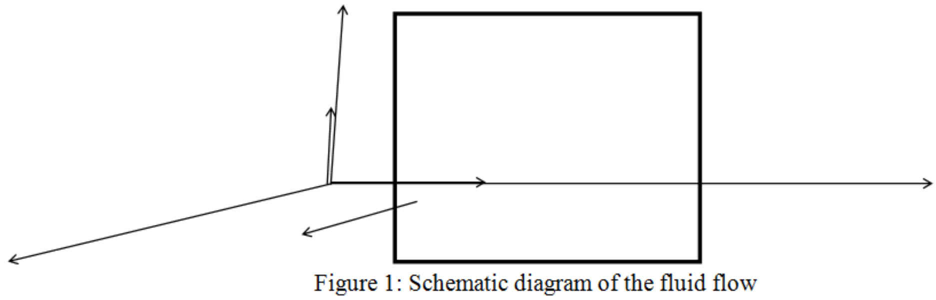

Figure 1: Schematic diagram of the fluid flow

Initially, for time, $t \leq 0$, the plate and the fluid are at rest and at the same free stream temperature T_∞ The temperature of the plate is raised simultaneously to T_w such that $T_w > T_\infty$ hence $T_w - T_\infty$ is the temperature difference which causes free convection currents near the plate, thus the hot plate becomes a constant heat source Q into the flowing fluid. There is no applied voltage of which it means absence of an electric field ($E = 0$). Induced magnetic field is considered negligible hence $\boldsymbol{H} = (0,0,H_0)$. The fluid flowing is subjected to a constant strong transverse magnetic field H_0 along a horizontal z-axis direction normal to the fluid flow. The x-y vertical plate is infinite in extend, such that, it has non-end effects and the derivatives with respect to x and y are identically zero. There is no z-component velocity then $\overline{w} = 0$, therefore, the flow variables are functions of z and t only.

$$\rho \frac{\partial \bar{v}}{\partial t} = -\frac{\partial \bar{p}}{\partial y} + \rho g_y + \mu \left(\frac{\partial^2 \bar{v}}{\partial z^2}\right) - \frac{\partial(\overline{v'w'})}{\partial z} + (J \times B)_y \quad (1)$$

$$\rho \frac{\partial u}{\partial t} = \mu \frac{\partial^2 \bar{u}}{\partial z^2} - \frac{\partial(\overline{u'w'})}{\partial z} + (J \times B)_x \quad (2)$$

$$\frac{\partial \bar{T}}{\partial t} = \frac{k}{\rho c_p}\left(\frac{\partial^2 \bar{T}}{\partial z^2}\right) + \frac{\mu}{\rho c_p}\left\{\left(\frac{\partial \bar{v}}{\partial z}\right)^2 + \left(\frac{\partial \bar{u}}{\partial z}\right)^2\right\} + \frac{J_x^2 + J_y^2}{\rho c_p \sigma} + \frac{Q_0(\bar{T} - \bar{T}_\infty)}{\rho c_p} - \frac{\partial(\overline{T'w'})}{\partial z} \quad (3)$$

The initial and boundary conditions given as;

$t < 0; u = 0, v = 0, T = T_\infty$ Everywhere (4a)

$t \geq 0; u = 0, v = 0, T = T_w$ At $z = 0$. (4b)

$u \to 0, v \to 0, T \to T_\infty$ As $z \to \infty$

(4c)

Solving the electromagnetic force term, $J \times B$:

$$(J \times B)_x = \sigma B_0^2 \left(\frac{mv - u}{1 + m^2}\right)$$

(5) $$(J \times B)_y = \sigma B_0^2 \left(\frac{mu + v}{1 + m^2}\right)$$

The terms $\overline{u'w'}$ and $\overline{v'w'}$ are the apparent turbulent shear stresses while $\overline{T'w'}$ is the apparent turbulent heat flux. The turbulent stress terms in momentum and energy equations were resolved using Prandtl mixing length hypothesis and turbulent Prandtl number respectively.

The apparent turbulent shear stress terms $\overline{u'w'}$ and $\overline{v'w'}$ are written as:

$$-\overline{vw} = -\kappa^2 z^2 \left(\frac{\partial \bar{v}}{\partial z}\right)^2 \quad and \quad -\overline{uw} = -\kappa^2 z^2 \left(\frac{\partial \bar{u}}{\partial z}\right)^2 \quad (6a)$$

The apparent turbulent heat flux $\overline{T'w'}$ is written as:

$$-\overline{wT} = \frac{\varepsilon_M}{\Pr_t}\frac{\partial \bar{T}}{\partial z}$$ where $$\varepsilon_M = -\kappa^2 z^2 \frac{\partial \bar{v}}{\partial z} \quad Or \quad \varepsilon_M = -\kappa^2 z^2 \frac{\partial \bar{u}}{\partial z}$$

(6b)

Thus, the partial derivatives of the turbulent stress terms, $\overline{u'w'}$, $\overline{v'w'}$ and $\overline{T'w'}$ are given as:

$$\frac{\partial(\overline{u'w'})}{\partial z} = 2\kappa^2\left[z\left(\frac{\partial \bar{u}}{\partial z}\right)^2 + z^2\left(\frac{\partial \bar{u}}{\partial z}\right)\left(\frac{\partial^2 \bar{u}}{\partial z^2}\right)\right] \quad (7a)$$

$$\frac{\partial(\overline{v'w'})}{\partial z} = 2\kappa^2\left[z\left(\frac{\partial \bar{v}}{\partial z}\right)^2 + z^2\left(\frac{\partial \bar{v}}{\partial z}\right)\left(\frac{\partial^2 \bar{v}}{\partial z^2}\right)\right] \quad (7b)$$

$$\frac{\partial(\overline{T'w'})}{\partial z} = \frac{\kappa^2}{\Pr_t}[2z\left(\frac{\partial \bar{v}}{\partial z}\frac{\partial T}{\partial z}\right) + z^2\left(\frac{\partial^2 \bar{v}}{\partial z^2}\frac{\partial T}{\partial z} + \frac{\partial \bar{v}}{\partial z}\frac{\partial^2 T}{\partial z^2}\right) \quad (7c)$$

Substituting equations (5), (6) and (7) into equation (1), (2) and (3), the final conservation equations reduce to:

$$\frac{\partial v}{\partial t} = -\frac{\sigma B_0^2}{\rho(1+m^2)}(mu + v) + g\beta(T - T_\infty) + \nu\frac{\partial^2 v}{\partial z^2} + 2\kappa\left[z\left(\frac{\partial v}{\partial z}\right)^2 + z^2\left(\frac{\partial v}{\partial z}\right)\left(\frac{\partial^2 v}{\partial z^2}\right)\right]$$

(8)

$$\frac{\partial u}{\partial t} = \frac{\sigma B_0^2}{\rho(1+m^2)}(mv - u) + \nu\frac{\partial^2 u}{\partial z^2} + 2\kappa\left[z\left(\frac{\partial u}{\partial z}\right)^2 + z^2\left(\frac{\partial u}{\partial z}\right)\left(\frac{\partial^2 u}{\partial z^2}\right)\right]$$

(9)

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial z^2}\right) + \frac{1}{\rho c_p}\mu\left\{\left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right\} + \frac{\sigma B_0^2\left[(mu + v)^2 + (mv - u)^2\right]}{\rho c_P\left(1+m^2\right)^2}$$

$$+ \frac{Q_0(T - T_\infty)}{\rho c_P} - \left\{\frac{\kappa^2}{\Pr_t}\left[2z\frac{\partial v}{\partial z}\frac{\partial T}{\partial z} + z^2\left(\frac{\partial^2 v}{\partial z^2}\frac{\partial T}{\partial z} + \frac{\partial v}{\partial z}\frac{\partial^2 T}{\partial z^2}\right)\right]\right\}$$

(10)

To non-dimensionalise equation (8), (9) and (10), the following scaling variables were applied:

$$\tau = \frac{tU_0^2}{\nu}, Z = \frac{zU_0}{\nu} = \frac{z}{L}, U = \frac{u}{U_0}, V = \frac{v}{U_0}, \theta = \frac{T - T_\infty}{T_w - T_\infty}$$

(11)

Using the scaling variables above yields to the model equations:

$$\frac{\partial V}{\partial \tau} = \frac{-M^2}{(1+m^2)}(mU + V) + Gr\,\theta + \frac{\partial^2 V}{\partial Z^2} + 2\kappa^2\left[Z\left(\frac{\partial V}{\partial Z}\right)^2 + Z^2\left(\frac{\partial V}{\partial Z}\right)\left(\frac{\partial^2 V}{\partial Z^2}\right)\right]$$

(12)

$$\frac{\partial U}{\partial \tau} = \frac{M^2}{(1+m^2)}(mV - U) + \frac{\partial^2 U}{\partial Z^2} + 2\kappa^2\left[Z\left(\frac{\partial U}{\partial Z}\right)^2 + Z^2\left(\frac{\partial U}{\partial Z}\right)\left(\frac{\partial^2 U}{\partial Z^2}\right)\right]$$

(13)

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Z^2} + Ec\left[\left(\frac{\partial U}{\partial Z}\right)^2 + \left(\frac{\partial V}{\partial Z}\right)^2\right] + \frac{M^2 Ec}{(1+m^2)^2}\left[(mU + V)^2 + (mV - U)^2\right] + \frac{S}{\Pr}\theta$$

$$-\frac{\kappa^2}{\Pr_t}\left\{2Z\frac{\partial V}{\partial Z}\frac{\partial \theta}{\partial Z} + Z^2\left(\frac{\partial^2 V}{\partial Z^2}\frac{\partial \theta}{\partial Z} + \frac{\partial V}{\partial Z}\frac{\partial^2 \theta}{\partial Z^2}\right)\right\}$$

(14)

Where, $\Pr = \frac{\nu\rho c_p}{k}$, $Gr = \frac{g\beta\nu(T_w - T_\infty)}{U_0^3} = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2}$, $M^2 = \frac{\sigma B_0^2\nu}{\rho U_0^2}$, $Ec = \frac{U_0^2}{c_p(T_w - T_\infty)}$,

$m = \omega_e\tau_e$, $S = \frac{Q_0\nu^2}{kU_0^2}$ and $\Pr_t = \frac{\varepsilon_M}{\varepsilon_H}$

Corresponding initial and boundary conditions written as;

$t < 0; U = 0, V = 0, \theta = 0$ *everywhere*

$t \geq 0; U = 0, V = 0, \theta = 1$ $at Z = 0$

$U \to 0, V \to 0, \theta \to 0$ $as Z \to \infty$

(15)

The equivalent finite difference scheme for equations (12), (13) and (14) are respectively:

$$\frac{V_i^{j+1}-V_i^j}{\Delta\tau}=\frac{V_{i+1}^j-2V_i^j+V_{i-1}^j}{(\Delta Z)^2}+0.08\,i\Delta Z\left(\frac{V_{i+1}^j-V_{i-1}^j}{2\Delta Z}\right)^2+0.08\,(i\Delta Z)^2\frac{V_{i+1}^j-2V_i^j+V_{i-1}^j}{(\Delta Z)^2}\left(\frac{V_{i+1}^j-V_{i-1}^j}{2\Delta Z}\right)$$

$$+Gr\,\theta-\frac{M^2(mU_i^j+V_i^j)}{(1+m^2)}$$

(16)

$$\frac{U_i^{j+1}-U_i^j}{\Delta\tau}=\frac{U_{i+1}^j-2U_i^j+U_{i-1}^j}{(\Delta Z)^2}+0.08\,i\Delta Z\left(\frac{U_{i+1}^j-U_{i-1}^j}{(2\Delta Z)}\right)^2+0.08\,(i\Delta Z)^2\frac{U_{i+1}^j-2U_i^j+U_{i-1}^j}{(\Delta Z)^2}\left(\frac{U_{i+1}^j-U_{i-1}^j}{2\Delta Z}\right)$$

$$+\frac{M^2(mV_i^j-U_i^j)}{(1+m^2)}$$

(17)

$$\frac{\theta_i^{j+1}-\theta_i^j}{\Delta\tau}=\frac{1}{\Pr}\left[\frac{\theta_{i+1}^j-2\theta_i^j+\theta_{i-1}^j}{(\Delta Z)^2}\right]+Ec\left\{\left(\frac{U_{i+1}^j-U_{i-1}^j}{2\Delta Z}\right)^2+\left(\frac{V_{i+1}^j-V_{i-1}^j}{2\Delta Z}\right)^2\right\}$$

$$+\frac{M^2\cdot Ec}{(1+m^2)^2}\left\{\left(mU_i^j+V_i^j\right)^2+\left(mV_i^j-U_i^j\right)^2\right\}+\frac{S}{\Pr}\theta_i^j$$

$$-\frac{\kappa^2}{\Pr_t}\left\{\begin{aligned}&2i\Delta Z\left(\frac{V_{i+1}^j-V_{i-1}^j}{2\Delta Z}\right)\left(\frac{\theta_{i+1}^j-\theta_{i-1}^j}{2\Delta Z}\right)\\&+\left(i\Delta Z\right)^2\left[\frac{V_{i+1}^j-2V_i^j+V_{i-1}^j}{\left(\Delta Z\right)^2}\left(\frac{\theta_{i+1}^j-\theta_{i-1}^j}{2\Delta Z}\right)+\frac{\theta_{i+1}^j-2\theta_i^j+\theta_{i-1}^j}{\left(\Delta Z\right)^2}\left(\frac{V_{i+1}^j-V_{i-1}^j}{2\Delta Z}\right)\right]\end{aligned}\right\}$$

(18)

Where, i and j refers to Z and τ respectively. The values of $\kappa=0.2, \Delta Z=0.1 \, and \, \Delta\tau=1\times10^{-8}$
The initial and boundary conditions take the form;

$$j<0; U_i^j=0, V_i^j=0, \theta_i^j=0 \qquad everywhere$$
$$j\ge 0; U_i^j=0, V_i^j=0, \ \theta_i^j=1 \qquad for \quad i=0$$
$$U_i^j\to 0, V_i^j\to 0, \theta_i^j\to 0 \qquad for \quad i\to\infty$$

(19)

Using the boundary and initial conditions (19), values of consecutive grid points; V_i^{j+1},U_i^{j+1} and θ_i^{j+1} for primary and secondary velocities and temperature respectively were computed as:

$$V_i^{j+1}=V_i^j+\Delta\tau\,\frac{V_{i+1}^j-2V_i^j+V_{i-1}^j}{(\Delta Z)^2}+0.08\,i\Delta Z\Delta\tau\left(\frac{V_{i+1}^j-V_{i-1}^j}{2\Delta Z}\right)^2$$

$$+0.08\,(i\Delta Z)^2\Delta\tau\,\frac{V_{i+1}^j-2V_i^j+V_{i-1}^j}{(\Delta Z)^2}\left(\frac{V_{i+1}^j-V_{i-1}^j}{2\Delta Z}\right)+Gr\,\theta\Delta\tau-\frac{M^2(mU_i^j+V_i^j)}{(1+m^2)} \qquad (20)$$

$$U_i^{j+1}=U_i^j+\Delta\tau\,\frac{U_{i+1}^j-2U_i^j+U_{i-1}^j}{(\Delta Z)^2}+0.08\,i\Delta Z\Delta\tau\left(\frac{U_{i+1}^j-U_{i-1}^j}{(2\Delta Z)}\right)^2+0.08\,(i\Delta Z)^2\Delta\tau\,\frac{U_{i+1}^j-2U_i^j+U_{i-1}^j}{(\Delta Z)^2}\left(\frac{U_{i+1}^j-U_{i-1}^j}{2\Delta Z}\right)$$

$$+\Delta\tau\,\frac{M^2(mV_i^j-U_i^j)}{(1+m^2)}$$

(21)

$$\theta_i^{j+1} = \theta_i^j + \frac{\Delta\tau}{\Pr}\left[\frac{\theta_{i+1}^j - 2\theta_i^j + \theta_{i-1}^j}{(\Delta Z)^2}\right] + \Delta\tau Ec\left\{\left(\frac{U_{i+1}^j - U_{i-1}^j}{2\Delta Z}\right)^2 + \left(\frac{V_{i+1}^j - V_{i-1}^j}{2\Delta Z}\right)^2\right\}$$

$$+ \frac{M^2 \cdot Ec}{(1+m^2)^2}\Delta\tau\left\{\left(mU_i^j + V_i^j\right)^2 + \left(mV_i^j - U_i^j\right)^2\right\} + \frac{S}{\Pr}\theta_i^j\Delta\tau$$

$$- \Delta\tau\frac{\kappa^2}{\Pr_t}\left\{\begin{array}{l} 2i\Delta Z\left(\frac{V_{i+1}^j - V_{i-1}^j}{2\Delta Z}\right)\left(\frac{\theta_{i+1}^j - \theta_{i-1}^j}{2\Delta Z}\right) \\ + (i\Delta Z)^2\left[\frac{V_{i+1}^j - 2V_i^j + V_{i-1}^j}{(\Delta Z)^2}\left(\frac{\theta_{i+1}^j - \theta_{i-1}^j}{2\Delta Z}\right) + \frac{\theta_{i+1}^j - 2\theta_i^j + \theta_{i-1}^j}{(\Delta Z)^2}\left(\frac{V_{i+1}^j - V_{i-1}^j}{2\Delta Z}\right)\right]\end{array}\right\}$$

(22)

III. Discussion Of The Results

The numerical results were obtained from the finite difference scheme are represented figure 2, 3 and 4. The trends of various non-dimensional fluid parameters were discussed as they were obtained upon varying them for positive values of Gr corresponding to cooling of the plate.

The positive Grashof number (Gr>0) implies the fluid flow is at lower temperature than that of the plate. Therefore, the plate loses heat to the surrounding fluid through conduction. The fluid subsequently transfers heat through free convectional currents, that is, buoyancy driven past an infinite vertical plate.

3.1 Primary velocity (V)

From figure 2 it is observed that:

(i) Primary velocity decrease as the Grashof number (Gr) reduced.

(ii) There is a decrease in primary velocity as the magnetic parameter (M) is increased.

(iii) There is a decrease in the primary velocity as the Prandtl number (Pr) is increased. Prandtl number(Pr) is a ratio of kinematic viscosity to thermal diffusivity.

(iv) Primary velocity decrease as the heat parameter (S) is decreased.

(v) There is an increase in primary velocity as Hall parameter is increased.

3.2 Secondary velocity ($\boldsymbol{U}$)

From figure 3 it is observed that:

(i) Secondary velocity showed a more significant increase with the increase of the magnetic parameter.

(ii) There is a decrease in secondary velocity as Hall parameter is increased.

(iii) There is a decrease in secondary velocity when Grashof number is decreased.

(iv) An increase in Prandtl number (Pr) leads to a decrease in secondary velocity.

(v) A decrease in heat parameter (S) leads to a decrease in secondary velocity.

3.3 Temperature ($\boldsymbol{\theta}$)

From figure 4 it is observed that:

(i) There is a decrease in temperature as the Prandtl number (Pr) is increased.

(ii) There is no effect on temperature profile as magnetic parameter(M) is increased.

(iii) A decrease in heat parameter (S) leads to a decrease in temperature.

(iv) There is no effect on temperature profile as Hall parameter (m) and Grashof number (Gr) are increased.

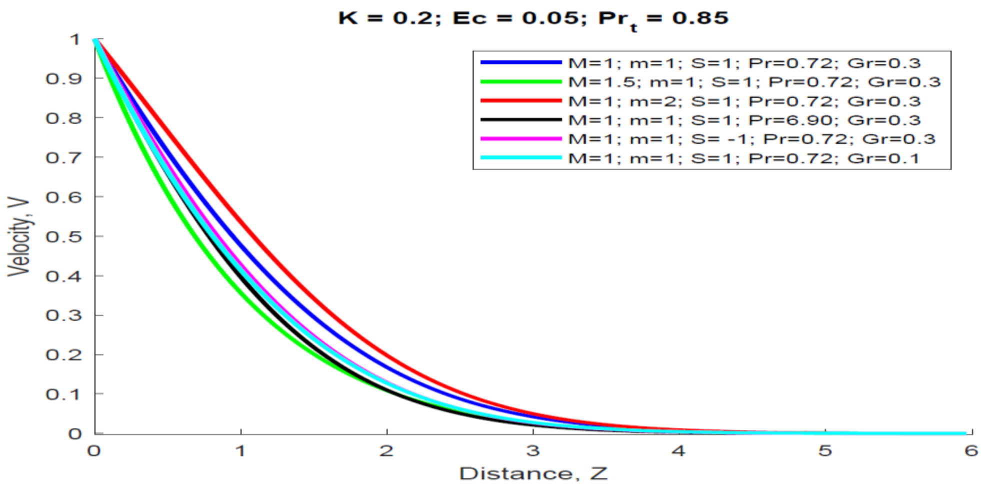

Figure2: Primary velocity profiles with cooling of the plate

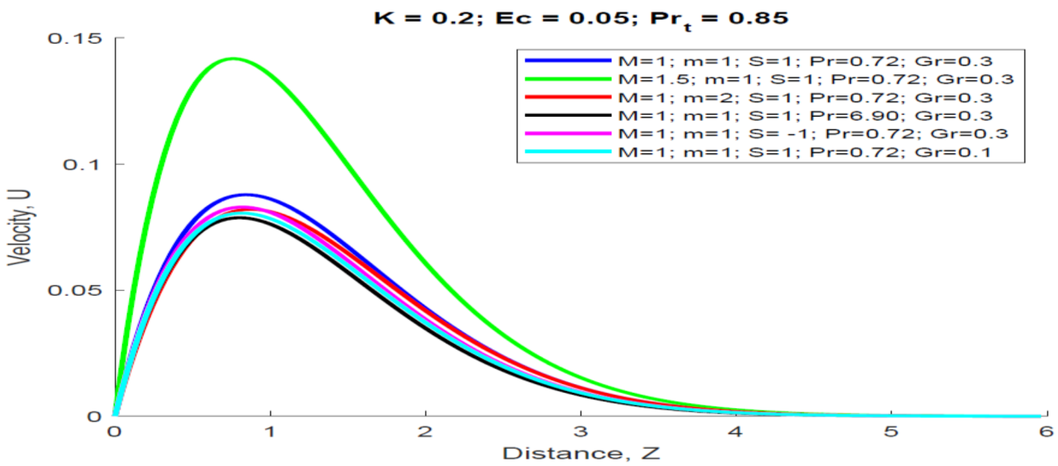

Figure3: Secondary velocity profiles with cooling of the plate

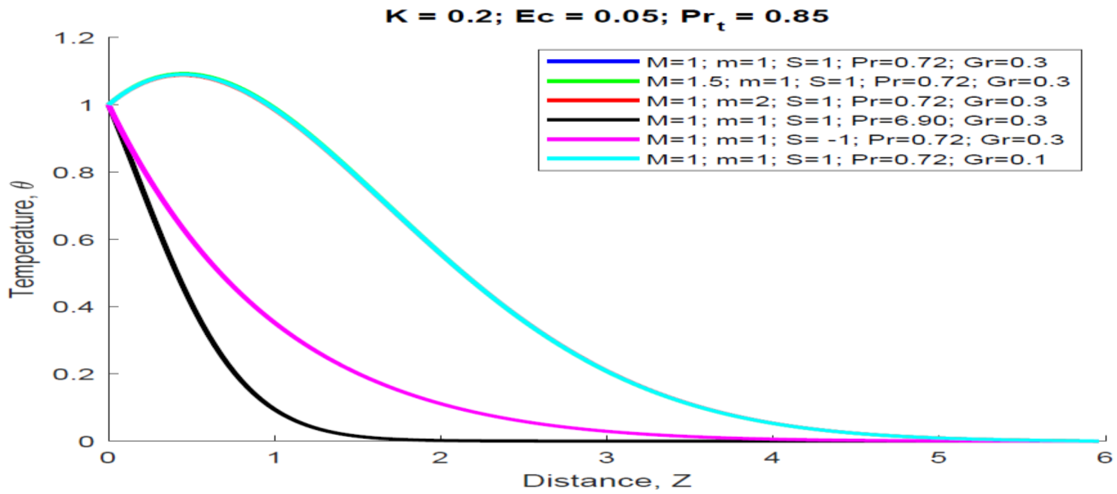

Figure 4: Temperature profiles with cooling of the plate

IV. Conclusion

In conclusion it is established that when $Gr > 0$, the primary velocity(V) increased with an increase in Grashof number(Gr) and Hall parameter (m) with decrease in heat parameter(S) but decreases with an increase in magnetic parameter(M). While, the secondary velocity increases with the increase in magnetic parameter(M), Grashof number$(Gr)$and decrease in heat parameter(S), but decreases with the increase in Hall parameter(m) and prandtl number(Pr). There is no effect on temperature profile, during cooling of the plate with increase of the magnetic parameter(M), Hall parameter (m) and Grashof number (Gr) but it decreases as Prandtl number (Pr) is increased and heat parameter(S) is decreased.

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