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An Eco-epidemic model with disease in Plant populations and Pesticides as control measure

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Abstract
An eco-epidemic model with diseases in populations of plants is proposed and analysed in the present study, and pesticides are used as a control mechanism. Pesticides are assumed to be sprayed to populations of both susceptible and infected plant species. Linear responses between susceptible and infective, between susceptible and pesticides, and between infective and pesticides are also considered. The positivity and boundedness of the system have been proven. The conditions for the existence of all possible equilibrium points and local stability have been carried out. The Routh-Hurwitz Criterion has been used to demonstrate the model's endemic equilibrium point. In order to address the analytical results, numerical simulations were finally conducted.
Keywords: *Eco- epidemic model, Pesticides, Jacobian matrix, Stability analysis, Routh-Hurwitz Criterion.*

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I. Introduction

Ecology is the study of how organisms interact with their surroundings. Ecosystems are a subfield of ecology and are made up of both biotic (living things like plants, animals, and other species) and abiotic (non-living things components) in the environment [1]. Eugene Odum defined ecosystem as "A unit that includes all the organisms, i.e., the community in a given area interacting with the physical environment so that a flow of energy leads to clearly defined trophic structure, biotic diversity, and material cycles, i.e., exchange of materials between living and non-living, within the system".

Two crucial subfields of biological mathematics and applied mathematics are mathematical ecology and mathematical epidemiology. The amount of research in this area has increased over time, and a new field known as mathematical eco-epidemiology has formed [2]. For many years, the impact of pests on the agricultural sector has been a major disaster not only for farmers, but also on the ecosystems, human health and the environment in general. For the purpose of preventing and managing diseases brought on by pests to agricultural crops, the use of pesticides and other natural enemies of pests are becoming more and more crucial. As a result, mathematical models have emerged as a crucial tool for studying the transmission and management of such diseases. Mathematical modeling is the process of transforming real-world problems into mathematical equations and mathematically solving these equations, and the resulting solutions are then transformed into real-world phenomena [3].

Alfred Hugo et al. analysed an environmental epidemiological model with optimal control strategies for infected prey [4]. They noticed that spreading disease within the population tended to decline when the control rate of contaminated prey increased. To prevent contamination, it is advised to keep the infected population separate from the susceptible population. A Prey-Predator Model with Two-Stage Infection in Prey combined with pest control was developed by Swapan Kumar Nandi et al. [5]. They used the prey population as pests, and the chosen pests are then eaten by predators. Additionally, they draw the conclusion that providing food for the insect population's natural enemies has a significant impact on the pest population's eradication.

An eco-epidemiological model with a Z-type control mechanism was presented and examined by AK Alzahrani et al. [6]. They looked at a predator-prey paradigm in which the prey is exposed to disease transmission and has a Holling type II functional response. They noted that the disease's ability to produce chaotic vibrations can destabilised the system. They created a Poincare map and compute the Lyapunov exponent in order to demonstrate the existence of chaos. They also noticed that chaos and disease may be removed from a system by applying an indirect z-controller to a population of predators.

An infectious disease that is present in that environment at a specific period of time is depicted mathematically by the Eco-epidemic model, which represents the ecosystem of interacting populations. [7-10].

The structure of the paper is as follows: An eco-epidemiological model and assumptions with disease in plant populations was developed in Section 2. Section 3 provides a discussion of positivity and boundedness. The stability study of each equilibrium point has been covered in sections 4 and 5. The analytical results are clarified using numerical simulations in Section 6. In Section 7, concise conclusions are provided. Advanced software like Mathematica, Matlab, and MatCont was used for analytical calculation, numerical simulation, etc.

II. Model Formulation and Assumptions

Let $x(t)$ and $y(t)$ be the susceptible population and infective population of the total plant populations respectively. Let $z(t)$ be the amount of pesticide used to control the infectious diseases in plant populations. The following assumptions are taken into account while formulating the mathematical eco-epidemiological model:

i. In the presence of disease, the total plant population is composed of two classes: the susceptible population $x(t)$ and the infected population $y(t)$. Therefore, the total plant population at any point in time t is given by $x(t) + y(t) = N(t)$.
ii. In the absence of the disease, the plant populations grows at a growth rate of r.
iii. The pesticide $z(t)$ is applied to both the susceptible and the infected populations.
iv. A linear response for βxy between susceptible and infective is considered.
v. Additionally, linear responses for cxz and dyz are taken into account between susceptible and pesticides, and between the infective and pesticides, respectively.
vi. All the model parameters are assumed to be positive.

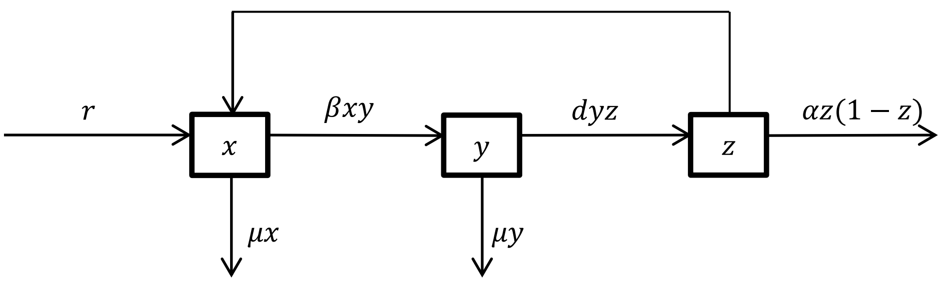

Figure 1 Transfer diagram of model (2.1)

Under the above assumptions, the following mathematical model is formed:

$$\frac{dx}{dt} = \mathrm{r} - \beta xy - \mu x - cxz$$
$$\frac{dy}{dt} = \beta xy - dyz - \mu y$$
$$\frac{dz}{dt} = \alpha z(1 - z) \qquad (2.1)$$

With initial conditions $x(0) \equiv x_0 > 0, y(0) \equiv y_0 > 0 \; and \; z(0) \equiv z_0 > 0.$ (2.2)

Where $\frac{dx}{dt}, \frac{dy}{dt}$ and $\frac{dz}{dt}$ represent the rate of change of the quantities $x(t), y(t)$ and $z(t)$ respectively.
Here, r is the constant growth rate of the plant populations, β is the susceptible and infective contact rate, μ is the natural death rate of plant populations, c is the susceptible and pesticides contact rate, d is the infective and pesticides contact rate and α is the amount of pesticides used.

III. Positivity and Boundedness

3.1. Positivity

Theorem 1: All solutions of the system represented by (2.1) that start in R^3 remain positive at R^3_+ for all $t \geq 0$.

Proof: Considering (2.1) in a matrix form,
$\dot{X} = F(X)$ where, $X = [x, y, z]^T$ and $\bar{X}(0) = [x_0, \; y_0, z_0]^T \epsilon \; {R_+}^3$ and

$$F(X) = \begin{bmatrix} \mathrm{r} - \beta xy - \mu x - cxz \\ \beta xy - dyz - \mu y \\ \alpha z(1 - z) \end{bmatrix}$$

It is observed that, for $\bar{X}(0) \epsilon \; {R_+}^3$, whenever and $\bar{X}(0) = [0, \; 0, 0]^T$, $F(X) \geq 0$, So the solution of (2.1) will always lie in ${R_+}^3$.
Hence, the theorem is proved.

3.2. Boundedness

Theorem 2: All solutions of the system (2.1) that start in $R_+^{\,3}$ are uniformly bounded in the solution set $\Omega = \left\{(x,y,z): 0 \le x \le \frac{r}{\mu}, 0 \le y \le \frac{r}{\mu},\ 0 \le z \le 1,\ 0 \le x+y \le \frac{r}{\mu}\right\}$.

Proof: Let $x(t), y(t), z(t))$ be a solution of the system (2.1).

Let $w = x + y$

$$\frac{dw}{dt} = \frac{dx}{dt} + \frac{dy}{dt}$$

$$\frac{dw}{dt} = \mathrm{r} - \beta xy - \mu x - cxz + \beta xy - \mathrm{dyz} - \mu y$$

$$\frac{dw}{dt} + \mu w = r - (cx + dy)z$$

Therefore, $\frac{dw}{dt} + \mu w \le r \Rightarrow w \le \frac{r}{\mu} + \mathrm{C}e^{-\mu t}$ and $w \le \frac{r}{\mu} + (w_0 - \frac{r}{\mu})e^{-\mu t}$

As t tends to infinity, $e^{-\mu t}$ tends to 0 implies W tends to $\frac{r}{\mu}$

Thus $w(t) \le \frac{r}{\mu}$ and hence w is bounded.

Now $\frac{dz}{dt} = \alpha z(1-z) \Rightarrow z = \frac{1}{1-Ce^{-\alpha t}}$

As t tends to infinity, z tends to 1 and hence z is bounded for any initial value and for all t.

Therefore, x, y, z are bounded.

Hence, from Theorem 1 and Theorem 2, we can conclude that the model is biologically valid and well defined.

IV. Equilibria

For finding the equilibrium points, we set the right-hand side of the system (2).1 equals zero. The system (2.1) has the following equilibrium points:

(i) The trivial equilibrium $E_0(0{,}0{,}0)$.

(ii) The equilibrium point $E_1(0{,}0{,}1)$.

(iii) The equilibrium point $E_2(x_1, 0{,}0)$ where $x_1 = \frac{r}{\mu}$ which always exist.

(iv) The equilibrium point $E_3(x_2, 0, z_2)$where $x_2 = \frac{r}{c+\mu}$ and $z_2 = 1$.

(v) The equilibrium point $E_4(x_3, y_3, z_3)$ where $x_3 = \frac{\mu}{\beta}$, $y_3 = \frac{1}{\mu}(r - \frac{\mu^2}{\beta})$ and $z_3 = 0$.

(vi) The endemic equilibrium point $E_5(x_4, y_4, z_4)$ where $x_4 = \frac{d+\mu}{\beta}$, $y_4 = \frac{r}{d+\mu} - \frac{(c+\mu)}{\beta}$ and $z_4 = 1$.

V. Stability Analysis

For analysing the stability properties of the model, we take the help of a Jacobian matrix. The Jacobian matrix J of the system (2.1) is reported as follows:

$$J = \begin{bmatrix} J_{11} & -\beta x & -cx \\ \beta y & J_{22} & -dy \\ 0 & 0 & J_{33} \end{bmatrix}$$

Where $J_{11} = -\beta y - \mu - cz$

$J_{22} = \beta x - dz - \mu$

$J_{33} = \alpha - 2\alpha z.$

The following theorems discuss the stability of six equilibrium points of the system (2.1).

Theorem 3 The trivial equilibrium point $E_0(0{,}0{,}0)$ is unstable.

Proof: The Jacobian matrix of E_0 is given by $J_{E_0} = \begin{bmatrix} -\mu & 0 & 0 \\ 0 & -\mu & 0 \\ 0 & 0 & \alpha \end{bmatrix}$

Eigenvalues of the above matrix are $\lambda_1 = -\mu, \lambda_2 = -\mu, \lambda_3 = \alpha$.

Eigenvalues $\lambda_1,\ \lambda_2$ are always negative and λ_3 is positive,

Hence, E_0 is unstable.

Theorem 4: The equilibrium point $E_1(0,0,1)$ is locally asymptotically stable.

Proof: The Jacobian matrix of E_1 is given by $J_{E_1} = \begin{bmatrix} -\mu - c & 0 & 0 \\ 0 & -d-\mu & 0 \\ 0 & 0 & -\alpha \end{bmatrix}$

Eigenvalues of the above matrix are $\lambda_1 = -(c+\mu), \lambda_2 = -(d+\mu), \lambda_3 = -\alpha$.
Clearly, the eigenvalues $\lambda_1 < 0, \lambda_2 < 0, \lambda_3 < 0$ and therefore $\lambda_1, \lambda_2, \lambda_3$ have negative real parts
Hence, the equilibrium E_1 is locally asymptotically stable.

Theorem 5: The equilibrium point $E_2(x_1, 0,0)$ is unstable.

Proof: The Jacobian matrix of E_2 is given by $J_{E_2} = \begin{bmatrix} -\mu & -\frac{\beta r}{\mu} & -\frac{cr}{\mu} \\ 0 & \frac{\beta r}{\mu} - \mu & 0 \\ 0 & 0 & \alpha \end{bmatrix}$

Eigenvalues of the above matrix are $\lambda_1 = -\mu, \lambda_2 = \frac{\beta r}{\mu} - \mu, \lambda_3 = \alpha$.
Clearly, eigenvalue λ_3 is positive.
Hence E_2 is unstable.

Theorem 6: The equilibrium point $E_3(x_2, 0,1)$ is locally asymptotically stable if $A_0 < 1$,
where $A_0 = \frac{\beta r}{(c+\mu)(d+\mu)}$.

Proof: The Jacobian matrix of E_3 is given by $J_{E_3} = \begin{bmatrix} -\mu - c & -\frac{\beta r}{c+\mu} & -\frac{cr}{c+\mu} \\ 0 & \frac{\beta r}{c+\mu} - d - \mu & 0 \\ 0 & 0 & -\alpha \end{bmatrix}$

Eigenvalues of the above matrix are $\lambda_1 = -(c+\mu), \lambda_2 = \frac{\beta r}{\mu + c} - d - \mu\,, \lambda_3 = -\alpha$.

Here, the eigenvalues $\lambda_1 < 0, \lambda_3 < 0 \Rightarrow \lambda_1, \lambda_3$ have negative real parts.
Now for stability, we must have: $\lambda_2 < 0$ i.e. $\frac{\beta r}{\mu+c} - d - \mu < 0 \Rightarrow \frac{\beta r}{(c+\mu)(d+\mu)} < 1 \Rightarrow A_0 < 1$.

Hence, the equilibrium E_3 is locally asymptotically stable if $A_0 < 1$, where $A_0 = \frac{\beta r}{(c+\mu)(d+\mu)}$.

Theorem 7: The equilibrium point $E_4(x_3, y_3, z_3)$ is unstable.

Proof: The Jacobian matrix of E_4 is given by $J_{E_4} = \begin{bmatrix} -\frac{\beta r}{\mu} & -\mu & -\frac{c\mu}{\beta} \\ \frac{\beta r}{\mu} - \mu & 0 & \frac{-d}{\mu}(r - \frac{\mu^2}{\beta}) \\ 0 & 0 & \alpha \end{bmatrix}$

Eigenvalues of the above matrix are $\lambda_1 = \frac{\frac{-\beta r}{\mu} - \sqrt{\left(\frac{\beta r}{\mu}\right)^2 - 4\mu(\frac{\beta r}{\mu} - \mu)}}{2}, \lambda_2 = \frac{\frac{-\beta r}{\mu} + \sqrt{\left(\frac{\beta r}{\mu}\right)^2 - 4\mu(\frac{\beta r}{\mu} - \mu)}}{2}$ and $\lambda_3 = \alpha$.
Clearly, the eigenvalue λ_3 is positive.
Hence E_4 is unstable.

Theorem 8: The endemic equilibrium point $E_5(x_4, y_4, z_4)$ is locally asymptotically stable if and only if the following condition holds:

I. $\beta r > (c+\mu)(d+\mu)$.
II. $\beta r + \alpha(d+\mu) > 0$.
III. $\beta r\left(1 + \frac{\alpha}{d+\mu}\right) > (c+\mu)(d+\mu)$.
IV. $\left(\frac{\beta r}{d+\mu} + \alpha\right)\left[\beta r\left(1 + \frac{\alpha}{d+\mu}\right) - (c+\mu)(d+\mu)\right] > \alpha[\beta r - (c+\mu)(d+\mu)]$.

Proof: We will use the Routh-Hurwitz Criterion to analysed the stability of the equilibrium point E_5.

The Jacobian matrix of E_5 is given by $J_{E_5} = \begin{bmatrix} -\frac{\beta r}{d+\mu} & -(d+\mu) & -\frac{c(d+\mu)}{\beta} \\ \frac{\beta r}{d+\mu}-(c+\mu) & 0 & -d\left[\frac{r}{d+\mu}-\frac{(c+\mu)}{\beta}\right] \\ 0 & 0 & -\alpha \end{bmatrix}$

Here,

$$w_1 = -\det(J_{E_5}) = \alpha[\beta r - (c+\mu)(d+\mu)]$$

$$w_2 = -tr(J_{E_5}) = \frac{\beta r}{d+\mu} + \alpha$$

$$w_3 = det\begin{bmatrix} -\frac{\beta r}{d+\mu} & -(d+\mu) \\ \frac{\beta r}{d+\mu}-(c+\mu) & 0 \end{bmatrix} + det\begin{bmatrix} -\frac{\beta r}{d+\mu} & -\frac{c(d+\mu)}{\beta} \\ 0 & -\alpha \end{bmatrix} + det\begin{bmatrix} 0 & -d\left[\frac{r}{d+\mu}-\frac{(c+\mu)}{\beta}\right] \\ 0 & -\alpha \end{bmatrix}$$

$$= \beta r\left(1+\frac{\alpha}{d+\mu}\right) - (c+\mu)(d+\mu)$$

According to Routh-Hurwitz Criterion, the real parts of all eigenvalues of J_{E_5} are negative if and only if
$w_i > 0$ for $i = 1,2,3$ and $w_2 w_3 > w_1$ (8.1)
Now, from equation (8.1), we have;

I. $w_1 > 0 \Rightarrow \alpha[\beta r - (c+\mu)(d+\mu)] > 0 \Rightarrow \beta r > (c+\mu)(d+\mu)$.

II. $w_2 > 0 \Rightarrow \frac{\beta r}{d+\mu} + \alpha > 0$.

III. $w_3 > 0 \Rightarrow \beta r\left(1+\frac{\alpha}{d+\mu}\right) - (c+\mu)(d+\mu) > 0 \Rightarrow \beta r\left(1+\frac{\alpha}{d+\mu}\right) > (c+\mu)(d+\mu)$.

IV. $w_2 w_3 > w_1 \Rightarrow \left(\frac{\beta r}{d+\mu}+\alpha\right)\left[\beta r\left(1+\frac{\alpha}{d+\mu}\right) - (c+\mu)(d+\mu)\right] > \alpha[\beta r - (c+\mu)(d+\mu)]$.

Hence, the equilibrium point E_5 is locally asymptotically stable if and only if the above condition holds.

VI. Numerical Analysis

The proposed model is investigated numerically to observe the behaviour of the spread of disease and the role of control measures on the decline of the disease. Numerical analysis is done on Matlab 2018a.

6.1 Parameters and Initial Conditions

$x(0) = 100$ (100% of plant population), $y(0) = 1$(10% of plant population infected), $z(0) = 20$(proportion of pesticide used). $r = 0.01, \beta = 0.001, \mu = 0.001, c = 0.001, d = 0.007, \alpha = 0.01$.

6.2 Numerical simulation

Using the above parameters and initial values, a model (2.1) with $t = 700$ is simulated for the proposed eco-epidemiological model. Figure 2 shows a diagram without control measures and Figure 3 shows a diagram with control measures. Figure 4 shows a diagram of the amount of pesticide used as a control measure.

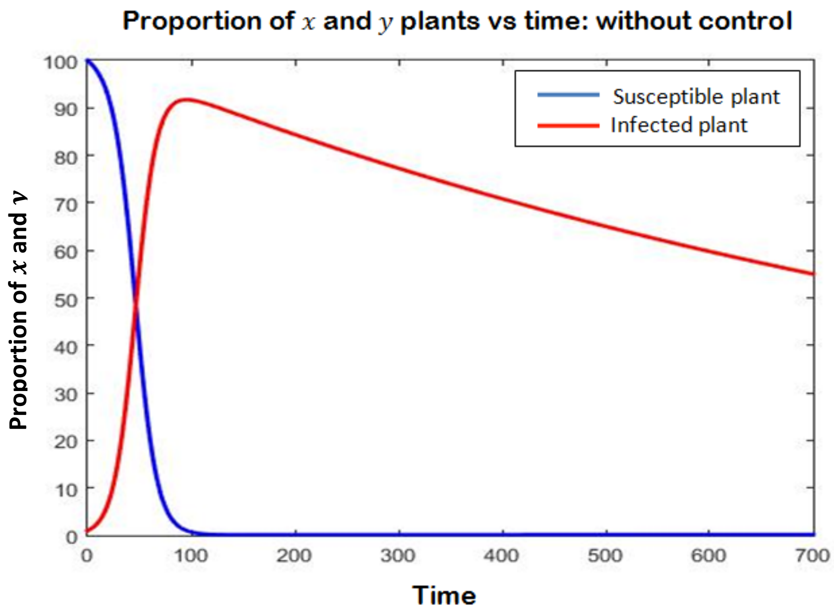

Figure 2: x-y model without control

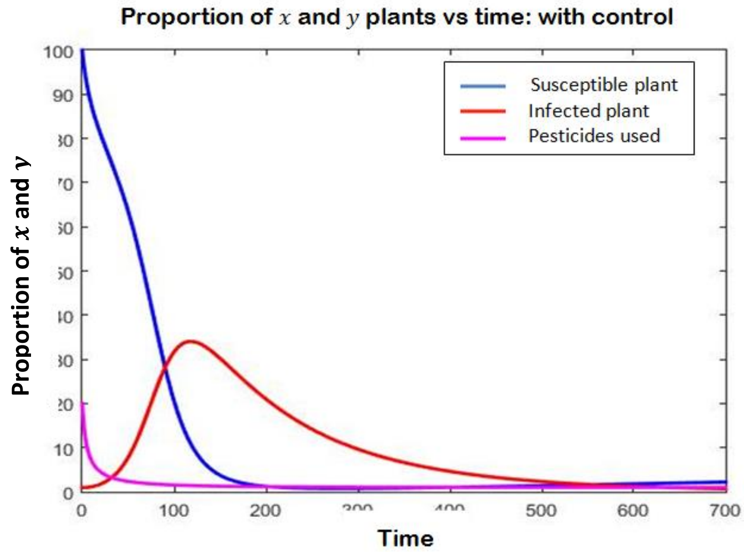

Figure 3: x-y model with pesticide as control measure

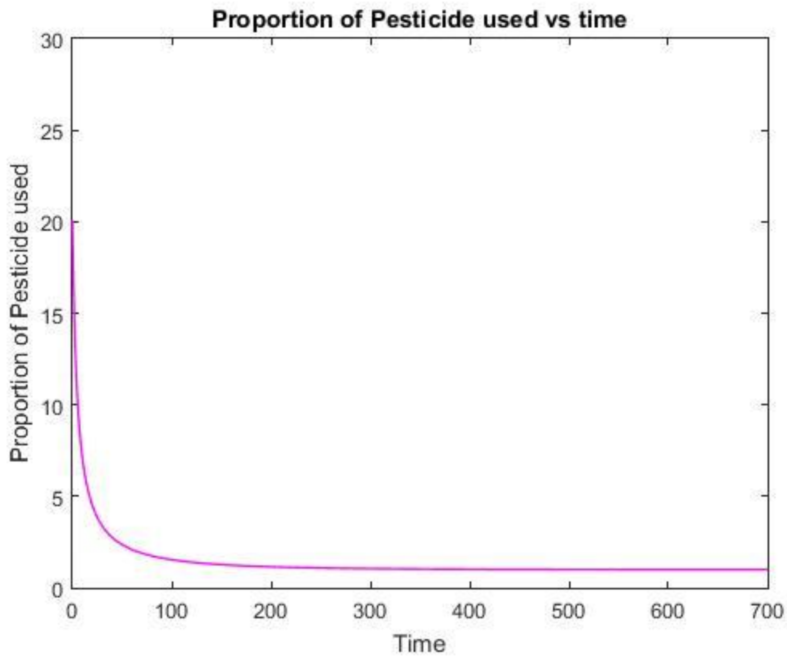

Figure 4: Amount of pesticides used

VII. Conclusion

The proposed mathematical model given by (2.1) and (2.2) have been analysed analytically and numerically. For local stability analysis, six points of equillibria have been obtained. It has been observed that:

- The equilibrium point $E_4(x_3, y_3, z_3)$ exists only if $r \geq \frac{\mu^2}{\beta}$
- $E_5(x_4, y_4, z_4)$ exists only if $\frac{r}{d+\mu} \geq \frac{(\mu+c)}{\beta}$

The stability analysis of the model around equilibrium point has been carried out using Eigenvalue theorem and Routh Hurwitz criteria. It is observed that:

- The trivial equilibrium point $E_0(0,0,0)$, $E_2\left(\frac{r}{\mu}, 0,0\right)$ and $E_4\left(\frac{\mu}{\beta}, \frac{1}{\mu}(r - \frac{\mu^2}{\beta}),0\right)$ are unstable.
- The equilibrium point $E_1(0,0,1)$ is locally asymptotically stable.

Also, the rest of the equilibria are locally asymptotically stable under certain parametric conditions as described below:

- The equilibrium point $E_3\left(\frac{r}{\mu+c}, 0,1\right)$ is locally asymptotically stable if $\frac{\beta r}{(c+\mu)(d+\mu)} < 1$.
- The endemic equilibrium point $E_5\left(\frac{d+\mu}{\beta}, \frac{r}{d+\mu} - \frac{(\mu+c)}{\beta}, 1\right)$ is locally asymptotically stable if and only if

I. $\beta r > (c+\mu)(d+\mu)$.

II. $\beta r + \alpha(d+\mu) > 0$.

III. $\beta r\left(1+\frac{\alpha}{d+\mu}\right) > (c+\mu)(d+\mu)$.

IV. $\left(\frac{\beta r}{d+\mu}+\alpha\right)\left[\beta r\left(1+\frac{\alpha}{d+\mu}\right)-(c+\mu)(d+\mu)\right] > \alpha[\beta r-(c+\mu)(d+\mu)]$.

The numerical analysis of the model with suitable parameter values as described in Figure2-Figure4 using MatLab2018a, it is observed from Figure 2 and Figure 3 that:

- Susceptible plant population decline and infectives increases without the presence of Pesticide.
- Both susceptible and infectives decline with the use of pesticide as per the assumption of the model that use of pesticide will not only act on infectives but also on plant without infection.

The numerical analysis validates the model assumptions.

References

[1]. Omotosho, L.O., Olayiwola M.O., Olaleye V.O. & Ozoh P. (2020). Simulation of Mathematical Model for Ecological Surveillance of Prey-Predator. Islamic University Multidisciplinary JournalIUMJ, vol. 7(1).

[2]. Kar, T.K. (2012). A Mathematical Study on the Dynamics of an Eco-Epidemiological Model in the Presence of Delay. Applications and Applied Mathematics: An International Journal (AAM), 7(1), 300 – 333.

[3]. Kapur. J.N. (1998). Mathematical modelling. New Age International (P) Limited, Publishers.

[4]. Hugo, A. & Simanjilo, E. (2019). Analysis of an Eco-Epidemiological Model under Optimal Control Measures for Infected Prey. Applications and Applied Mathematics: An International Journal (AAM), 14(10, 117-138.

[5]. Nandi, S.K, Mondal, P.K, Jana, S., Haldar, P. & Kar, T.K. (2015). Prey-Predator Model with Two-Stage Infection in Prey: Concerning Pest Control. Journal of Nonlinear Dynamics, Volume 2015, Article ID 948728, 13 pages.

[6]. Alzahrani, A.K., Alshomrani, A.S., Pal, N. & Samanta, S. (2018). Study of an eco-epidemiological model with Z-type control. Chaos, Solitons & Fractals, 113, 197-208.

[7]. Abraha, T., Basir, F.A., Obsu, L.L. & Tores, D.F.M. (2021). Pest control using farming awareness: Impact of time delays and optimal use of biopesticides. Chaos, Solitons & Fractals, 146, 110869.

[8]. Beltrami, E. & Carroll, T.O. (1994). Modelling the role of viral disease in recurrent phytoplankton blooms. Journal of Mathematical Biology, 32, 857–863.

[9]. Hall, C.A.S. & Day, J.W. (1990). Ecosystem Modeling in Theory and Practice: An Introduction with Case Histories. University Press of Colorado, 7–8.

[10]. Kermack, W.O. & McKendrick, A.G. (1927). A Contribution to the Mathematical Theory of Epidemics. Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character, 115(772), 700–721.

[11]. Wang, L., Zhang, F. & Jin, C. (2017). Analysis of an eco-epidemiological model with disease in the prey and predator International Journal of Mathematical Research, 6(1), 22-28.

[12]. Bezabih, A.F., Edessa, G.K. & Rao, K.P. (2020), Mathematical Eco-Epidemic Model on Prey-Predator System. IOSR Journal of Mathematics (IOSR-JM), 16 (1), 22-34.

[13]. Hugo1, A., Massawe, E.S. & Makinde O.D. (2012). An eco-epidemiological mathematical model with treatment and disease infection in both prey and predator population. Journal of Ecology and the Natural Environment, 4(10), 266-279.

[14]. Kendall, D.G. (1956). Deterministic and stochastic epidemics in closed populations. Proceedings of the Third Berkeley Symposium on Mathematical Statistics and Probability: Contributions to Biology and Problems of Health, 4, 149–165.

[15]. Hilker, F.M. (2008). Schmitz K. Disease-induced stabilization of predator–prey oscillations. Journal of Theoretical Biology, 255(3), 299-306.

[16]. Saenz, R.A. & Hethcote, H.W. (2006). Competing species models with an infectious disease. Mathematical Biosciences and Engineering, 3, 219–235.

[17]. Kumar, R., Chatterjee, A., Singh, M.K. & Singh, V.P. (2020). Mathematical modelling to establish the influence of pesticides on groundwater contamination. Arabian Journal of Geosciences, 13:603.

[18]. Dubey, B. (2007). A Prey-Predator Model with a Reserved Area. Nonlinear Analysis: Modelling and Control, 12(4), 479–494.

[19]. Venturino, E. (2002). Epidemics in predator-prey models. Disease in the predators: IMA Journal of Mathematics Applied in Medicine and Biology, 19, 185–205.

[20]. Bhattacharjee, D., Kashyap, A.Y., Sarmah, H.K. & Paul, R. (2021). Dynamics in a ratio-dependent eco-epidemiological predator-prey model having cross species disease transmission. Communications in Mathematical Biology and Neuroscience, , 2021:15.

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