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Inverse Trigonometric Functions

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Abstract
To Find the Principal Value of an Inverse Trigonometric Functions using a Unit Circle, and analysing the domain and range of Inverse Trigonometric Functions.

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I. Introduction

Inverse Trigonometric Functions are simply defined as the inverse functions of the basic trigonometric functions which are sine, cosine, tangent, cotangent, secant, and cosecant functions.

They Are also called "**Arc Functions**" since, for a given value of trigonometric functions, they produce the length of arc needed to obtain that particular value. The inverse trigonometric functions perform the opposite operation of the trigonometric functions such as sine, cosine, tangent, cosecant, secant, and cotangent. We know that trigonometric functions are especially applicable to the right angle triangle.

These six important functions are used to find the angle measure in the right triangle when two sides of the triangle measures are known.

The inverse trigonometry functions have major applications in the field of engineering, physics, geometry, and navigation.

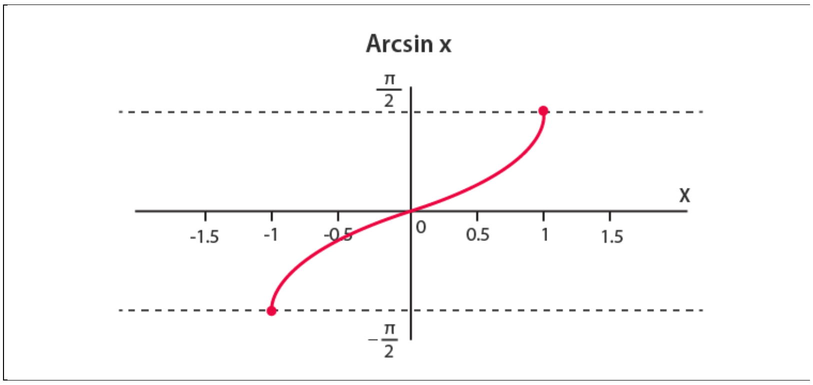
There are particularly six inverse trig functions for each trigonometric ratio. The inverse of six important trigonometric functions are:

- Arcsine
- Arccosine
- Arctangent
- Arccotangent
- Arcsecant
- Arccosecant

Domain and Range

Arcsine Function:

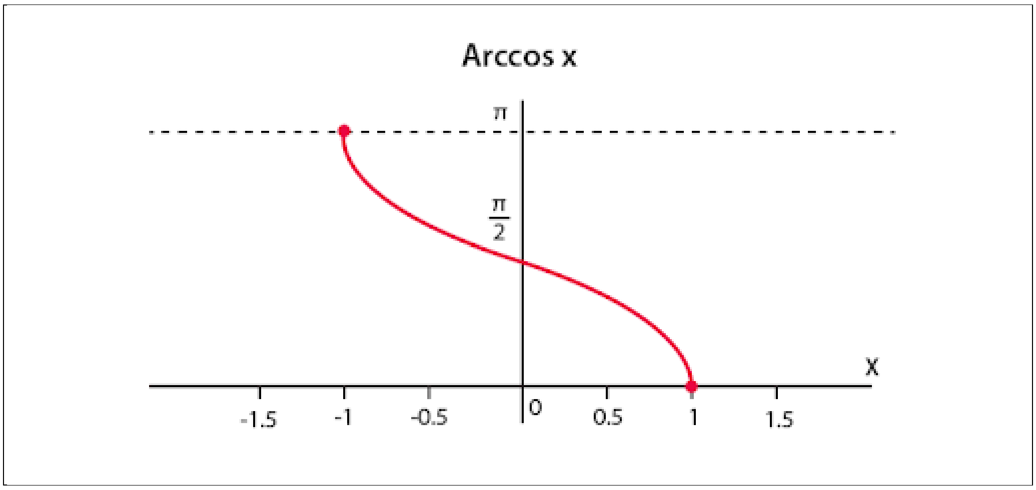
Arcsine function is an inverse of the sine function denoted by $\sin^{-1}x$. It is represented in the graph as shown below:



Domain & Range of arcsine function:

Domain	$-1 \le x \le 1$
Range	$-\pi/2 \le y \le \pi/2$

Arccosine Function:
Arccosine function is the inverse of the cosine function denoted by $\cos^{-1}x$. It is represented in the graph as shown below:



Therefore, the inverse of cos function can be expressed as **$y = \cos^{-1}x$ (arccosine x)**

Domain & Range of arccosine function:

Domain	$-1 \le x \le 1$
Range	$0 \le y \le \pi$

Inverse Trigonometric Functions Table:

Function Name	Notation	Definition	Domain of x	Range
Arcsine or inverse sine	$y = \sin^{-1}x$	x=sin y	$-1 \le x \le 1$	• $-\pi/2 \le y \le \pi/2$ • $-90° \le y \le 90°$
Arccosine or inverse cosine	$y=\cos^{-1}x$	x=cos (y)	$-1 \le x \le 1$	• $0 \le y \le \pi$ • $0° \le y \le 180°$
Arctangent or Inverse tangent	$y=\tan^{-1}x$	x=tan y	All real numbers	• $-\pi/2 < y < \pi/2$ • $-90° < y < 90°$
Arccotangent or Inverse Cot	$y=\cot^{-1}x$	x=cot y	All real numbers	• $0 < y < \pi$ • $0° < y < 180°$
Arcsecant or Inverse Secant	$y =\sec^{-1}x$	x=sec y	$x \le -1$ or $1 \le x$	• $0 \le y < \pi/2$ or $\pi/2 < y \le \pi$ • $0° \le y < 90°$ or $90° < y \le 180°$
Arccosecant	$y=\mathrm{cosec}^{-1}x$	x=cosec y	$x \le -1$ or $1 \le x$	• $-\pi/2 \le y < 0$ or $0 < y \le \pi/2$ • $-90° \le y < 0°$ or $0° < y \le 90°$

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Principal Value

The principal value of a trig function f(x) for x > 0, is the length of the arc of a unit circle centered at the origin which subtends an angle at the center whose value is x. For this reason, $\sin^{-1} x$ is also denoted by arcsin x. Similarly, $\cos^{-1}x$, $\tan^{-1}x$, $\mathrm{cosec}^{-1}x$ $\sec^{-1}x$, and $\cot^{-1}$ are denoted by arccos x, arctan x, arcsc x, arcsec x, arcot x.

The inverse functions $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$, $\mathrm{cosec}^{-1}x$, $\sec^{-1}x$, $\cot^{-1}x$ are called inverse circular functions. For the function y=sinx, there are infinitely many angles x which satisfy sin x = t, $-1 \le t \le 1$. Of these infinite sets of values, there is one which lies in the interval $[-\pi/2, \pi/2]$.

This angle is called the principal angle and denoted by $\sin^{-1} t$. The principal value of an inverse function is the general value which is numerically least. It may be positive or negative.

When there are two values, one is positive and the other is negative such that they are numerically equal, then the principal value is the positive one.

The trigonometric functions aren't really invertible, because they have multiple inputs that have the same output. For example, sin0=sin π= 0. So what should be $\sin^{-1}0$?

In order to define the inverse functions, we have to restrict the domain of the original functions to an interval where they are invertible. These domains determine the range of the inverse functions.

The value from the appropriate range that an inverse function returns is called **the principal value** of the function

Degree to Radians

Degrees and radians are both ways to write out angle measurements. They are ways of measuring angles. A radian is equal to the amount an angle would have to be open to capture an arc of the circle's circumference of equal length to the circle's radius.

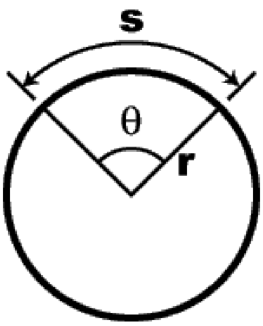
360° (360 degrees) is equal to 2π radians. Degrees are more common in general: there are 360 degrees in a whole circle, 180 degrees in a half-circle, and 90 degrees in a quarter of a circle (a right angle)

A radian is the amount an angle has to open such that the length of the section of the circle's circumference it captures is equal to the length of the radius.

From the latter, we obtain the equation $1°=\pi/180$ radians. This leads us to the rule to convert degree measure to radian measure. To convert from degrees to radians, multiply the degrees by $\pi/180°$ radians.

A radian is defined by an arc of a circle. The length of the arc is equal to the radius of the circle. Because of this the radian is a fixed size no matter what the size of the circle is.

Recall that the circumference of a circle is $2\pi R$, so that means there are 2π, or roughly 6.28 radians in a full circle. Because a full circle is also exactly 360°, each radian comes out to approximately 57.296°.



	Exact	Approximate
Given degrees (D)	$\pi D/180$ radians	0.017D radians
Given radians (R)	$180R/\pi$ degrees	57.3R degrees

There are two reasons for using radians over degrees:
The radian is defined to be (see the diagram) the ratio of the length of the arc of a circle (indicated by s in the diagram) to the length of the radius of the circle (indicated by r in the diagram), where **each length is measured in the same unit**. Therefore, when you divide s by r to get the radian measure of the angle, the units for the two lengths cancel, and you end up with a measure that has no units:

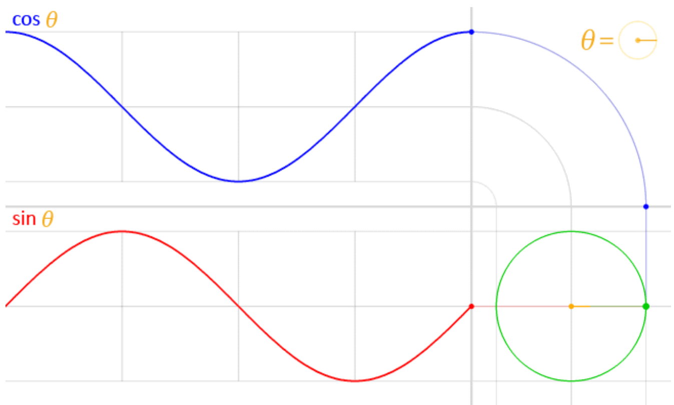
If you are working with the derivative of a trigonometric function, then it is preferable to use radian measure for angles, because then derivative formulas (and limit formulas) are easier. For example, using radians, the derivative formulas for sine and cosine are:

$$\frac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x \quad \text{and} \quad \frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$$

However, if degrees are used, the derivative formulas for sine and cosine are:

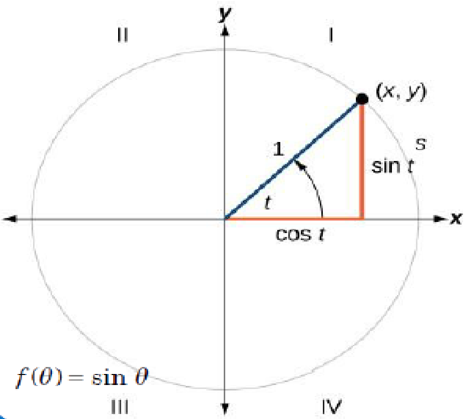
$$\frac{\mathrm{d}}{\mathrm{d}x}\sin x = \left(\frac{\pi}{180}\right)\cos x \quad \text{and} \quad \frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\left(\frac{\pi}{180}\right)\sin x$$

Trig Functions using Unit Circle



The x- and y-axes divide the coordinate plane (and the unit circle, since it is centered at the origin) into four quarters called quadrants. We label these quadrants to mimic the direction a positive angle would sweep. The four quadrants are labeled I, II, III, and IV.

For any length coordinates, (x,y) , we can get the positive angle sweep as $\sin^{-1}t$ *or* $\cos^{-1}t$*. x and y will be the outputs of the trigonometric functions* $\cos^{-1}x = t$ *and* $\sin^{-1}y = t$ *respectively.*



This means:

$x = \cos t$ *and* $y = \sin t$

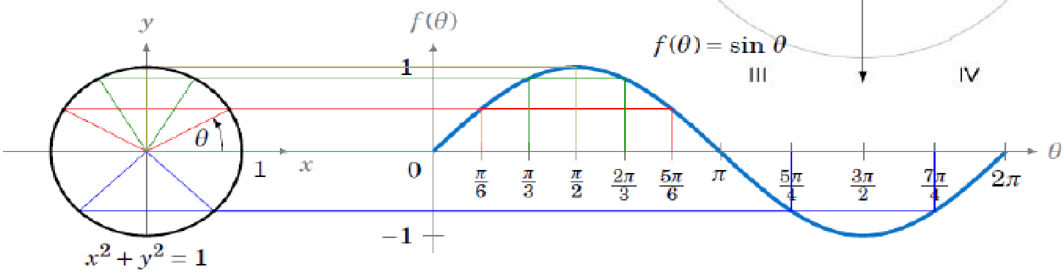


Fig:Unit Circle and sinθ

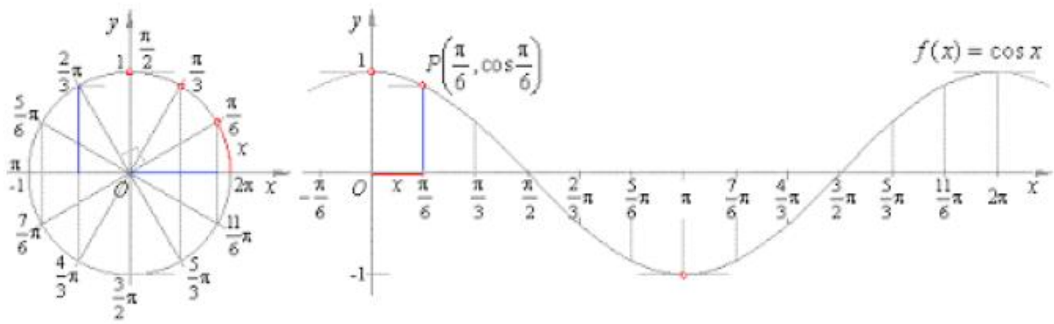


Fig:Unit Circle and cosθ

Calculations

For sin $^{-1}$(arcsin):

- *Let us take the point (1,1)*

That means sin t = 1 and cos t = 1

sin t = 1
t = $\sin^{-1}$ 1
t = $\pi/2$

- Let us take the point (0,0)

That means sin t = 0 and cos t = 0

sin t=0
t = $\sin^{-1}$0
t = 0

- Let us take the point (-1,-1)

That means sin t = -1 and cos t = -1

sin t = -1
t = $\sin^{-1}$ -1
t = $-\pi/2$

For cos $^{-1}$(arccos):

- Let us take the point (1,1)

That means sin t = 1 and cos t = 1

cos t = 1

$t = \cos^{-1} 1$
$t = 0$

- Let us take the point (0,0)

  That means sin t = 0 and cos t = 0

  $\cos t = 0$
  $t = \cos^{-1} 0$
  $t = \pi/2$

- Let us take the point (-1,-1)

  That means sin t = -1 and cos t = -1

  $\cos t = -1$
  $t = \cos^{-1} -1$
  $t = \pi$

II. Conclusion

The Principal values of $\sin^{-1}$(arcsin) and $\cos^{-1}$(arccos) are :

t	**0**	**1**	**-1**
$\sin^{-1} t$	0	$\pi/2$	$-\pi/2$
$\cos^{-1} t$	$\pi/2$	0	π

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