

On Nirmala indices of carbon nanocone $C_4[2]$

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Abstract

Nirmala index of a graph is recently introduced degree based topological index which is defined as $N(G) = \sum_{uv \in E(G)} \sqrt{(d_{(G)}(u) + d_{(G)}(v))}$, where G is finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$ and d_v is the degree of vertex $v \in V(G)$. In this paper different versions of Nirmala index of carbon nanocone $C_4[2]$ are investigated.

Keywords: Carbon nanocone $C_4[2]$, degree, multiplication degree, Nirmala index, reduced inverse Nirmala index, sum connectivity index, vertex degree sum.

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I. Introduction

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree of vertex $v \in V(G)$, d_v is the number of edges incident with v . A topological index is a numerical parameter mathematically derived from the graph structure. The function

$F(x, y) = (x + y)^\lambda$ for the general sum connectivity index where λ is an adjustable parameter were discussed by I. Gutman and J. Tosovic for testing quality of topological indices [1]. The topological indices of Vitamin D_3 are computed by M.R.R. Kanna et al. in [2].

The molecular graph of carbon nanocones $CNCk[n]$ have conical structures with a cycle of length k at its center and n layers of hexagons placed at the conical surface around its center [3]. Carbon nanocones are one of the forms of carbon nanostructures and these have been proposed as possible molecular gas storage devices [4]. V.R. Kulli introduced Nirmala index, first and second inverse Nirmala indices of a graph G [5-6] as

$$\text{Nirmala index } N(G) = \sum_{uv \in E(G)} \sqrt{(d_{(G)}(u) + d_{(G)}(v))},$$

$$\text{first inverse Nirmala index } IN_1(G) = \sum_{uv \in E(G)} \left(\frac{1}{d_u} + \frac{1}{d_v}\right)^{\frac{1}{2}},$$

$$\text{and second inverse Nirmala index } IN_2(G) = \sum_{uv \in E(G)} \left(\frac{1}{d_u} + \frac{1}{d_v}\right)^{-\frac{1}{2}}.$$

We introduce reduced first and second inverse, δ -first and δ -second inverse and average Nirmala indices of a graph G as

$$\text{reduced first inverse Nirmala index } RIN_1(G) = \sum_{uv \in E(G)} \left(\frac{1}{d_u-1} + \frac{1}{d_v-1}\right)^{\frac{1}{2}},$$

$$\text{reduced second inverse Nirmala index } RIN_2(G) = \sum_{uv \in E(G)} \left(\frac{1}{d_u-1} + \frac{1}{d_v-1}\right)^{-\frac{1}{2}},$$

$$\delta\text{-first inverse Nirmala index } \delta IN_1(G) = \sum_{uv \in E(G)} \left(\frac{1}{d_u-\delta(G)+1} + \frac{1}{d_v-\delta(G)+1}\right)^{\frac{1}{2}},$$

$$\text{and } \delta\text{-second inverse Nirmala index } \delta IN_2(G) = \sum_{uv \in E(G)} \left(\frac{1}{d_u-\delta(G)+1} + \frac{1}{d_v-\delta(G)+1}\right)^{-\frac{1}{2}},$$

where $\delta(G) \geq 2$, $\delta(G)$ is the minimum degree among the vertices of G .

Recently introduced degree based and mostly studied topological index is Sombor index

$SO(G) = \sum_{uv \in E(G)} \sqrt{(d_u^2 + d_v^2)}$, where d_v is degree of vertex v in graph G [7].

The average Sombor index is introduced and studied for graphs in [8-9] as

$$SO_{\text{avg}}(G) = \sum_{uv \in E(G)} \left[(d_u - \frac{2m}{n})^2 + (d_v - \frac{2m}{n})^2 \right]^{\frac{1}{2}}, \text{ where } |V(G)| = n \text{ and } |E(G)| = m.$$

Like average Sombor index we propose average Nirmala index as

$$N_{\text{avg}}(G) = \sum_{uv \in E(G)} \left[(d_u - \frac{2m}{n}) + (d_v - \frac{2m}{n}) \right]^{\frac{1}{2}}, \text{ where } |V(G)| = n \text{ and } |E(G)| = m.$$

Different versions of Nirmala index of certain chemical structures were studied by V.R. Kulli in [10]. Inverse degree, Randic index and Harmonic index of graphs were discussed by K.C. Daset al. [11]. In [12] different versions of Harmonic indices of certain nanotubes are discussed by authors wherein six types of Harmonic

indices were defined and computed. Sixth version of Harmonic index is Q_u type Harmonic index. Harmonic index in terms of Q_u parameter is

$$H_{gen}(G) = \sum_{uv \in E(G)} \frac{2}{Q_u + Q_v},$$

where Q_u is the unique parameter which is acquired from the vertex $u \in V(G)$, we call it Q_u type Harmonic index for multiplication degree of vertices [13] which is

$$H_{Q_u}(G) = \sum_{uv \in E(G)} \frac{2}{M_u + M_v}, \text{ where } M_v = \prod_{u \in N(v)} \text{deg}(u).$$

We introduce Q_u type Nirmala index as

$$N_{Q_u}(G) = \sum_{uv \in E(G)} (M_u + M_v)^{\frac{1}{2}}, \text{ where } M_v = \prod_{u \in N(v)} \text{deg}(u).$$

R-degree Nirmala index by M-polynomials for carbon nanocone $C_4[2]$ was studied by N.K. Raut in [14]. The sum connectivity index was studied by [15-18] which is defined as

$$\square = \sum_{uv \in E(G)} (d_u + d_v)^{-\frac{1}{2}}.$$

Multiplicative sum connectivity index was investigated by Y.C. Kwun et al. in [19]. The reduced sum connectivity index and reduced product connectivity indices of $TUC_4C_8[S]$ was computed by N.K. Raut [20]. The reduced reciprocal Randic index defined by M.K. Jamil in [21] as

$$RRR(G) = \sum_{uv \in E(G)} \sqrt{(d_u - 1) + (d_v - 1)}.$$

Like reduced Bahhatti-Sombor index or reduced Sombor index, increased Sombor index was introduced by W. Ning [22-23] as

$$SO^1(G) = \sum_{uv \in E(G)} \sqrt{(d_u + 1)^2 + (d_v + 1)^2}.$$

We propose first and second increased inverse Nirmala indices as [24]

$$IN_1^1(G) = \sum_{uv \in E(G)} \left(\frac{1}{d_u + 1} + \frac{1}{d_v + 1}\right)^{\frac{1}{2}} \text{ and } IN_2^1(G) = \sum_{uv \in E(G)} \left(\frac{1}{d_u + 1} + \frac{1}{d_v + 1}\right)^{-\frac{1}{2}} \text{ respectively.}$$

By using the first derivative of the Schultz, modified Schultz polynomials of Jahangir graph $J_{3,m}$ (evaluated at $x = 1$) one can compute the Schultz, modified Schultz indices [25] as

$$SC(J_{3,m}) = \left. \frac{\partial Sc(J_{3,m}, x, y)}{\partial x} \right|_{x=1}.$$

Considering the Nirmala index, we define the R-degree Nirmala index as

$$N_R(G) = \sum_{uv \in E(G)} \sqrt{r(u) + r(v)}, \text{ where } M_v = \prod_{u \in N(v)} \text{deg}(u) \text{ and } S_v = \sum_{u \in N(v)} \text{deg}(u) \text{ and } r(v) = M_v + S_v.$$

The terms and notations used in this paper are standard and mainly taken from books of graph theory [26-28].

In this paper reduced first and second inverse Nirmala indices ($RIN_1(G)$ and $RIN_2(G)$), δ -first and δ -second inverse Nirmala indices ($\delta IN_1(G)$ and $\delta IN_2(G)$), average Nirmala index ($N_{avg}(G)$), Q_u type Nirmala index ($N_{Q_u}(G)$), R-degree Nirmala index ($N_R(G)$), first and second increased inverse Nirmala indices ($IN_1^1(G)$ and $IN_2^1(G)$) are investigated for carbon nanocone $C_4[2]$.

II. Materials and Methods

A molecular graph is a simple and connected graph. The 2-dimensional graph of carbon nanocone $C_4[2]$ is shown in figure 1. Let the graph of carbon nanocone $C_4[2]$ be denoted by G . There are three edges in G given by $E_1 = 4, E_2 = 16, E_3 = 28$ as $E_1 = \{uv \in E(G) | d_u = d_v = 2\}, E_2 = \{uv \in E(G) | d_u = 2 \text{ and } d_v = 3\}$ and $E_3 = \{uv \in E(G) | d_u = d_v = 3\}$. It is observed from figure that the vertex set = 36 and edge set = 48. Edge set of $C_4[2]$ are $|E_{(5,7)}|, |E_{(7,9)}|, |E_{(5,5)}|, |E_{(6,7)}|$ and $|E_{(9,9)}|$ in sum degree of vertices and $|E_{(6,12)}|, |E_{(12,27)}|, |E_{(6,6)}|, |E_{(9,12)}|$ and $|E_{(27,27)}|$ in multiplication degree of vertices. To compute Q_u type and R-degree Nirmala indices, multiplication degree of vertices and sum-multiplication degree of vertices are used respectively. The edge partition of carbon nanocone $C_4[2]$ is used to study $RIN_1(G), RIN_2(G), \delta$ -first and δ -second inverse, $N_{avg}(G)$, first and second increased inverse Nirmala indices and for Q_u type Nirmala index, R-degree Nirmala index, multiplication degree and sum-multiplication degree. In this study $RIN_1(G), RIN_2(G), \delta IN_1(G), \delta IN_2(G), N_{avg}(G), N_{Q_u}(G), N_R(G), IN_1^1(G)$ and $IN_2^1(G)$ are computed.

III. Results and Discussion

3.1 Reduced first, second, δ -first, δ -second inverse Nirmala indices and average Nirmala index of carbon nanocone $C_4[2]$

The molecular graph of carbon nanocone $C_4[2]$ is shown in figure 1. Let the graph of carbon nanocone $C_4[2]$ be denoted by G .

The 2-D graph of carbon nanocone $C_4[2]$ has 48 edges and 36 vertices and degree of vertices 2 and 3. The edge partition of carbon nanocone $C_4[2]$ is given in table 1 and sum degree and multiplication degree of vertices in table 2.

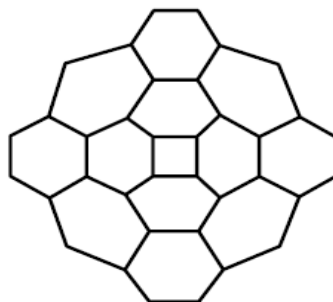


Figure 1. 2-D graph of carbon nanocone $C_4[2]$.

(d_u, d_v)	(2,2)	(2,3)	(3,3)
Number of edges	4	16	28

Table 1. Edge partition of carbon nanocone $C_4[2]$.

(S_u, S_v)	(5,7)	(7,9)	(5,5)	(6,7)	(9,9)
(M_u, M_v)	(6,12)	(12,27)	(6,6)	(9,12)	(27,27)
Number of edges	8	8	4	8	20

Table 2. The sum degree and multiplication degree of vertices of carbon nanocone $C_4[2]$.

Theorem 1. The first reduced inverse Nirmala index of carbon nanocone $C_4[2]$ is 53.

Proof. Consider a molecular graph of carbon nanocone $C_4[2]$ as shown in figure 1. Let $E_{(u,v)}$ denote the edge connecting the vertices of u and v . The graph contains $E_{(2,2)}$, $E_{(2,3)}$ and $E_{(3,3)}$ edges. Using edge partition from table 1 we get first reduced inverse Nirmala index.

$$\begin{aligned} \text{RIN}_1(G) &= \sum_{uv \in E(G)} \left(\frac{1}{d_u-1} + \frac{1}{d_v-1} \right)^{\frac{1}{2}} \\ &= |E_{(2,2)}| \left(\frac{1}{2-1} + \frac{1}{2-1} \right)^{\frac{1}{2}} + |E_{(2,3)}| \left(\frac{1}{2-1} + \frac{1}{3-1} \right)^{\frac{1}{2}} + |E_{(3,3)}| \left(\frac{1}{3-1} + \frac{1}{3-1} \right)^{\frac{1}{2}} \\ &= 4(2)^{\frac{1}{2}} + 16(1.5)^{\frac{1}{2}} + 28(1)^{\frac{1}{2}} \\ &= 53. \end{aligned}$$

Theorem 2. The second reduced inverse Nirmala index of carbon nanocone $C_4[2]$ is 44.

Proof. Consider a molecular graph of carbon nanocone $C_4[2]$ as shown in figure 1. Let $E_{(u,v)}$ denote the edge connecting the vertices of u and v . The graph contains $E_{(2,2)}$, $E_{(2,3)}$ and $E_{(3,3)}$ edges. Using edge partition from table 1 we get second reduced inverse Nirmala index.

$$\begin{aligned} \text{RIN}_2(G) &= \sum_{uv \in E(G)} \left(\frac{1}{d_u-1} + \frac{1}{d_v-1} \right)^{-\frac{1}{2}} \\ &= |E_{(2,2)}| \left(\frac{1}{2-1} + \frac{1}{2-1} \right)^{-\frac{1}{2}} + |E_{(2,3)}| \left(\frac{1}{2-1} + \frac{1}{3-1} \right)^{-\frac{1}{2}} + |E_{(3,3)}| \left(\frac{1}{3-1} + \frac{1}{3-1} \right)^{-\frac{1}{2}} \\ &= 4(2)^{-\frac{1}{2}} + 16(1.5)^{-\frac{1}{2}} + 28(1)^{-\frac{1}{2}} \\ &= 44. \end{aligned}$$

Theorem 3. The δ -first inverse Nirmala index of carbon nanocone $C_4[2]$ is 53.

Proof. Using edge partition from table 1 we get δ -first inverse Nirmala index.

Here $\delta(G) \geq 2$, where $\delta(G)$ is the minimum degree among vertices. In graph G degree of vertices are 2 and 3, so $\delta(G) = 2$.

$$\begin{aligned} \text{The } \delta\text{-first inverse Nirmala index } \delta\text{IN}_1(G) &= \sum_{uv \in E(G)} \left(\frac{1}{d_u-\delta(G)+1} + \frac{1}{d_v-\delta(G)+1} \right)^{\frac{1}{2}} \\ &= |E_{(2,2)}| \left(\frac{1}{2-2+1} + \frac{1}{2-2+1} \right)^{\frac{1}{2}} + |E_{(2,3)}| \left(\frac{1}{2-2+1} + \frac{1}{3-2+1} \right)^{\frac{1}{2}} + |E_{(3,3)}| \left(\frac{1}{3-2+1} + \frac{1}{3-2+1} \right)^{\frac{1}{2}} \\ &= 4(2)^{\frac{1}{2}} + 16(1.5)^{\frac{1}{2}} + 28(1)^{\frac{1}{2}} \\ &= 53. \end{aligned}$$

Theorem 4. The δ -second inverse Nirmala index of carbon nanocone $C_4[2]$ is 44.

Proof. Using edge partition from table 1 we get δ -second inverse Nirmala index.

Here $\delta(G) \geq 2$, where $\delta(G)$ is the minimum degree among vertices. In graph G degree of vertices are 2 and 3, so $\delta(G) = 2$.

$$\text{The } \delta\text{-second inverse Nirmala index } \delta\text{IN}_2(G) = \sum_{uv \in E(G)} \left(\frac{1}{d_u-\delta(G)+1} + \frac{1}{d_v-\delta(G)+1} \right)^{-\frac{1}{2}}$$

$$\begin{aligned}
 &= |E_{(2,2)}| \left(\frac{1}{2-2+1} + \frac{1}{2-2+1}\right)^{-\frac{1}{2}} + |E_{(2,3)}| \left(\frac{1}{2-2+1} + \frac{1}{3-2+1}\right)^{-\frac{1}{2}} + |E_{(3,3)}| \left(\frac{1}{3-2+1} + \frac{1}{3-2+1}\right)^{-\frac{1}{2}} \\
 &= 4(2)^{-\frac{1}{2}} + 16(1.5)^{-\frac{1}{2}} + 28(1)^{-\frac{1}{2}} \\
 &= 44.
 \end{aligned}$$

Theorem 5. The average Nirmala index of carbon nanocone $C_4[2]$ is $4\sqrt{-1.332} + 16\sqrt{-0.332} + 28\sqrt{0.668}$.

Proof. Using edge partition from table 1 we get average Nirmala index.

Here $|V(G)| = n = 36$ and $|E(G)| = m = 48$, hence $\frac{2m}{n} = \frac{2 \cdot 48}{36} = 2.666$.

$$\begin{aligned}
 N_{\text{avg}}(G) &= \sum_{uv \in E(G)} \left[\left(d_u - \frac{2m}{n}\right) + \left(d_v - \frac{2m}{n}\right) \right]^{\frac{1}{2}} \\
 &= |E_{(2,2)}| \left[(2 - 2.666) + (2 - 2.666) \right]^{\frac{1}{2}} + |E_{(2,3)}| \left[(2 - 2.666) + (3 - 2.666) \right]^{\frac{1}{2}} + |E_{(3,3)}| \left[(3 - 2.666) + (3 - 2.666) \right]^{\frac{1}{2}} \\
 &= 4\sqrt{-1.332} + 16\sqrt{-0.332} + 28\sqrt{0.668}.
 \end{aligned}$$

3.2 Q_u type, R-degree, first and second increased inverse Nirmala indices of carbon nanocone $C_4[2]$

In the following section Q_u type, R-degree, first and second increased inverse Nirmala indices are computed.

Theorem 6. The Q_u type Nirmala index of carbon nanocone $C_4[2]$ is 281.3.

Proof. Consider a molecular graph of carbon nanocone $C_4[2]$ as shown in figure 1. Let $M_{u,v}$ denote the multiplication degree of vertices u and v in G . The graph contains $M_{(6,12)}, M_{(12,27)}, M_{(6,6)}, M_{(9,12)}$ and $M_{(27,27)}$ edges. Using edge partition from table 2, we get Q_u type of Nirmala index.

$$\begin{aligned}
 N_{Q_u}(G) &= \sum_{uv \in E(G)} (M_u + M_v)^{\frac{1}{2}}, \text{ where } M_v = \prod_{u \in N(v)} \text{deg}^{\text{in}}(u) \\
 &= |E_{(6,12)}| (M_u + M_v)^{\frac{1}{2}} + |E_{(12,27)}| (M_u + M_v)^{\frac{1}{2}} + |E_{(6,6)}| (M_u + M_v)^{\frac{1}{2}} + |E_{(9,12)}| (M_u + M_v)^{\frac{1}{2}} + |E_{(27,27)}| (M_u + M_v)^{\frac{1}{2}} \\
 &= 8(6 + 12)^{\frac{1}{2}} + 8(12 + 27)^{\frac{1}{2}} + 4(6 + 6)^{\frac{1}{2}} + 8(9 + 12)^{\frac{1}{2}} + 20(27 + 27)^{\frac{1}{2}} \\
 &= 281.3.
 \end{aligned}$$

Theorem 7. The R-degree Nirmala index of carbon nanocone $C_4[2]$ is 338.9.

Proof. Consider a molecular graph of carbon nanocone $C_4[2]$ as shown in figure 1. Let $M_{u,v}$ denote the multiplication degree and $S_{u,v}$ denote sum degree of vertices of u and v in G .

$$\begin{aligned}
 \text{The R-degree Nirmala index } N_R(G) &= \sum_{uv \in E(G)} \sqrt{r(u) + r(v)}, \\
 \text{where } M_v &= \prod_{u \in N(v)} \text{deg}^{\text{in}}(u), S_v = \sum_{u \in N(v)} \text{deg}^{\text{in}}(u) \text{ and } r(v) = M_v + S_v.
 \end{aligned}$$

$$\begin{aligned}
 \text{The R-degree Nirmala exponential is } N_R(G, x) &= \sum_{uv \in E(G)} x^{\sqrt{r(u) + r(v)}} \\
 &= 8x^{\sqrt{11+19}} + 8x^{\sqrt{19+36}} + 4x^{\sqrt{11+11}} + 8x^{\sqrt{15+19}} + 20x^{\sqrt{36+36}}.
 \end{aligned}$$

$$\begin{aligned}
 \text{R-degree Nirmala index } N_R(G) &= \left. \frac{\partial (G, x)}{\partial x} \right|_{x=1} \\
 &= 8\sqrt{30} + 8\sqrt{55} + 4\sqrt{22} + 8\sqrt{34} + 20\sqrt{72} \\
 &= 338.9.
 \end{aligned}$$

Theorem 8. The first increased inverse Nirmala index of carbon nanocone $C_4[2]$ is 35.

Proof. Consider a molecular graph of carbon nanocone $C_4[2]$ as shown in figure 1. Let $E_{(u,v)}$ denote the edge connecting the vertices of u and v in G . The graph contains E_1, E_2 and E_3 edges. Using edge partition from table 1 we get first increased inverse Nirmala index.

$$\begin{aligned}
 IN_1^{-1}(G) &= \sum_{uv \in E(G)} \left(\frac{1}{d_u+1} + \frac{1}{d_v+1}\right)^{\frac{1}{2}} \\
 &= |E_{(2,2)}| \left(\frac{1}{2+1} + \frac{1}{2+1}\right)^{\frac{1}{2}} + |E_{(2,3)}| \left(\frac{1}{2+1} + \frac{1}{3+1}\right)^{\frac{1}{2}} + |E_{(3,3)}| \left(\frac{1}{3+1} + \frac{1}{3+1}\right)^{\frac{1}{2}} \\
 &= 4\left(\frac{1}{3} + \frac{1}{3}\right)^{\frac{1}{2}} + 16\left(\frac{1}{3} + \frac{1}{4}\right)^{\frac{1}{2}} + 28\left(\frac{1}{4} + \frac{1}{4}\right)^{\frac{1}{2}} \\
 &= 35.
 \end{aligned}$$

Theorem 9. The second increased inverse *Nirmala index* of carbon nanocone $C_4[2]$ is 65.

Proof. Using edge partition from table 1 we get second increased inverse *Nirmala index*.

$$\begin{aligned} IN_2^{-1}(G) &= \sum_{uv \in E(G)} \left(\frac{1}{d_u+1} + \frac{1}{d_v+1} \right)^{-\frac{1}{2}} \\ &= |E_{(2,2)}| \left(\frac{1}{2+1} + \frac{1}{2+1} \right)^{-\frac{1}{2}} + |E_{(2,3)}| \left(\frac{1}{2+1} + \frac{1}{3+1} \right)^{-\frac{1}{2}} + |E_{(3,3)}| \left(\frac{1}{3+1} + \frac{1}{3+1} \right)^{-\frac{1}{2}} \\ &= 4 \left(\frac{1}{3} + \frac{1}{3} \right)^{-\frac{1}{2}} + 16 \left(\frac{1}{3} + \frac{1}{4} \right)^{-\frac{1}{2}} + 28 \left(\frac{1}{4} + \frac{1}{4} \right)^{-\frac{1}{2}} \\ &= 65. \end{aligned}$$

IV. Conclusion

We have introduced and computed some *Nirmala indices*. The reduced first inverse *Nirmala index* and δ -first inverse *Nirmala index* is equal to 53 and the second inverse *Nirmala index* and δ -second inverse *Nirmala index* is equal to 44 for carbon nanocone $C_4[2]$. Average *Nirmala index*, Q_u type *Nirmala index*, R -degree *Nirmala index*, first and second increased inverse *Nirmala indices* are investigated for carbon nanocone $C_4[2]$.

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