IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 18, Issue 4 Ser. III (Jul. – Aug. 2022), PP 10-11
www.iosrjournals.org

The Relative Importance of Parameter Values in the Use of Mathematics Instructional Materials With Sensitivity Analysis

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Abstract
In this paper, we perform sensitivity analysis on a mathematical model which describes the evolution of users of mathematics instructional materials over time. We use ODE23 and ODE45 sensitivity analyses to select the important parameters. Our results indicate that the total number of potential users in the learning population is the dominant most sensitive parameter.

Keywords: *Sensitivity Analysis, Mathematics Instructional Materials, ODE23, ODE45, Parameters Potential Users, Learning Population*

Date of Submission: 15-07-2022 Date of Acceptance: 31-07-2022

I. Introduction

The application of a deterministic fundamental diffusion model in different fields of research has received extensive reports (Dickerson and Gentry, 1983; Ekaka-a, 2010; Gatignon et al., 1989; Mahajan et al., 1990; Olshavsky, 1980; Rogers, 1962; Vijay et al., 1979; Vijay et al., 1985; Ziemer, 1985). While the bulk of these research reports has applications in several aspects of the discipline of marketing, it is rare to find the inclusion of sensitivity analysis which measures the relative importance of each model parameter. It is against this primary limitation that we propose to determine the important parameters in the use of mathematics instructional materials. The model which describes the evolution of users of mathematics instructional materials over time (Rogers, 1962).

II. Mathematical Formulation

Following Rogers (1962), the fundamental diffusion model is described by

$$\frac{dN(t)}{dt} = g(t)\big(K - N(t)\big) \tag{2.1}$$

With the boundary condition $N(t = t_0) = N_0 > 0$

Here, $N(t)$ stands for the cumulative number of users of mathematics instructional materials at time t, K stands for the total number of potential users in the learning population at time t otherwise called the carrying capacity for this population of users, $\frac{dN(t)}{dt}$ stands for the rate of diffusion at time t, $g(t)$ stands for the coefficient of diffusion and N_0stands for the cumulative number of users at the initial time. For the purpose of this simulation, we will consider the scenario when the coefficient of diffusion takes the functional structure of $a + bN(t)$.

The diffusion model clearly shows that the rate of diffusion of mathematics instructional materials at any time t is directly proportional to the difference between the total number of users existing at that time and the number of previous users at that time which is hereby represented by $\big(K - N(t)\big)$. A possible implication of this diffusion model shows that as the cumulative number of prior users $N(t)$ approaches the total number of potential users in the learning population, K, the rate of diffusion will decrease. Following Rogers (1962), we will assume that the interaction between the rate of diffusion and the number of potential users existing at time t, $\big(K - N(t)\big)$, can be controlled by $g(t)$. The value of $g(t)$ is likely to change when certain features of the diffusion process such as the nature of the instructional materials and the communication channels are implemented. In this paper, we consider the function $g(t)$ to represent the probability of usage at time t. In this context, the expression $g(t)\big(K - N(t)\big)$ will represent the expected number of users at time t.

III. Methodology Of Sensitivity Analysis

The method of sensitivity analysis over a continuous time is described by three steps (Bass, 1969). In the first step, the given model of Rogers (1962) is coded as a single model equation without varying its parameters. In the second step, the modified version of the given model is coded. In the third step, a sub-model which calls the models in step 1 and step 2 and calculates the cumulative percentage change due to each parameter variation is coded in a MATLAB programming language using both the ODE23 and ODE45 in-built functions which are based on the Runge-Kutta formulations of orders 2-3 and 4-5 respectively. Our sensitivity calculations for only the ODE45 is presented since the ODE23 show similar sensitivity calculations.

IV. Discussion Of Results

In this section, we will present and discuss our key results. These results will be presented in the form of four co-ordinates. The first co-ordinate presents the sensitivity of parameter K while the second co-ordinate presents the sensitivity of time t. Similarly, the third co-ordinate presents the sensitivity of parameter a while the fourth co-ordinate presents the sensitivity of time b. We consider parameter variations of 1 percent, 2 percent, 3 percent, 4 percent and 5 percent.

In this context, the 1-norm cumulative percentage effects due to these parameter variations are (98.95, 0.071, 0.042, 0.044), (97.95, 0.037, 0.045, 0.037), (96.95, 0.0413, 0.043, 0.040), (95.95, 0.0398, 0.0440, 0.0385) and (94.95, 0.0395, 0.0438, 0.0392).
The 2-norm cumulative percentage effects due to these parameter variations are (98.998, 0.3021, 0.0576, 0.0745), (97.998, 0.0485, 0.0590, 0.0500), (96.998, 0.0546, 0.0573, 0.0528), (95.998, 0.0508, 0.0566, 0.0529) and (94.998, 0.0528, 0.0573, 0.0521).
The infinity-norm cumulative percentage effects due to these parameter variations are (99.001, 3.2415, 0.1932, 0.5821), (98.002, 0.1420, 0.1873, 0.1645), (97.004, 0.1712, 0.1592, 0.1606), (96.004, 0.1266, 0.1590, 0.1649) and (95.004, 0.1443, 0.1506, 0.1516).

In summary, our present sensitivity analysis shows that the total number of potential users in the learning population is the dominant most sensitive parameter irrespective of the mathematical norm which is used to calculate the sensitivity values.

V. Conclusion

In this study, we have used the tool of sensitivity analysis to determine the important parameter in the use of mathematics instructional materials. In our context, we have identified and selected the total number of potential users in the learning population at time as the most sensitive parameter. The implication of this key result is that the total number of potential users in the learning population will need to be estimated efficiently in order for us to obtain reliable results. The other model parameters will only need to be considered as high estimates. In terms of application and educational policy, varying the most sensitive parameter a little and obtaining biggest effect in the solution trajectory proposes that the size the users learning population will need to be small to enhance proper learning and articulation of mathematics instructional materials.

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NAFO N. M. "The Relative Importance of Parameter Values in the Use of Mathematics Instructional Materials With Sensitivity Analysis." *IOSR Journal of Mathematics (IOSR-JM),* 18(4), (2022): pp. 10-11.