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Inventory Model for Deteriorating Items with Quadratic DemandRate and constant Deterioration under two parameter Weibull Distribution

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Abstract-*In this paper, we have analysed an inventory model for deteriorating items with time dependent quadratic demand rate and time dependent holding cost. Two parameter Weibull distribution is considered for time to deterioration and shortages are not allowed to occur.The objective is to minimize the average total inventory cost.The solution of the discussed model is illustrated by a numerical example and sensitivity analysis is carried out to understand the effect of change of the significant parameters. The aim of this paper is to derive optimum results to help the retailer to take inventory management decisions for an economic order quantity model.*

Keywords **:***Inventory model, deteriorating items, quadratic demand, Weibull distribution.*

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I. Introduction

Inventory management is becoming more and more crucial in today's time with rapid growing technology and ever-rising competition across industries, especially with 2 categories of goods- perishable products like food items such as eggs, milk, green vegetables and seasonal products such as woolen apparels, new technological devices etc. A lot of research was done in late 1990s to develop inventory models to figure out the optimum inventory level in order to fulfil customer's demand on time but that assumed constant demand function. In real scenario, important factors like time and price hugely affect the demand and so any model with a constant demand rate is less real. Hence, many researchers then discussed models with time-varying demand rate. Linear demand rate implies steady positive or negative trend in demand of the product which is very rare in the market. In contrast, other spiking items like latest mobile phones, computer chips, software etc. show rapid change in demand with time. While some researchers modeled this kind of demand with exponential rate, others suggested that exponential demand is extra-ordinarily high and all variations in demand may not be drastically growing or dropping. The best way to represent such demand is by means of a quadratic demand rate. Inventory management system is also largely affected by deterioration of physical goods; specially in case of perishable goods like bread, fish, food grains etc. Decay or spoilage of such items cannot be overlooked as these incur a lot of inventory holding costs. Hence, it makes sense to consider time-varying holding cost in the model to bring it bearer to real-world situations.

A partial backlogging inventory model with time-dependent demand as well as holding cost is proposed in [3]. An EOQ model with parabolic time changeable holding cost and quadratic time-sensitive demand rate with salvage value is established in [15]. An economic lot size inventory model with non-linear cumulative holding cost dependent on both quantity and time, stock level dependent demand rate and without any shortages is modeled in [11].

The payment terms affect the buying behavior of distributors and retailers. [5] dealt with the inventory problem for exponentially deteriorating items when supplier allowed delay in payments under constant demand rate. An EOQ model is discussed in [7] with quadratic time dependent demand rate with permissible delay in payment.

Sometimes, excess demand causes shortages in the inventory system which incurs lost sale cost and affects the total profit. An EOQ model with trapezoidal type, time dependent demand rate with shortages is developed in [10]. An inventory model with demand rate as a function of selling price with time dependent holding cost is suggested in [16]. An inventory model for stock-dependent demand and time varying holding cost under different trade credits is studied in [14]. Another order-level inventory model with demand rate represented as a continuous, quadratic function of time is presented in [6]. A deterministic inventory model with quadratic demand rate and holding cost as a linear function of time is presented in [12], here time to deterioration follows exponential distribution.

An investigation of multi-warehouse inventory problem with different deterioration effects under inflation has been done in [1] A multi-item inventory problem and multi-objective optimization problem was proposed in [2]. Similarly, a multi-criteria decision making approach for a multi-item inventory model over infinite horizon with price dependent demand rate and without shortages is discussed in [9].

Many researchers developed inventory models with time dependent holding cost with or without shortages. An economic order quantity model with deterioration considered as Pareto distribution with linear demand and shortages is presented in [13]. A deterministic inventory model with time dependent demand and time dependent holding cost where shortages are allowed and partially backlogged is developed in [4]. An EPQ Model under constant amelioration with exponential demand rate and completely backlogged shortages is suggested in [8].

II. Notations

We consider the following notations for the mathematical model used:

1. I(t) : On-hand inventory level at time t ($0 \leq t \leq T$)
2. R(t) :Quadratic demand rate varying over time
3. $\theta(t)$:Rate of deterioration per unit per unit time
4. A : Ordering cost per order that is known and constant.
5. C_p : Purchasing cost per unit
6. C_d : Deterioration cost per unit per unit time
7. T : Fixed length of each cycle
8. θ : Deterioration rate (θ>0)
9. TC : Total cost per unit time

III. Assumptions

Following is the list of assumptions we have taken to develop the inventory model:

1. The inventory system deals with single item.
2. The annual demand rate is a quadratic function of time and it is R(t) = $a + bt + ct^2$ (a,b,c > 0)
3. $\theta(t) = \alpha\beta t^{\beta-1}$is the two parameter Weibull deterioration rate in the time interval [0, T]; where α is scale parameter ($0 < \alpha << 1$) and β is shape parameter.
4. Holding cost is linear function of time given by h + rt (h, r > 0)
5. Replenishment rate is infinite and instantaneous.
6. The lead time is zero.
7. Shortages are not allowed.
8. Time horizon is infinite.
9. No repair or replacement of the deteriorated items takes place during a given cycle.
10. Total inventory cost is a real, continuous function and is convex to the origin.

IV. Mathematical model formulation and solution

With these assumptions, consider this inventory cycle as shown in the figure below that starts with its on-hand and maximum inventory level I(t) at time t = 0. This inventory then starts depleting partially due to demand and partially due to two-parameter Weibull distribution detrioration rate in the time period [0, T]. The inventory level drops down to zero at time t= T. At this point, inventory is replenished in the cycle since shortages are not allowed. This inventory cycle keeps repeating. The objective of this inventory problem is to find out the optimal order quantity such that the average total cost is minimum.

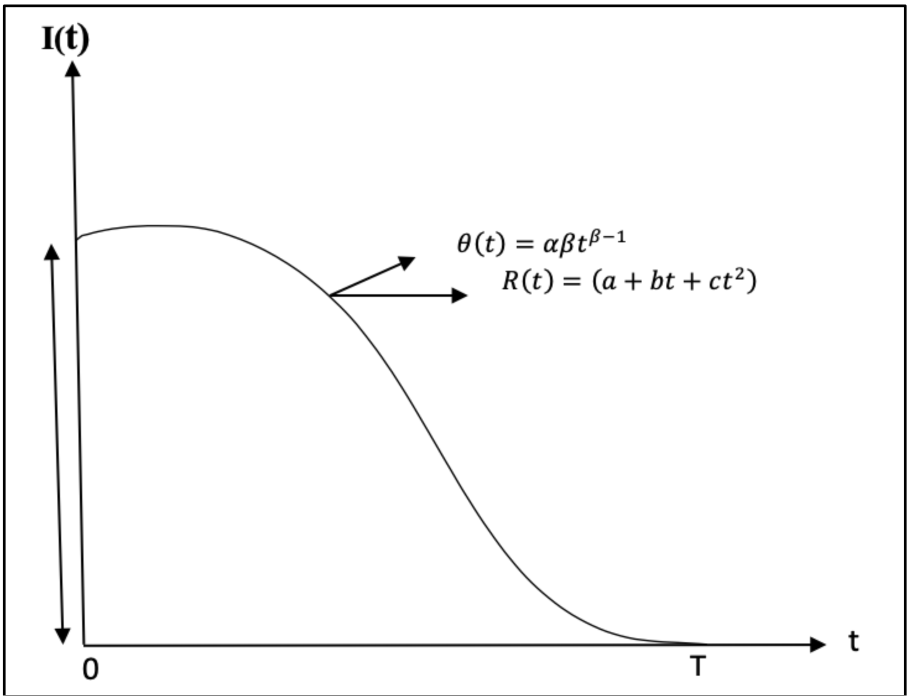

Fig. 1 Graphical Representation of the inventory level & time relationship

The differential equation governing the level of inventory over the inventory cycle due to demand rate as well as deterioration rate is given by

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = -(a + bt + ct^2); \qquad 0 \le t \le T \qquad (1)$$

Using the boundary condition I(0) = Q, the solution of equation (1) is

$$I(t) = Q(1 - \alpha t^{\beta}) - \left[at + \frac{bt^2}{2} + \frac{ct^3}{3} - \alpha\beta\left(\frac{at^{\beta+1}}{\beta+1} + \frac{bt^{\beta+2}}{2(\beta+2)} + \frac{ct^{\beta+3}}{3(\beta+3)}\right)\right]; \qquad 0 \le t \le T \qquad (2)$$

Substituting I(T)=0 in equation (2), we get order quantity

$$Q = \frac{aT + b\frac{T^2}{2} + c\frac{T^3}{3} - \alpha\beta\left(a\frac{T^{\beta+1}}{\beta+1} + b\frac{T^{\beta+2}}{2(\beta+2)} + c\frac{T^{\beta+3}}{3(\beta+3)}\right)}{(1 - \alpha T^{\beta})} \qquad (3)$$

Based on the assumptions and description of the model, the total cost comprises of the following components:

1. Ordering Cost: Ordering cost OC = A (4)

2. Purchasing Cost: Purchase cost per cycle PC is given by:

$PC = C_p Q$

$$PC = \frac{C_p}{1 - \alpha T^{\beta}}\left[aT + \frac{bT^2}{2} + \frac{cT^3}{3} - \alpha\beta\left(\frac{aT^{\beta+1}}{\beta+1} + \frac{bT^{\beta+2}}{2(\beta+2)} + \frac{cT^{\beta+3}}{3(\beta+3)}\right)\right] \qquad (5)$$

3. Deterioration Cost: Deterioration cost per cycle DC is given by:

$$DC = C_d \int_0^T \alpha\beta t^{\beta-1} I(t)dt$$

$$DC = \alpha\beta C_d\left(\left(\frac{aT+\frac{bT^2}{2}+\frac{cT^3}{3}-\alpha\beta\left(\frac{aT^{\beta+1}}{\beta+1}+\frac{bT^{\beta+2}}{2(\beta+2)}+\frac{cT^{\beta+3}}{3(\beta+3)}\right)}{1-\alpha T^{\beta}}\right)\left(\frac{T^{\beta}}{\beta}\right)-\frac{aT^{\beta+1}}{\beta+1}-\frac{bT^{\beta+2}}{2(\beta+2)}-\frac{cT^{\beta+3}}{3(\beta+3)}\right) \qquad (6)$$

4. Holding Cost: Holding cost per cycle IHC is given by

$$IHC = \int_0^T (h+rt)I(t)dt$$

$$IHC = h\left(\begin{array}{c}\left(\frac{aT+\frac{bT^2}{2}+\frac{cT^3}{3}-\alpha\beta\left(\frac{aT^{\beta+1}}{\beta+1}+\frac{bT^{\beta+2}}{2(\beta+2)}+\frac{cT^{\beta+3}}{3(\beta+3)}\right)}{1-\alpha T^{\beta}}\right)\left(T-\alpha\left(\frac{T^{\beta+1}}{\beta+1}\right)\right)-\frac{aT^2}{2}-\frac{bT^3}{6}-\frac{cT^4}{12}\\ +\alpha\beta\left(a\left(\frac{T^{\beta+2}}{(\beta+1)(\beta+2)}\right)+b\left(\frac{T^{\beta+3}}{2(\beta+2)(\beta+3)}\right)+c\left(\frac{T^{\beta+4}}{3(\beta+3)(\beta+4)}\right)\right)\end{array}\right)$$
$$+r\left(\begin{array}{c}\left(\frac{aT+\frac{bT^2}{2}+\frac{cT^3}{3}-\alpha\beta\left(\frac{aT^{\beta+1}}{\beta+1}+\frac{bT^{\beta+2}}{2(\beta+2)}+\frac{cT^{\beta+3}}{3(\beta+3)}\right)}{1-\alpha T^{\beta}}\right)\left(\frac{T^2}{2}-\frac{\alpha T^{\beta+2}}{\beta+2}\right)-\frac{aT^3}{3}-\frac{bT^4}{8}-\frac{cT^5}{15}\\ +\alpha\beta\left(a\left(\frac{T^{\beta+3}}{(\beta+1)(\beta+3)}\right)+b\left(\frac{T^{\beta+4}}{2(\beta+2)(\beta+4)}\right)+c\left(\frac{T^{\beta+5}}{3(\beta+3)(\beta+5)}\right)\right)\end{array}\right) \qquad (7)$$

Total Cost

Thus, the total average cost TC per cycle is given by

$TC = \frac{1}{T}(OC + DC + PC + IHC)$

$$= \frac{1}{T}\left[\begin{array}{c}A+\alpha\beta C_d\left[\left(\left(\frac{aT+\frac{bT^2}{2}+\frac{cT^3}{3}-\alpha\beta\left(\frac{aT^{\beta+1}}{\beta+1}+\frac{bT^{\beta+2}}{2(\beta+2)}+\frac{cT^{\beta+3}}{3(\beta+3)}\right)}{1-\alpha T^{\beta}}\right)\left(\frac{T^{\beta}}{\beta}\right)-\frac{aT^{\beta+1}}{\beta+1}-\frac{bT^{\beta+2}}{2(\beta+2)}-\frac{cT^{\beta+3}}{3(\beta+3)}\right)\right]\\ +C_p\left[\left(\frac{aT+\frac{bT^2}{2}+\frac{cT^3}{3}-\alpha\beta\left(\frac{aT^{\beta+1}}{\beta+1}+\frac{bT^{\beta+2}}{2(\beta+2)}+\frac{cT^{\beta+3}}{3(\beta+3)}\right)}{1-\alpha T^{\beta}}\right)\right]\\ +h\left[\begin{array}{c}\left(\frac{aT+\frac{bT^2}{2}+\frac{cT^3}{3}-\alpha\beta\left(\frac{aT^{\beta+1}}{\beta+1}+\frac{bT^{\beta+2}}{2(\beta+2)}+\frac{cT^{\beta+3}}{3(\beta+3)}\right)}{1-\alpha T^{\beta}}\right)\left(T-\alpha\left(\frac{T^{\beta+1}}{\beta+1}\right)\right)-\frac{aT^2}{2}-\frac{bT^3}{6}-\frac{cT^4}{12}\\ -\alpha\beta\left(a\left(\frac{T^{\beta+2}}{(\beta+1)(\beta+2)}\right)+b\left(\frac{T^{\beta+3}}{2(\beta+2)(\beta+3)}\right)+c\left(\frac{T^{\beta+4}}{3(\beta+3)(\beta+4)}\right)\right)\end{array}\right]\\ +r\left[\begin{array}{c}\left(\frac{aT+\frac{bT^2}{2}+\frac{cT^3}{3}-\alpha\beta\left(\frac{aT^{\beta+1}}{\beta+1}+\frac{bT^{\beta+2}}{2(\beta+2)}+\frac{cT^{\beta+3}}{3(\beta+3)}\right)}{1-\alpha T^{\beta}}\right)\left(\frac{T^2}{2}-\frac{\alpha T^{\beta+2}}{\beta+2}\right)-\frac{aT^3}{3}-\frac{bT^4}{8}-\frac{cT^5}{15}\\ -\alpha\beta\left(a\left(\frac{T^{\beta+3}}{(\beta+1)(\beta+3)}\right)+b\left(\frac{T^{\beta+4}}{2(\beta+2)(\beta+4)}\right)+c\left(\frac{T^{\beta+5}}{3(\beta+3)(\beta+5)}\right)\right)\end{array}\right]\end{array}\right] \qquad (8)$$

The objective is to compute the optimum value of T such that the cost function TC is minimized. An appropriate software is used to find the optimum value of T which is the solution of the equation $\frac{\partial TC}{\partial T} = 0$ satisfying the condition $\frac{d^2TC}{dT^2} > 0$ (9)

V. Numerical example

To numerically illustrate the developed model, consider the input parameter values in appropriate units:
α = 0.0001, β = 2.1, h = 3, r = 0.9, a = 40, b = 1.2, c = 0.01, A = 180, C_d = 0.9 and C_p = 2.1

On solving equation (8)and satisfying the condition stated in equation (9), we obtain the values of optimum order quantity (Q) as 123.4850120 and optimal total cost (TC) as392.8168086.

VI. Sensitivity analysis

Partial sensitivity analysis is carried out to understand the effect of change in the values of the system parameters on the optimal solution of the model. In order to do this, we change the value of each parameter in the range of -20% to +20% keeping the values of other parameters unchanged and repeating the process for each parameter. The table below represents the result:

Table 1: Partial Sensitivity Analysis

INPUT DATA			OUTPUT DATA		
Parameter	**Change in parameter**	**Parameter Value**	**T**	**TC**	**Q**
C_d = 0.9	-20	0.72	2.951336146	392.8143931	123.4850120
	-10	0.81	2.951336146	392.8156010	123.4850120
	+10	0.99	2.951336146	392.8180162	123.4850120
	+20	1.08	2.951336146	392.8192238	123.4850120
C_p = 2.1	-20	1.68	2.951336146	375.2438510	123.4850120
	-10	1.89	2.951336146	384.0303296	123.4850120
	+10	2.31	2.951336146	401.6032876	123.4850120
	+20	2.52	2.951336146	410.3897662	123.4850120
A = 180	-20	144	2.951336146	380.6189436	123.4850120
	-10	162	2.951336146	386.7178761	123.4850120
	+10	198	2.951336146	398.9157411	123.4850120
	+20	216	2.951336146	405.0146736	123.4850120
h = 3	-20	2.4	2.951336146	355.2114643	123.4850120
	-10	2.7	2.951336146	374.0141364	123.4850120
	+10	3.3	2.951336146	411.6194808	123.4850120
	+20	3.6	2.951336146	430.4221526	123.4850120
r = 0.9	-20	0.72	2.951336146	381.6320288	123.4850120
	-10	0.81	2.951336146	387.2244185	123.4850120
	+10	0.99	2.951336146	398.4091984	123.4850120
	+20	1.08	2.951336146	404.0015881	123.4850120
α = 0.0001	-20	0.00008	2.951336146	392.7124759	123.4610645
	-10	0.00009	2.951336146	392.7646380	123.4730371
	+10	0.00011	2.951336146	392.8689877	123.4969892
	+20	0.00012	2.951336146 0	392.9211756	123.5089687
β = 2.1	-20	1.68	2.951336146	392.6114077	123.4412484
	-10	1.89	2.951336146	392.7022632	123.4606480
	+10	2.31	2.951336146	392.9612496	123.5156119
	+20	2.52	2.951336146	393.1433980	123.5540471

a = 40	-20	32	2.951336146	330.0480641	99.8513841
	-10	36	2.951336146	361.4324364	111.6681981
	+10	44	2.951336146	424.2011804	135.3018259
	+20	48	2.951336146	455.5855533	147.1186399
b = 1.2	-20	0.96	2.951336146	389.2843228	122.4387503
	-10	1.08	2.951336146	391.0505652	122.9618811
	+10	1.32	2.951336146	394.5830510	124.0081428
	+20	1.44	2.951336146	396.3492944	124.5312737
c = 0.01	-20	0.008	2.951336146	392.7523300	123.4678571
	-10	0.009	2.951336146	392.7845690	123.4764345
	+10	0.11	2.951336146	392.8490476	123.4935894
	+20	0.12	2.951336146	392.8812865	123.5021668

VII. Graphical representation of sensitivity analysis

Figures (2) and (3) represent the above sensitivity analysis graphically.

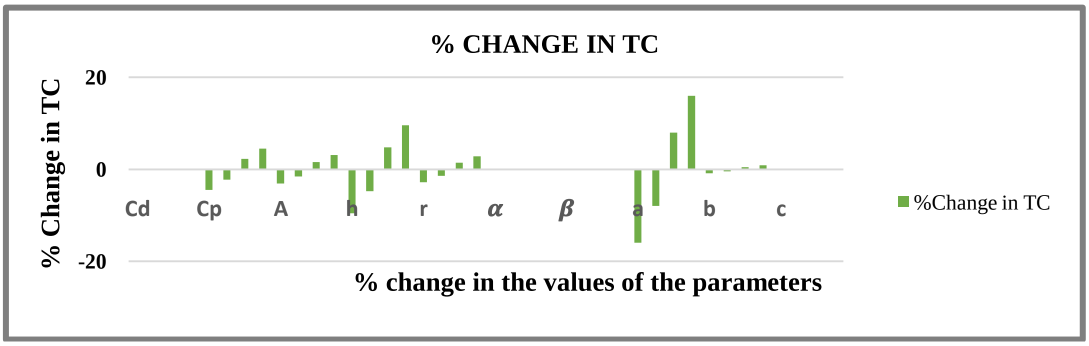

Fig. (2)

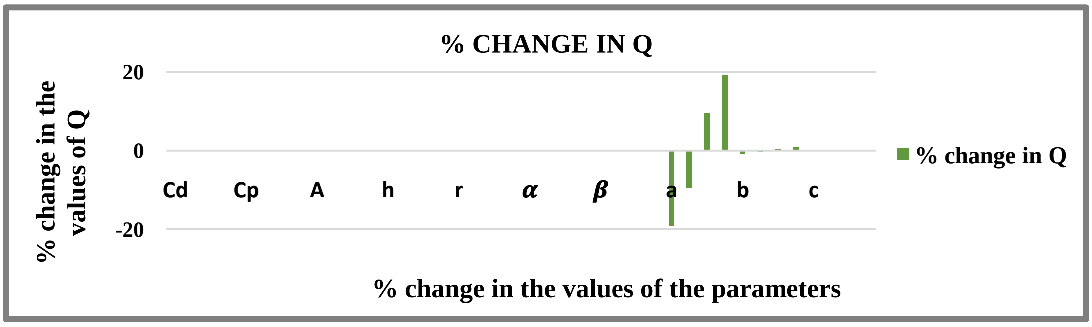

Fig. (3)

VIII. Observations & conclusion

From the sensitivity analysis table and graphical representation figures, we observe that:

1. Effect due to change in the demand constants (a) and (b): Any increase in the demand constants increases the average total cost (TC) as well as the optimal order quantity Q

2. Effect due to change in Holding cost parameter (h) and (r) : Any increase in the value of holding cost parameters leads to increase in the average total cost TC

3. Effect due to change in Purchasing cost parameter (C_p): A slight increase in the value of the purchase cost parameter leads to an increase in the average total cost TC

4. Effect due to change in the Ordering Cost (A): An increase in the value of the ordering cost lease to a slight increase in the average total cost TC

Based on the above observations, it can be concluded that:

The impact of the variation in the demand constants is most significant on the inventory model. Additionally, the holding cost parameters, purchasing cost and ordering cost also significantly affect the behaviour of the inventory system. This proposed model can be used in industries that deal with seasonal products like perishable goods considering the nature of demand rate and holding cost function. The study can be extended to analyse effects of profit function and salvage function.

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References

[1]. Bhunia, A. K., and Shaikh, A. A. (2016). Investigation of two-warehouse inventory problems in interval environment under inflation via particle swarm optimization. Mathematical and Computer Modelling of Dynamical Systems, 22(2), 160-179.
[2]. Dutta, D., and Kumar, P. (2015a). Application of fuzzy goal programming approach to multi- objective linear fractional inventory model. International Journal of Systems Science, 46(12), 2269- 2278.
[3]. Dutta, D., and Kumar, P. (2015b). A partial backlogging inventory model for deteriorating items with time-varying demand and holding cost: An interval number approach. Croatian Operational Research Review, 6(2), 321-334.
[4]. Gothi U. B. & Kirtan Parmar (2015), "Order level lot size inventory model for deteriorating items under quadratic demand with time dependent IHC and partial backlogging", Research Hub - International Multidisciplinary Research Journal IMRRHJ), Vol 2, Issue 2.
[5]. Hwang H, and Shinn S. W. (1997). Retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payments. Computers and Operations Research, Vol. 24, Issue 6. pp 539-547
[6]. Khanra, S., and Chaudhuri, K. S. (2003). A note on order level inventory model for a deteriorating item with time dependent quadratic demand. Computers and Operations Research, 30, 1901-1916.
[7]. Khanra, S., Ghosh, S. K., and Chaudhuri, K. S. (2011). An EOQ model for a deteriorating item with time dependent quadratic demand under permissible delay in payments. Applied Mathematics and Computation, 62(4), 1-9.
[8]. Khatri, Pooja D. and . U.B. Gothi. (2018). An EPQ Model under Constant Amelioration, Different Deteriorations with Exponential Demand Rate and Completely Backlogged Shortages. International Journal of Scientific Research in Mathematical and Statistical Sciences. Vol. 5 Issue 2, pp 21-28.
[9]. Kumar, P., and Dutta, D. (2015). Multi-objective linear fractional inventory model of multi- products with price-dependant demand rate in fuzzy environment. International Journal of Mathematics in Operations Research, 7(5), 547-565.
[10]. Mishra. U. (2018) Optimizing a three-rates-of-production inventory model under market selling price and advertising cost with deteriorating items. International Journal of Management Science and Engineering Management 13:4, pages 295-305.
[11]. Pando, V, San-Jos ́e, L. A., Garcıa-Laguna, J., and Sicilia, J. (2013). An economic lot-size model with non-linear holding cost hinging on time and quantity. International Journal of Production Economics, 145(1), 294-303.
[12]. R. Palani and M. Maragatham, (2016). EOQ model for controllable deterioration rate and time dependent demand and Inventory holding cost. International Journal of Mathematics Trends and Technology (IJMTT) – Vol. 39 No. 4, pp 245-251.
[13]. Sindhuja S. and Arathi P. (2022). An economic order quantity model for Pareto distribution deterioration with linear demand under linearly time-dependent shortages. European Journal of Industrial Engineering, Vol. 16, No. 4, pp 418-44.
[14]. Tripathi,R.P., Singh,D., and Aneja,S. (2018). Inventory models for stock-dependent demand and time varying holding cost under different trade credits. Yugoslav Journal of Operations Research, 28(1), 139- 151.
[15]. Tripathi, R. P., and Tomar, S. S. (2018). Establishment of EOQ Model with Quadratic Time- Sensitive Demand and Parabolic-Time Linked Holding Cost with Salvage Value. International Journal of Operations Research, 15(3), 135-144.
[16]. Tripathy C. K, and Mishra U. (2010). An Inventory Model for Weibull Deteriorating Items with Price Dependent Demand and Time-Varying Holding Cost. Applied Mathematical Sciences, Vol. 4, no. 44, 2171 – 2179.

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