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Facility Location Problems by Considering the Traveling Salesman Problem

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Abstract:*The selection of strategic facility locations is a very important aspect to minimize the distance and total allocation costs. This study aims to locate the optimal facility location to serve a number of customers with a minimum total distance. TSP is used to find facility routes to serve a number of customers. An illustrative example is presented to implement the proposed method. The illustrative results show that the proposed method can provide a good solution to this problem.*

Keywords*: facility location, location-allocation problem, traveling salesman problems.*

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I. Introduction

The selection of a strategic location for a public facility or warehouse is a very important aspect. This problem is related to how to serve and supply optimally to the destinations that have been set and are known to be located. In this problem it must be considered the number of facilities, location of facilities and capacities that will serve a particular set of purposes [1].

The facility location problem is to define the position of a set of facility points in a given location space based on the distribution of customer demand points to be allocated to facilities. A popular model in location studies is Weber's multi-source problem or location-allocation problem with the aim of finding m facilities and allocating n customers to these facilities to minimize total transportation costs [2]. Location problems were described by several researchers including[3], [4], [5] and [6].

Traveling Salesman Problem (TSP) is designed to find the best salesman route to visit all cities and return to the starting point that minimizes travel costs and visits each city exactly once [7]. TSP can be solved with the exact method. However, the application to everyday problems is usually computationally difficult both in terms of size and the computational time needed to get a solution. Therefore, the approximation method is used to solve the TSP problems. There are many approximation methods that can be used to solve the TSP problems, one of which is the artificial bee colony (ABC) algorithm. The problem of traveling salesmen has been described by several researchers including [8], [9] and [10].

In this article, we discuss the combination of location problems and traveling salesman problems. The location problem is presented by considering the problem of near optimal placement of facilities that serve a number of customers who use the facility. To determine the optimal location, the Weiszfelditerative method is be used. The TSP associates a set of existing customers using an artificial bee colony algorithm.

II. Material And Methods

Location-allocation problem

The location-allocation problem is to find a new set of facilities so that the total cost of transportation from the facility to the customer is minimized and the optimal number of facilities must be located in the area of interest to meet customer demand. The objective function and constraints of the location-allocation problem model are as follows [11]:

$$\min \sum_{i=1}^{m}\sum_{j=1}^{n} w_{ij}\, d\left(X_i, a_j\right) \tag{1}$$

subject to

$$\sum_{i=1}^{m} w_{ij} = r_j, \qquad j = 1, 2, \dots, n \tag{2}$$

$$w_{ij} \geq 0 \tag{3}$$

where $d(X_i, a_j)$ is the Euclidean distance from facility i to customer j, w_{ij} is the flow from facility i to customer j, X_i is the coordinates of the location of facility i, a_j is the coordinates of customer location j and r_j is customer demand and $i = 1,2, \dots, m$ and , $j = 1,2, \dots, n$. Based on the above model, it is known that the goal is to obtain a minimum distance between facilities and customers. Constraints (2) ensure all customers are served. Constraint (3) guarantees that the feasible variable is non-negative. This model assumes that the distance between customers and facilities is Euclidean.

The optimal location-allocation problem is usually very resistant to exact solutions, so the solution is obtained by using a heuristic approximation method. One way is to define subsets of customers and facilities and solve for each subset using the Weiszfeld method by considering each subset as a location-allocation problem for a single facility [12]. The problem of locating a single facility is to find a facility that will serve a set of customers optimally, which means minimizing the distance traveled by customers [13]. Based on this, this study examine the determination of the location of facilities using the Weiszfeld iterative method.

The Weiszfeld's Iterative Method

The Weiszfeld's iterative method is known as follows[14]:

$$X^{(k+1)} = \frac{\sum_{i=1}^{n} \frac{w_i a_i}{d_i(x^{(k)}, y^{(k)})}}{\sum_{i=1}^{n} \frac{w_i}{d_i(x^{(k)}, y^{(k)})}} \quad Y^{(k+1)} = \frac{\sum_{i=1}^{n} \frac{w_i b_i}{d_i(x^{(k)}, y^{(k)})}}{\sum_{i=1}^{n} \frac{w_i}{d_i(x^{(k)}, y^{(k)})}} \tag{4}$$

Initial values $(X^0, Y^0)$are required to obtain (X^1, Y^1). The values $(X^1, Y^1)$are then used to obtain (X^2, Y^2) etc. The points obtained will converge to the optimal point in certain subsets. All customer fixed points will be allocated to the nearest facility when the optimal point is obtained. Settlement using this method aims to obtain the optimum location of the facility and allocate customers to the facility. In this research, a coordinate system is constructed over a feasible set using Euclidean distance calculations. The customer point is defined as a fixed point and the facility point is determined by random$rand(0, x_{max})$ and $rand(0, y_{max})$ where x_{max} and y_{max} are obtained from the customer's coordinates.

Traveling Salesman Problem

Traveling Salesman Problem (TSP) is a problem in combinatorial optimization where a salesman has to visit a number of cities starting from and returning to his hometown by only visiting each city once. Given that n is the number of cities to be visited, and the total number of possible routes that cover all cities is called the TSP solution set with $(n - 1)!/2$[15]. Suppose d_{ij} is the distance between city i and city j andx_{ij} is a decision variable that shows the salesman's journey from city i to city j, then the TSP model is defined as follows:

$$\min \sum d_{ij} x_{ij} \tag{5}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = 1 , \qquad j = 1,2,\ldots,n \tag{6}$$

$$\sum_{i=1}^{n} x_{ij} = 1 , \qquad i = 1,2,\ldots,n \tag{7}$$

$$\sum_{i,j \in S} x_{ij} \le |S| - 1 \qquad 2 \le |S| \le n-2,\ S \subset \{1,2,\ldots,n\} \tag{8}$$

$$x_{ij} \in (0,1), \qquad i, j = 1,2,\ldots,n \qquad i \ne j \tag{9}$$

The objective function (5) means the minimum total distance; constraint (6) means that a salesperson can only depart from city i once; constraint (7) means that a salesperson can only visit city j once. Constraints (6) and (7) guarantee that sales visit each city once. Constraint (8) requires that there are no repetitions in any of the city subsets formed by sales.

***Artificial Bee Colony* Algorithm**

Artificial bee colony algorithm is a population-based metaheuristic approach inspired by the intelligent behavior of honey bees looking for food. The main steps in the artificial bee colony algorithm are as follows [16]:

1. Send scout bees to the initial food source.
2. Send employed bees to the food source and determine the fitness value with the following equation:

$$\text{fitness}_i = \begin{cases} \dfrac{1}{1+f(x_i)} & \text{if} \quad f(x_i) \ge 0, \\ 1+|f(x_i)| & \text{if} \quad f(x_i) < 0. \end{cases} \tag{10}$$

3. Calculate the probability value of the source favored by onlooker bees.
4. Send onlooker bees to the food source and determine the amount of nectar.
5. Stop the process of exploiting food sources that are depleted by bees.
6. Send scout bees to new search areas to find new food sources randomly.
7. Memorize the best food sources that have been found.
8. Repeat steps 2-6 until the requirements are met.

III. Result and Discussion

We illustrate the method with an example to solve the facility location problem by taking into account the traveling salesman problem. Given seven customer locations namely $C_1(8,18)$, $C_2(8,26)$, $C_3(11,20)$, $C_4(17,15)$, $C_5(17,22)$, $C_6(24,17)$, and $C_7(31,19)$ which will be served by 2 facilities. The initial location of the facility are randomly selected, namely facility 1 at $F_1(22,15)$ and facility 2 at $F_2(12,23)$. This illustration is calculated using Notebook Core™ i3-6006U, CPU 20GHz and OS Windows 10.

The first step is to calculate the distance of each facility to all customers by using Euclidean distance calculations and gathering customers with their closest facilities. Based on the distance that has been obtained, it is obtained that facility 1 serves customer 4, customer 6, and customer 7, and facility 2 serves customer 1, customer 2, customer 3, and customer 5. Figure 1 shows the location of the facilities and the allocation of customers which are divided into two subsets.

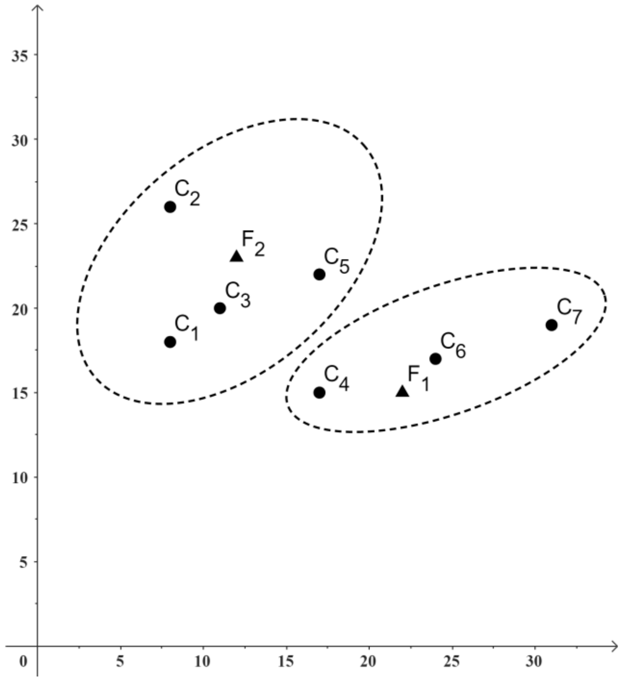

Figure 1: Facility Section with Customers

The second step is to determine the local optimum location of each facility. Determining the optimum locations are carried out using the Weiszfeld's iterative method as Equation (4), so the results can be seen in Table 1.

Table 1: Computational Results with Weiszfeld's Iterative Method

Iteration	Facility 1		Facility 2	
	X	*Y*	*X*	*Y*
1	22.00000	15.00000	12.00000	23.00000
2	22.94784	16.69938	11.45317	21.09054
3	23.75251	16.92929	11.24219	20.38715
4	23.98364	16.99532	11.10176	20.13104
5	23.99992	16.99998	11.03647	20.04403
6	-	-	11.01237	20.01470
7	-	-	11.00413	20.00489
8	-	-	11.00137	20.00162
9	-	-	11.00045	20.00054

Based on these calculations, the local optimum location for facility 1 is (23.99992, 16.99998) and the local optimum location for facility 2 is (11.00045, 20.00054). Figure 2 shows the optimum location of the facilities in each subset.

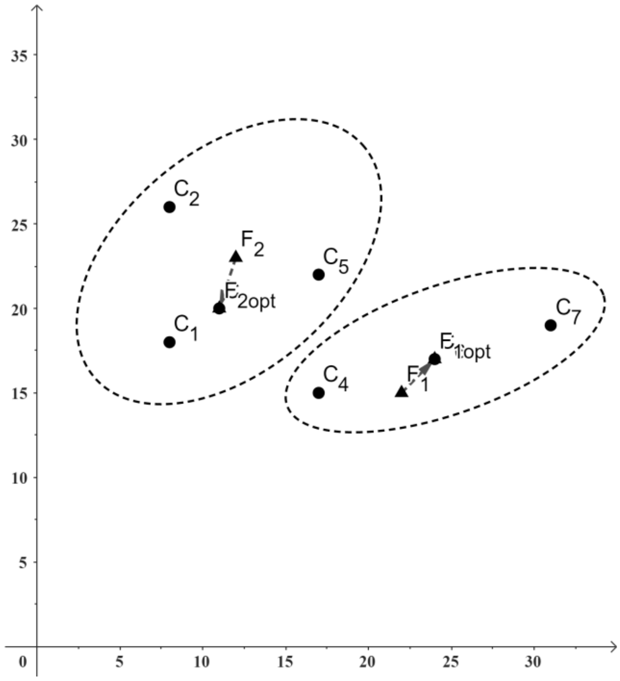

Figure 2: Local Optimum Locations of Facilities

The local optimum location for the facility has been obtained, so the next solution is the traveling salesman problem for each set of facilities with customers using the artificial bee colony algorithm. The initialization of the parameters used can be seen in Table 2.

Table 2: Parameters of Artificial Bee Colony

Parameter	Facility 1	Facility 2
Colony size	2	3
Maksimum iterasi	1	1
Size problem	4	5
Limit	8	10
Nse	8	10

In the next step, the initial solution is obtained randomly with random permutations of the predetermined colony size. The initial solution can be seen in Table 3.

Table 3: Initial Solutions

Facility 1						Facility 2						
Solution	Route				Total Distance	Solution	Route					Total Distance
X_1	F_1	C_1	C_2	C_3	29.12044	X_1	F_2	C_2	C_3	C_1	C_4	45.18039
X_2	F_1	C_3	C_2	C_1	29.12044	X_2	F_2	C_4	C_1	C_2	C_3	44.03286
						X_3	F_2	C_2	C_4	C_3	C_1	43.76259

After the initial solution for each facility is obtained, The overall fitness value of the solution in the population are evaluated. The fitness value is calculated by Equation (10) whose results are presented in Table 4.

Table 4: Initial Fitness Value

Facility 1						Facility 2						
Solution	Route				*Fitness*	Solution	Route					*Fitness*
X_1	F_1	C_1	C_2	C_3	0.03320	X_1	F_2	C_2	C_3	C_1	C_4	0.02165
X_2	F_1	C_3	C_2	C_1	0.03320	X_2	F_2	C_4	C_1	C_2	C_3	0.02221
						X_3	F_2	C_2	C_4	C_3	C_1	0.02234

The next step is the worker bee phase. In this phase, improvements will be made to worker bees with neighborhood operators, namely swap operator (SO) and swap sequence (SS). Improvement with SO is carried out for each solution in the population with the SO index determined randomly. The selected solution as a result of improvement with SO is presented in Table 5.

Table 5: Selected Solution fromSO Improvement

Facility 1							Facility 2							
Solution	Route				*Fitness*	Trials	Solution	Route					*Fitness*	Trials
X_1	F_1	C_1	C_2	C_3	0.03320	1	X_1	F_2	C_2	C_3	C_1	C_4	0.02165	1
X_2	F_1	C_3	C_2	C_1	0.03320	1	X_2	F_2	C_3	C_1	C_2	C_4	0.02292	0
							X_3	F_2	C_2	C_4	C_3	C_1	0.02234	1

Furthermore, improvement with SS is carried out for each SO solution with a SO index determined randomly as much as Nse for each solution. The selected solution as a result of improvement with SS can be seen in Table 6.

Table 6: Selected Solutions from SS Improvement

Facilty 1							Facility 2							
Solution	Route				*Fitness*	Trials	Solution	Route					*Fitness*	Trials
X_1	F_1	C_1	C_2	C_3	0.03320	9	X_1	F_2	C_3	C_1	C_4	C_2	0.02325	7
X_2	F_1	C_3	C_2	C_1	0.03320	9	X_2	F_2	C_3	C_1	C_2	C_4	0.02292	10
							X_3	F_2	C_2	C_4	C_1	C_3	0.02325	7

The solution in the worker bee phase which is selected to proceed to the observer bee phase is selected using a roulette wheel selection. In this roulette wheel selection, the solution is chosen by looking at the cumulative probability and *rand*[0,1]. The selected solution that proceeds to the observer bee phase from the roulette wheel selection results can be seen in Table 7.

Table 7: Selected Solutions from Roulette Wheel Selection

Facility 1							Facility 2							
Solution	Route				*Fitness*	Trials	Solution	Route					*Fitness*	Trials
X_1	F_1	C_1	C_2	C_3	0.03320	9	X_1	F_2	C_3	C_1	C_4	C_2	0.02325	7
X_2	F_1	C_1	C_2	C_3	0.03320	9	X_2	F_2	C_3	C_1	C_2	C_4	0.02292	10
							X_3	F_2	C_3	C_1	C_2	C_4	0.02292	10

In the observer bee phase, improvements are made to the observer bees with neighborhood operators, namely the insert operator (IO) and the insert sequence (IS). Improvement with IO is carried out for each solution in the population with the IO index determined randomly. The selected solution as a result of improvement with IO can be seen in Table 8.

Table 8: Selected Solutions from IO Improvement

Facility 1							Facility 2							
Solution	Route				*Fitness*	Trials	Solution	Route					*Fitness*	Trials
X_1	F_1	C_1	C_2	C_3	0.03320	10	X_1	F_2	C_3	C_1	C_4	C_2	0.02325	8
X_2	F_1	C_1	C_2	C_3	0.03320	10	X_2	F_2	C_3	C_1	C_2	C_4	0.02292	11
							X_3	F_2	C_3	C_1	C_2	C_4	0.02292	11

Furthermore, improvement with IS is carried out for each solution resulting from IO with an IO index determined randomly as much as Nse for each solution. The selected solution as a result of improvement with IS can be seen in Table 9.

Table 9: Selected Solutions from IS Improvement

Facility 1							Facility 2							
Solution	Route				*Fitness*	Trials	Solution	Route					*Fitness*	Trials
X_1	F_1	C_1	C_2	C_3	0.03320	18	X_1	F_2	C_3	C_1	C_4	C_2	0.02325	18
X_2	F_1	C_1	C_2	C_3	0.03320	18	X_2	F_2	C_2	C_4	C_1	C_3	0.02325	5
							X_3	F_2	C_3	C_1	C_2	C_4	0.02292	21

The next step is the scout bee phase. In this phase, the maximum trial value (M) is checked against the limit used to form a new solution for the worker bee phase in the next iteration. If$M > limit$and there is a solution that does not improve, then the solution is deleted and replaced with a new random solution, and the trial value is reset to 0.If $M > limit$and there is a solution that has improved, then the solution does not need to be replaced with a new solution and the trial value reset to 0. However, if$M < limit$then the entire solution does not need to be replaced and the trial value does not need to be reset. Therefore, the solutions obtained for the next iteration are presented in Table 10.

Table 10: New Solutions for Next Iteration

Facility 1						Facility 2						
Solution	Route				Trials	Solution	Route					Trials
X_1	F_1	C_3	C_1	C_2	0	X_1	F_2	C_3	C_1	C_4	C_2	0
X_2	F_1	C_1	C_2	C_3	0	X_2	F_2	C_2	C_4	C_1	C_3	0
						X_3	F_2	C_1	C_1	C_2	C_4	0

Because the maximum iteration that has been set is equal to 1, the iteration is stopped. For the best solution, it can be seen from the fitness value of the observer bee solution. The solution with the highest fitness value is selected as the best solution. Therefore, the best solution for determining the location problem by considering the traveling salesman problem is to build facility 1 at (23.99992,16.99998) with the route F_1-C_1-C_2-C_3-F_1 with a total distance of 29.12044 and build facility 2 at (11.00045,20.00054) with the route F_2-C_2-C_4-C_1-C_3-F_2 with a total distance of 41.99779.

IV. Conclusion

In this article, the Weiszfeld'siterative method and the artificial bee colony algorithm are used to solve the facility location problem by considering the TSP. The experimental results show that both methods provide a good solution for the given illustration of this problem. In subsequent developments, it is possible to modify the model for determining the location of facilities and the TSP method.

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