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A proposed method for solving assignment problems without anyiterations and comparative study with the existence methods

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Abstract
In this paper I have proposed a new algorithm to solve minimization and maximization assignment problem which gives the optimal solution directly without any iteration contrary to previous methods such as Hungarian methods. This method can be applied for both balanced and unbalanced assignment problems without using dummy cells as in Hungarian method. By this new approach we acheive the goal with less number of computational steps. We also provide some examples to illustrate the proposed method.
Keywords: *Assignment problems, Balanced and unbalanced assignment problems, Dummy cell, Hungarian method, Optimal solution.*

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I. Introduction

Assignment problem is a special type of linear programming program in which the objective is to find the optimal allocation (solution) of a number of tasks (jobs) to an equal number of facilities (persons). Here we make the assumptions that each person can perform each job but with varying degree of efficiency. Assignment problem finds many applications in allocation for example in assignment to different roots plans or pilots to different commercial flights, telecommunication, economic ... etc.

n persons can be assigned to n jobs in $n!$ possible ways. One method may be to find all possible $n!$ assignments and evaluate total costs in all cases. Then the assignment with minimum cost (as required) will give the optimal assignment. But this method is extremely laborious.

In (1995), Kuhn developed the Hungarian method of the assignment problem, the reason for naming it, is because its basis lies by the effort of the Hungarian mathematician Egervaryin the year 1931 [2].

Mathematical model known as assignment problem tries to provide the optimal costs in assignment system. Some well known and long use algorithms to solve assignment problems are Hungarian method. Afterwards many researchers provide many algorithms to solve assignment problems. Some of the methods and algorithms that the current research has gone through are: 'Revised ones assignment method for solving assignment problem' [4]; 'Solving the assignment problem directly without any iteration' [6]; 'A approach to solve unbalanced transportation problem' [7] ; H. Basirzadeh, 'Ones assignment method for solving assignment problems' [1] ; M.D.H. Gamal ' A Note On Ones Assignment Method' [3].

The above mentioned algorithms are beneficial to find the optimal solution to solve assignment problems besides, the current research also presents a useful algorithm which gives a better optimal solution in this topic. In this paper we achieved exact optimal solution which is same as that of Hungarian method.

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2 Mathematical Formulation of assignment problems

The assignment problem can be stated in the form of n×n, matrix $[C_{ij}]$ called the cost matrix (effectiveness matrix), where C_{ij} is the cost of assigning i^{th} facility(person) to the j^{th} job.

		Jobs				
		1	2	3	...	n
Persons	1	C_{11}	C_{12}	C_{13}	...	C_{1n}
	2	C_{21}	C_{22}	C_{23}	...	C_{2n}
	.	.	.	.	.	.
	n	C_{n1}	C_{n2}	C_{n3}	...	C_{nn}

Table 1 Effectiveness matrix

3 Some assumption of assignment problem

Minimize the total cost $Z = \sum_{i=1}^{n} \sum_{j=1}^{m} C_{ij} X_{ij}$

$$\text{Where } X_{ij} = \begin{cases} 1, & \text{if } i^{th} \text{ person is assigned to the } j^{th} \text{ job} \\ 0, & \text{if } i^{th} \text{ person is not assigned to the } j^{th} \text{ job} \end{cases}$$

X_{ij} = Subject to the conditions

(i) $\sum_{i=1}^{n} X_{ij} = 1, j = 1, 2, 3, \ldots, n$

which means that only one job is done by the i^{th} person, $i = 1, 2, 3, \ldots, n$

(ii) $\sum_{j=1}^{n} X_{ij} = 1, i = 1, 2, 3, \ldots, n$

which means that only one person should be assigned to the j^{th} job, $j = 1, 2, 3, \ldots, n$

Reduction theorem

If $X_{ij} = X_{ij}$, minimizes $Z = \sum_{i=1}^{n} \sum_{j=1}^{m} C_{ij} X_{ij}$ over all X_{ij} such that $\sum_{i=1}^{n} X_{ij} = 1 = \sum_{j=1}^{n} X_{ij}$ and $X_{ij} \geq 0$ then $X_{ij} = X_{ij}$ also minimizes $Z' = \sum_{i=1}^{n} \sum_{j=1}^{m} C'_{ij} X_{ij}$ where $C'_{ij} = C_{ij} \pm a_i \pm b_j$, a_i and b_j are constants, $i, j = 1, 2, 3, \ldots, n$

proof For proof see [5].

Theorem If all $C_{ij} \geq 0$ and there exists a solution $X_{ij} = X_{ij}$ such that $\sum_{i=1}^{n} \sum_{j=1}^{m} C_{ij} X_{ij} = 0$ then this solution is an optimal solution.

proof For proof see [5].

4 Algorithm of the problem

The new algorithm is as follows:
Step-I If the problem is minimization, then select the greatest element(cost) from each row and subtract it from each element of that row.
Step-II If the problem is maximization then select the smallest element (cost) from each row and subtract each element of that row from smallest element.
Step-III Adding (-1) in each element(cost) of the matrix.
Step-IV After adding (-1) all the elements of cost matrix are negative.
we begin the assignment as follows:
(A) If the problem is to be minimized or maximised we begin to assign from the most negative value of the matrix and after assigning most negative value delete the corresponding row and column.
(B) During row and column assignment if some most negative value may tie, then in this case select those values which has minimum cost (Maximum cost) in the original cost matrix if problem is minimization (Maximization).
(C) If minimum cost (Maximum cost) also tie in the original cost matrix then write all possible optimal assignment(Maximization).
Step-V Repeat the Step-IV to obtain each person associated with only one task.
Step-VI Write optimal assignment and total cost which is called row cost (R.C).
Step-VII Similarly, for column, we write the optimal assignment and total cost which is called column cost (C.C).
Step-VIII
(A) If problem is minimization, then the total cost = $Min\{R \cdot C, C \cdot C\}$.
(B) If problem is maximization, then the total cost = $Max\{R \cdot C, C \cdot C\}$.

5 Some numerical examples to compare the proposed method with existing methods.

Example 1. Consider the minimal assignment problem

jobs

persons		1	2	3	4
	A	12	30	21	15
	B	18	33	9	31
	C	44	25	24	21
	D	23	30	28	14

Determine the optimal assignment and minimum assignment cost.
Solution : For rows
Step-I Since problem is minimization then select the greatest element from each row and subtract it from each element of that row, the reduced is as follows:

	1	2	3	4
A	-18	0	-9	-15
B	-15	0	-24	-2
C	0	-19	-20	-23
D	-7	0	-2	-16

Step-II Now add (−1) to all element.

	1	2	3	4
A	-19	-1	-10	-16
B	-16	-1	-25	-3
C	-1	-20	-21	-24
D	-8	-1	-3	-17

Step-III (i) Now make assignment first we assign most negative value(cell) [C_{23}] of the matrix and after assigning the most negative value (cell) delete the corresponding row and column.

	1	2	3	4
A	-19	-1	-10	-16
B	-16	-1	-25	-3
C	-1	-20	-21	-24
D	-8	-1	-3	-17

(ii) Next the most negative value (cell)[C_{34}] of the matrix assign the C person to 4th job Similarly assign A person to 1 job and the last one is to assign D person to 2 job.

	1	2	4
A	-19	-1	-16
C	-1	-20	-24
D	-8	-1	-17

Thus, the optimal assignment is

Persons	Jobs
A	1
B	3
C	4
D	2

Hence optimal solution cost is = 12 + 9 + 21 +30 = 72 (which is called row cost ($R \cdot C$)).
Similarly for columns
Step-I Since problem is minimization then select the greatest element from each column and subtract it from each element of that column the reduce cost matrix is

	1	2	3	4
A	-32	-3	-7	-16
B	-26	0	-19	0
C	0	-8	-4	-10
D	-21	-3	0	-17

Step-II Now add (-1) to all element.

	1	2	3	4
A	-33	-4	-8	-17
B	-27	-1	-20	-1
C	-1	-9	-5	-11
D	-22	-4	-1	-18

Step-III Now make assignment first we assign most negative cell [C_{11}] of the matrix and after assigning most negative cell delete the corresponding row and column.

	1	2	3	4
A	-33	-4	-8	-17
B	-27	-1	-20	-1
C	-1	-9	-5	-11
D	-22	-4	-1	-18

Assign A person to 1 Job.

Reduced Matrix

	2	3	4
B	-1	-20	-1
C	-9	-5	-11
D	-4	-1	-18

Assign B person to 3 Job.

Reduced Matrix

	2	4
C	-9	-11
D	-4	-18

Assign D person to 4 Job.

Reduced Matrix

	2
C	-9

Assign C person to 2 Job.

Therefore optimal assignment is

persons	jobs
A	1
B	3
C	2
D	4

optimal value is = 12 + 9 + 25 + 14

= 60 (which is called column cost ($C\,C$)) .

Hence final minimized optimal solution cost is

$$= Min\{R \cdot C, C \cdot C\}$$

$$= Min\{72, 60\}$$

$$= 60$$

Minimized cost is 60 RS .

Example 2. Minimization unbalanced assignment problem .

Men

Tasks		1	2	3
	A	9	26	15
	B	13	27	6
	C	35	20	15
	D	18	30	20

Solution : For rows

Step-I since problem is minimization then select the greatest element from each row and subtract it from each element of that row the reduced matrix is

	1	2	3
A	-17	0	-11
B	-14	0	-21
C	0	-15	-20
D	-12	0	-10

Step-II Now add (-1) to all element.

	1	2	3
A	-18	-1	-12
B	-15	-1	-22
C	-1	-16	-21
D	-13	-1	-11

Step-III

(i) Now make assignment first we assign most negative value (cell)[c_{23}] of the matrix and after assigning most negative value (cell) and delete the corresponding row and column.

	1	2	3
A	-18	-1	-12
B	-15	-1	-22
C	-1	-16	-21
D	-13	-1	-11

Assign task B to Men 3

(ii) Next most negative value (cell)[c_{11}] of the matrix assign the task A to men 1

	1	2
A	-18	-1
C	-1	-16
D	-13	-1

Assign task 1 to Men 1

similarly assign task C to men 2

	2
C	-16
D	-1

Therefore optimal assignment is

Tasks	Men
A	1
B	3
C	2

Hence optimal value is = 9 + 6 + 20 = 35 (which is called Row cost).

Similarly for columns

Step-I since problem is minimization then select the greatest element from each column and subtract it from each element of that column the reduce cost matrix is

	1	2	3
A	-26	-4	-5
B	-22	-3	-14
C	0	-10	-5
D	-17	0	0

Step-II Now add (-1) to all element.

	1	2	3
A	-27	-5	-6
B	-23	-4	-15
C	-1	-11	-6
D	-18	-1	-1

Step-III Now make assignment first we assign most negative value [C_{11}] of the matrix and after assigning most negative value (cell) delete the corresponding row and column.

	1	2	3
A	-27	-5	-6
B	-23	-4	-15
C	-1	-11	-6
D	-18	-1	-1

Assign task A to Men 1

Reduced cost matrix is

	2	3
B	-4	-15
C	-11	-6
D	-1	-1

Assign task B to Men 3

Reduced cost matrix is

	2
C	-11
D	-1

Assign task C to Men 2

Therefore optimal assignment is

Tasks	Men
A	1
B	3
C	2

optimal value is = 9 + 6 + 20 = 35 (which is called column cost).

Hence final minimized optimal solution : hour is = $M\{n\ R\ C, C\ C\ \}$

$= Min\{35, 35\}$

$= 35$

Minimized Hour is 35.

Note: Above problem is solved without using dummy cell.

6 Solve the following assignment problems using Hungarian method as mention in [5].

7 Comparison of optimal value with Hungarian method

example	Hungarian method	proposed method	optimal value
01	60	60	60
02	35	35	35

8 Ones assignment method fails to get an optimal Solution : of the following assignment problem as mention in [3], but our proposed method solve the following problem.

Example 3. Consider the minimal assignment problem

jobs

Machines	1	2	3	4
A	4	5	2	5
B	3	1	1	4
C	13	1	7	4
D	12	6	5	9

Solution : Solve this problem by proposed method, Then after conducting the Steps introduced by proposed method we have Row cost is 14 and Column cost is 15

Hence minimized optimal cost is $= Min\{R\cdot C, C\cdot C\}$

$= Min\{14, 15\}$

= 14 (Which is same as obtained by Hungarian method)

Example 4. Consider the minimal assignment problem

jobs

persons	1	2	3	4	5
A	7	8	4	15	12
B	7	9	1	14	10
C	9	1	1	6	7
D	7	6	14	6	10
E	1	6	12	10	6

Solution : Then after conducting the Steps introduced by new proposed method, We have Row cost is 21 and Column cost is also 21

$= Min\{R\cdot C, C\cdot C\}$

$= Min\{21, 21\}$

= 21 (Which is same as obtained by Hungarian method)

.

9 An effective algorithm to solve assignment problems fails to get an optimal solution of this assignment problem as discussed in [8], but our proposed method solve this problem.

Example 5. Find the optimal assignment for the problem.

jobs

		1	2	3	4
Machines	A	5	3	1	8
	B	7	9	2	6
	C	6	4	5	7
	D	5	7	7	6

Solution Solve this problem by proposed method, then after conducting the Steps introduced by new proposed method, We have Row cost 16 and Column cost is also 16

$$= Min\{R \cdot C, C \cdot C\}$$
$$= Min\{16, 16\}$$
$= 16$ (Which is same as obtained by Hungarian method)

Conclusion

In this paper proposed method for solving assignment problem is faster than the known Hungarian method initially, we explained the proposed algorithm and showed the efficiency of it by numerical examples and we get optimal solution which is same as the optimal solution of Hungarian method. Therefore this paper introduces a different approach to solve assignment problem with less number of computational Step which reduces computational time.

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