Verification Of An Implicit Solution

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The relation $x^2 + y^2 = 25$ is an implicit solution of the nonlinear differential equation

$$\frac{dy}{dx} = -\frac{x}{y} \quad (1)$$

On the interval defined -5< x < 5 by implicit differentiation we obtain

$$2x + 2y\frac{dy}{dx} = 0$$

Solve the above said equation y in terms of x means we obtain, $y = \phi_1(x) = -\sqrt[2]{25 - x^2}$

$$y = \phi_2(x) = \sqrt[2]{25 - x^2}$$

Satisfy the relation (that is, $x^2 + \phi_1{}^2 = 25$ and $x^2 + \phi_2{}^2 = 25$ and are Explicit solutions defined on the interval (-5, 5). The solution curves given in **figure (b)** And (c) are segments of the graph of the implicit solution in Figure (a).

Any relation of the form $x^2 + y^2 - c = 0$ *formally* satisfies (1) for any constant *c*. However, It is understood that the relation should always make sense in the real number system; thus, for Example, we cannot say that $x^2 + y^2 + 25 = 0$ is an implicit solution of the equation.

Why not?

Because the distinction between an explicit solution and an implicit solution should be intuitively clear, we will not belabour the issue by always saying, "Here is an explicit (implicit) solution."

(a) Implicit solution
$x^2 + y^2 = 25$

(b) Explicit solution
$y_1 = \sqrt{25 - x^2},\ -5 < x < 5$

(c) Explicit solution
$y_2 = -\sqrt{25 - x^2},\ -5 < x < 5$