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On Fuzzy Baire Irresolute and Fuzzy Pseudo Irresolute Functions

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Abstract: *In this paper, the concepts of fuzzy Baire irresolute functions and fuzzy pseudo irresolute functions between fuzzy topological spaces are introduced and studied. A condition for a fuzzy topological space to become fuzzy semi-normal spaces is obtained by means of fuzzy Baire-separated spaces . Conditions for the inverse images of fuzzy pseudo -open sets to become fuzzy simply*-open sets, fuzzy resolvable sets are also obtained by means of fuzzy pseudo- irresolute functions.*

Keywords:*Fuzzy first category set, fuzzy residua set , fuzzy pseudo-open set, fuzzy Baire set, fuzzy Baire-separated space, fuzzy seminormal space, somewhat fuzzy nearly open function, fuzzy resolvable function.*

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I. INTRODUCTION
II.

In order to deal with uncertainties, the idea of fuzzy sets was introduced by **L.A. Zadeh** [30] in 1965. The potential of fuzzy notion was realized by the researchersand has successfully been applied in all branches of Mathematics. In 1968, **C. L. Chang** [**4**] applied basic concepts of general topology to fuzzy sets and introduced fuzzy topology. Based on this concept, many studies have been conducted in general theoretical areas and in different application sides.In the recent years, a considerable amount of research has been done on many types of fuzzy continuity in fuzzy topology.

The concept of Baire sets in classical topology was introduced and studied by **Andrzej Szymanski** [**1**].The notion of fuzzy Baire sets in fuzzy topological spaces was introduced and studied by **G.Thangaraj**and **R.Palani** [**14**] by means of fuzzy open sets and fuzzy residual sets. In 2020 ,the notion of fuzzy Baire continous functions between fuzzy topological spaces is introduced and studied by **G.Thangaraj** and **S.Senthil**[**20**]. In 2017, the notion of fuzzy pseudo-continuous functions between fuzzy topological spaces was introduced and studied by **G.Thangaraj** and **K.Dinakaran** [**17**]. The purpose of this paper is to introduce and study fuzzy Baire irresolute functions and fuzzy pseudo-irresolute functions between fuzzy topological spaces. The existence of such functions is ensured by establishing fuzzy continuous and somewhat fuzzy nearly open functions between fuzzy topological spaces. The conditions for the preservation of fuzzy sets being fuzzy first category under fuzzy continuous functions and for the preservation of fuzzy sets being fuzzy residual under fuzzy open functionsbetween fuzzy topological spaces are obtained. A condition for a fuzzy topological space to become a fuzzy semi-normal space is obtained by means of establishing fuzzy continuity and fuzzy openness between it and a fuzzy Baire- separated space. It is shown that fuzzy continuous and somewhat fuzzy nearly open function between topological spaces is a fuzzy Baire irresolute function. It is observed that the inverse image of a fuzzy Baire-separated space is a fuzzy Baire-separated space under fuzzy continuous and fuzzy open functions. The conditions for the inverse images of fuzzy pseudo -open sets to become fuzzy simply*open sets, fuzzy resolvable sets are obtained by means of fuzzy pseudo- irresolute functions.

III. PRELIMINARIES

Some basic notions and results used in the sequel are given in order to make the exposition self - contained. In this work by (X,T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I the unit interval [0,1]. A fuzzy set λ in X is a mapping from X into I. The fuzzy set 0_X is defined as $0_X(x) = 0$, for all $x \in X$ and the fuzzy set 1_X is defined as $1_X(x) = 1$, for all $x \in X$.

Definition 2.1[4] : A fuzzy topology is a family T of fuzzy sets in X which satisfies the following conditions :
(a). $0_X \in T$ and $1_X \in T$
(b). If $A, B \in T$, then $A \wedge B \in T$,
(c). If $A_i \in T$ for each $i \in J$, then $\vee_i A_i \in T$.

T is called a fuzzy topology for X, and the pair (X,T) is a fuzzy topological space, or fts for short. Members of T are called fuzzy open sets of X and their complements , fuzzy closed sets.

Definition 2.2 [4] : Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). The interior , the closure and the complement of λ are defined respectively as follows :
(i). $\text{int}(\lambda) = \vee \{ \mu / \mu \leq \lambda, \mu \in T \}$;
(ii). $\text{cl}(\lambda) = \wedge \{ \mu / \lambda \leq \mu, 1-\mu \in T \}$;
(iii). $\lambda'(x) = 1 - \lambda(x)$, for all $x \in X$.

For a family $\{\lambda_i / i \in I\}$ of fuzzy sets in (X,T), the union $\psi = \vee_i (\lambda_i)$ and intersection $\delta = \wedge_i (\lambda_i)$, are defined respectively as
(iv). $\psi(x) = \sup_i \{ \lambda_i(x) / x \in X\}$;
(v). $\delta(x) = \inf_i \{ \lambda_i(x) / x \in X\}$.

Lemma 2.1 [2] : For a fuzzy set λ of a fuzzy topological space X

(i). $1 - \text{int}(\lambda) = \text{cl}(1-\lambda)$ and (ii). $1 - \text{cl}(\lambda) = \text{int}(1-\lambda)$.

Definition 2.3 : A fuzzy set λ in the fuzzy topological space (X,T) is called a
(i). fuzzy regular-open set if $\lambda = \text{int cl}(\lambda)$; fuzzy regular- closed set if $\lambda = \text{cl int}(\lambda)$[**2**].
(ii).fuzzy G_δ -set in (X,T) if $\lambda = \wedge_{i=1}^{\infty} (\lambda_i)$, where $\lambda_i \in T$ for $i \in I$ and fuzzy F_σ -set in (X,T) if $\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, where $1 - \lambda_i \in T$ for $i \in I$ [**3**].
(iii).fuzzy dense set in (X,T)if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$. That is, $\text{cl}(\lambda) = 1$, in (X,T) [**9**] .
(iv).fuzzy nowhere dense set in (X,T)if there exists no non-zero fuzzy openset μ in (X,T) such that $\mu < \text{cl}(\lambda)$ That is, $\text{int cl}(\lambda) = 0$, in (X,T) [**9**].
(v). fuzzy first category set in (X,T) if $\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, where $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy second category[**9**] .
(vi). fuzzy residual set in (X,T) if $1 - \lambda$ is a fuzzy first category set in (X,T) [**11**].
(vii).fuzzy Baire set in (X,T) if $\lambda = \mu \wedge \delta$, where μ is a fuzzy open set and δ is a fuzzy residual set in (X,T) [**14**].
(viii).fuzzy pseudo-open set in (X,T)if $\lambda = \mu \vee \gamma$, where μ is a fuzzy open setand γ is a fuzzy first category set in (X,T)[**17**].
(ix). fuzzy somewhere dense set in (X,T) if there exists a non-zero fuzzy open set μ in (X, T) such that $\mu \leq \text{cl}(\lambda)$. That is, $\text{int cl}(\lambda) \neq 0$, in (X, T)[**10**].
(x). fuzzy simply open set in (X,T) if Bd(λ) is a fuzzy nowhere dense set in (X,T), where $\text{Bd}(\lambda) = \text{cl}(\lambda) \wedge \text{cl}(1-\lambda)$ [**15**].
(xi). fuzzy resolvable set in (X,T) if for each fuzzy closed set μ in (X,T), $\{ \text{cl}(\mu \wedge \lambda) \wedge \text{cl}(\mu \wedge [1-\lambda]) \}$ is a fuzzy nowhere dense set in (X,T) [**18**].
(xii).fuzzy simply* open set in (X,T) if $\lambda = \mu \vee \gamma$, where μ is a fuzzy open set and γ is a fuzzy nowhere dense set in (X,T) [**16**].

Definition 2.4[22] Let (X,T) be a fuzzy topological space. Then, a fuzzy set λ in X is said to have the property of fuzzy Baire if $\lambda = (\mu \wedge \eta) \vee \delta$, where μ is a fuzzy open set, η is a fuzzy residual set andδ is a fuzzy first category set in X.

Definition2.5 : A fuzzy topological space (X,T) is called a
(i).fuzzy Baire -separated space if for each pair of fuzzy closed sets μ_1 and μ_2 in (X,T) such that $\mu_1 \leq 1-\mu_2$, there exists a fuzzy Baire set η in (X,T) such that $\mu_1 \leq \eta \leq 1-\mu_2$ [**25**].
(ii). fuzzy seminormal space if given a fuzzy closed set λ and a fuzzy open set μ such

that $\lambda \leq \mu$, then there exists a fuzzy regular open set σ such that $\lambda \leq \sigma \leq \mu$ [**7**].

(iii). fuzzy Baire space if $\mathrm{int}\,(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X,T) [**11**].

(iv).fuzzy second category space if $\mathrm{int}\,(\vee_{i=1}^{\infty}(\lambda_i)) \neq 0$, where $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X,T) [**11**].

(v). fuzzy hyperconnected space if every non-null fuzzy open subset of (X, T) is fuzzy dense in (X, T) [**6**].

(vi).fuzzy nodef space if each fuzzy nowhere dense set is a fuzzy F_σ–set in (X,T) [**26**].

(vii).fuzzy extraresolvable space if whenever λ_i and λ_j ($i \neq j$) are fuzzy dense sets in (X,T), then $\lambda_i \wedge \lambda_j$ is a fuzzy nowhere dense set in (X,T) [**29**].

(viii).fuzzy Brown space if for any two non-zero fuzzy open sets λ and μ in (X,T), $cl(\lambda) \nleq 1 - cl(\mu)$, in (X,T) [**28**].

(ix).fuzzy hereditarily irresolvable space if there is no non-zero fuzzy resolvable set in (X,T) [**21**].

(x) .fuzzy D-Baire space if every fuzzy first category set in (X,T) is a fuzzy nowhere dense set in (X,T)[**12**].

(xi). fuzzy P-space if every non- zero fuzzy G_δ-set in (X,T) is fuzzy open in (X,T) [**8**].

(xii). fuzzy submaximal space if for each fuzzy set λ in (X,T) such that $cl\,(\lambda) = 1$, $\lambda \in T$ [**3**].

Definition 2.6 : Let (X,T) and (Y,S) be any two fuzzy topological spaces. A function f : (X,T) → (Y,S) is called a

(i) fuzzy continuous function if for every non-zero fuzzy open set λ in (Y,S), $f^{-1}(\lambda)$ is a fuzzy open set in (X,T) [**4**].

(ii). somewhat fuzzy nearly open function if for all $\lambda \in T$ and $f(\lambda) \neq 0$, there exists a non-zero fuzzy open set μ of (Y, S) such that $\mu \leq cl\,[\,f(\lambda)]$. That is, $\mathrm{int}\,cl\,[\,f(\lambda)\,] \neq 0$, in (Y, S) [**13**].

(iii).fuzzy pseudo-continuous function if for each fuzzy open set λ in (Y,S), $f^{-1}(\lambda)$ is a fuzzy pseudo- open set in (X,T) [**17**].

(iv).fuzzy Baire continuous function, if for every non-zero fuzzy open set λ in (Y,S), $f^{-1}(\lambda)$ is a fuzzy Baire set in (X,T) [**20**].

(v).fuzzy simply continuous function if for every non-zero fuzzy open set λ in (Y,S), $f^{-1}(\lambda)$ is a fuzzy simply open set in (X,T) [**15**].

(vi).fuzzy simply* continuous function if for every non-zero fuzzy open set λ in (Y,S), $f^{-1}(\lambda)$ is a fuzzy simply* open set in (X,T) [**16**].

(vii).fuzzy resolvable function if for every non-zero fuzzy open set λ in (Y,S), $f^{-1}(\lambda)$ is a fuzzy resolvable set in (X,T) [**19**].

Theorem 2.1 [25] : If λ is a fuzzy closed set and δ is a fuzzy open set in a fuzzy Baire - separated space (X,T) such that $\lambda \leq \delta$, then there exists a fuzzy Baire set η in (X,T) such that $\lambda \leq \eta \leq \delta$.

Theorem 2.2 [23]: If there exists a fuzzy Baire set in a fuzzy hyperconnected and fuzzy second category (but not fuzzy Baire) space (X,T), then (X,T) is not a fuzzy hereditarily irresolvable space.

Theorem 2.3 [24] : If δ is a fuzzy open set in a fuzzy hyperconnected and fuzzy nodef space (X,T), then δ is a fuzzy Baire set in (X,T).

Theorem 2.4 [28] : If λ is a fuzzy dense set in a fuzzy Brown and fuzzy nodef space (X,T), then λ is a fuzzy residual set in (X,T).

Theorem 2.5 [24] : If λ is a fuzzy residual set in a fuzzy extraresolvable space (X,T), then $\mathrm{int}\,(\lambda) = 0$, in (X,T).

Theorem 2.6 [17] : If λ is a fuzzy pseudo-open set in a fuzzy hyperconnected space (X,T), then λ is a fuzzy simply open set in (X,T).

Theorem 2.7 [17] : If λ is a fuzzy pseudo-open set in a fuzzy hyperconnected space (X,T), then λ is a fuzzy resolvable set in (X,T).

Theorem 2.8 [23] : If λ is a fuzzy Baire set in a fuzzy strongly hyperconnected space (X,T), then λ is a fuzzy open set in (X,T).

Theorem 2.9 [23] : If λ is a fuzzy Baire set in a fuzzy submaximal [fuzzy globally disconnected] and fuzzy P -space (X,T) , then for a fuzzy first category set μ in (X,T), $\lambda \vee \mu$ is a fuzzy pseudo-open set in (X,T).

Theorem 2.10 [23] : If λ is a fuzzy Baire set in a fuzzy strongly hyperconnected space (X,T) , then for a fuzzy first category set μ in (X,T), $\lambda \vee \mu$ is a fuzzy pseudo-open set in (X,T).

Theorem 2.11 [23] : If λ is a fuzzy pseudo-open set in a fuzzy D-Baire space (X,T), then λ is a fuzzy simply* open set in (X,T).

Theorem 2.12 [23] : If λ is a fuzzy pseudo-open set in a fuzzy hyperconnected and fuzzy D-Baire space (X,T), then λ is a fuzzy simply open set in (X,T).

Theorem 2.13 [5] : In any fuzzy topological space (X, τ), the following conditions are equivalent:
(i) (X,τ) is fuzzy hyperconnected space.
(ii). Every fuzzy subset of X is either fuzzy dense or fuzzy nowhere denseset there in.

IV. Fuzzy Baire Sets, Fuzzy Pseudo-Open Sets And Fuzzy Continuous Functions

If $f : (X,T) \to (Y,S)$ is a fuzzy continuous function from a fuzzy topological space (X, T) into another fuzzy topological space (Y,S) and λ is a fuzzy first category set in (Y, S), then does $f^{-1}(\lambda)$ necessarily be a fuzzy first category set in (X,T) ? The following example shows that inverse images of fuzzy first category sets under fuzzy continuous functions between fuzzy topological spaces need not be fuzzy first category sets.

Example 3.1 : Let X = { a, b }. Consider the fuzzy sets λ, μ and α defined on X as follows:
$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.5$; $\lambda(b) = 0.7$;
$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.8$; $\mu(b) = 0.4$;
$\alpha : X \to [0, 1]$ is defined as $\alpha(a) = 0.2$; $\alpha(b) = 0.6$.
Then, $T = \{ 0, \lambda, \mu, \alpha, \lambda\vee\mu, \mu\vee\alpha, \lambda\wedge\mu, \mu\wedge\alpha, (\alpha\vee[\lambda\wedge\mu]), 1\}$ and $S = \{ 0, \lambda, \mu, \lambda\vee\mu, \lambda\wedge\mu, 1 \}$ are fuzzy topologies on X. On computation, one can see that the fuzzy nowhere dense sets in (X,S) are $1-\lambda$, $1-\mu$, $1-(\lambda\vee\mu)$ and $1-(\lambda\wedge\mu) = [1-\lambda]\vee[1-\mu]\vee[1-(\lambda\vee\mu)]$ implies that $1-(\lambda\wedge\mu)$ is a fuzzy first category set in (X,S). Also on computation , the fuzzy nowhere dense sets in (X,T) are $1-\lambda, 1-(\lambda\vee\mu)$ and $(1-\lambda)\vee(1-(\lambda\vee\mu)) = 1-\lambda$, implies that $1-\lambda$ is a fuzzy first category set in (X,T).Define a function $f: (X, T) \to (X, S)$ by $f(a) = a$; $f(b) = b$. Clearly f is a fuzzy continuous function from (X,T) into (X,S).Now $f^{-1}(1-(\lambda\wedge\mu)) = \alpha\vee[\lambda\wedge\mu] \neq 1-\lambda$ and $f^{-1}(1-(\lambda\wedge\mu))$ is not a fuzzy first category set in (X,T), for the fuzzy first category set $1-(\lambda\wedge\mu)$ in (X,S).

The following proposition gives a condition for inverse images of fuzzy first category sets under fuzzy continuous functions between fuzzy topological spaces to become fuzzy first category sets.

Proposition 3.1 : If $f : (X,T) \to (Y,S)$ is a fuzzy continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S) in which $f(\lambda)$ is a fuzzy somewhere dense set in (Y,S) for each non-zero fuzzy open set λ in (X,T) and μ is a fuzzy first category set in (Y,S), then $f^{-1}(\mu)$ is a fuzzy first category set in (X,T).

Proof : Le μ be a fuzzy first category set in (Y,S). Then, $\mu = \vee_{i=1}^{\infty}(\mu_i)$, where $(\mu_i)'s$ are fuzzy nowhere dense sets in (Y,S). Now $f^{-1}(\mu) = f^{-1}(\vee_{i=1}^{\infty}(\mu_i)) = \vee_{i=1}^{\infty}(f^{-1}(\mu_i))$. Since the function f is a fuzzy continuous function from (X,T) into (Y,S) and $cl(\mu_i)$ is a fuzzy closed set in (Y,S), $f^{-1}(cl(\mu_i))$ is a fuzzy closed set in (X,T). Let $\lambda_i = int[f^{-1}(cl(\mu_i))]$. Suppose that $\lambda_i \neq 0$, in (X,T).Then, λ_i is a non-zero fuzzy open set in (X,T). Now $\lambda_i = int[f^{-1}(cl(\mu_i))] \leq f^{-1}(cl(\mu_i))$, implies that $f(\lambda_i) \leq f f^{-1}(cl(\mu_i)) \leq cl(\mu_i)$ and $int\, cl[f(\lambda_i)] \leq int\, cl[cl(\mu_i)] = int\, cl(\mu_i)$. By hypothesis, $f(\lambda_i)$ is a fuzzy somewhere dense set in (Y,S) and thus $int\, cl(f(\lambda_i)) \neq$

0 . This implies that $\operatorname{intcl}(\mu_i) \neq 0$, a contradiction to $(\mu_i)'s$ being fuzzy nowhere dense sets for which $\operatorname{intcl}(\mu_i)\ 0$, in (Y,S). Thus, it must be that $\lambda_i = 0$, in (X,T) and then $\operatorname{int}[f^{-1}(\operatorname{cl}(\mu_i))] = 0$. Since $f^{-1}(\operatorname{cl}(\mu_i))$ is fuzzy closed in (X,T), $f^{-1}(\operatorname{cl}(\mu_i)) = \operatorname{cl}[f^{-1}(\operatorname{cl}(\mu_i))]$ and $\operatorname{int}\operatorname{cl}[f^{-1}(\operatorname{cl}(\mu_i))] = \operatorname{int}[f^{-1}(\operatorname{cl}(\mu_i))] = 0$. Since $\operatorname{int}\operatorname{cl}[f^{-1}(\mu_i)] \leq \operatorname{int}\operatorname{cl}[f^{-1}(\operatorname{cl}(\mu_i))]$, $\operatorname{int}\operatorname{cl}[f^{-1}(\mu_i)] = 0$. Then, $[f^{-1}(\mu_i)]$'s are fuzzy nowhere dense sets in (X,T) and $f^{-1}(\mu) = \bigvee_{i=1}^{\infty}(f^{-1}(\mu_i))$.Thus, $f^{-1}(\mu)$ is a fuzzy first category set in (Y,S).

Corollary 3.1 : If $f : (X,T) \rightarrow (Y,S)$ is a fuzzy continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S) in which $f(\lambda)$ is a fuzzy somewhere dense set in (Y,S), for each non-zero fuzzy open set λ in (X,T) and μ is a fuzzy residual set in (Y,S), then$f^{-1}(\mu)$ is a fuzzy residual set in (X,T).

Proof : Let μ be a fuzzy residual set in (Y,S). Then, $1-\mu$ is a fuzzy first category set in (Y,S) and by Proposition **3.1**, $f^{-1}(1-\mu)$ is a fuzzy first category set in (Y,S). Now $f^{-1}(1-\mu) = 1 - f^{-1}(\mu)$, implies that $1 - f^{-1}(\mu)$ is a fuzzy first category set in (X,T) and hence $f^{-1}(\mu)$ is a fuzzy residual set in (X,T).

The following proposition gives a condition for images of fuzzy residual sets under fuzzy open functions between fuzzy topological spaces to become fuzzy residual sets.

Proposition 3.2 : If $f : (X,T) \rightarrow (Y,S)$ is a one- to- one , fuzzy open function from a fuzzy topological space (X,T) onto a fuzzy topological space (Y,S) in which $f^{-1}(\lambda)$ is a fuzzy somewhere dense set in (X,T) for each non-zero fuzzy open set λ in (Y,S) and μ is a fuzzy residual set in (X,T), then $f(\mu)$ is a fuzzy residual set in (Y,S).

Proof : Let μ be a fuzzy residual set in (X,T). Then, $1-\mu$ is a fuzzy first category set in (X,T) and thus $1-\mu = \bigvee_{i=1}^{\infty}(\mu_i)$, where $(\mu_i)'s$ are fuzzy nowhere dense sets in (X,T). Now $f(1-\mu) = f(\bigvee_{i=1}^{\infty}(\mu_i)) = \bigvee_{i=1}^{\infty}(f(\mu_i))$. Since the function f is a fuzzy open function from (X,T) into (Y,S) and $\operatorname{cl}(\mu_i)$ is a fuzzy closed set in (X,T), $f(\operatorname{cl}(\mu_i))$ is a fuzzy closed set in (Y,S). Let $\lambda_i = \operatorname{int}[f(\operatorname{cl}(\mu_i))]$. Suppose that $\lambda_i \neq 0$, in (Y,S). Then, λ_i is a non-zero fuzzy open set in (Y,S). Now $\lambda_i = \operatorname{int}[f(\operatorname{cl}(\mu_i))] \leq f(\operatorname{cl}(\mu_i))$, implies that $f^{-1}(\lambda_i) \leq f^{-1}(f(\operatorname{cl}(\mu_i)) = \operatorname{cl}(\mu_i))$ [since f is one-to-one] and $\operatorname{int}\operatorname{cl}(f^{-1}(\lambda_i)) \leq \operatorname{int}\operatorname{cl}(\operatorname{cl}(\mu_i)) = \operatorname{int}\operatorname{cl}(\mu_i)$. By hypothesis, $f^{-1}(\lambda_i)$ is a fuzzy somewhere dense set in (X,T) and thus $\operatorname{int}\operatorname{cl}(f^{-1}(\lambda_i)) \neq 0$. This implies that $\operatorname{intcl}(\mu_i) \neq 0$, a contradiction to $(\mu_i)'s$ being fuzzy nowhere dense sets for which $\operatorname{intcl}(\mu_i) = 0$, in (X,T). Thus, it must be that $\lambda_i = 0$, in (Y,S) and then $\operatorname{int}[f(\operatorname{cl}(\mu_i))] = 0$. Since $f(\operatorname{cl}(\mu_i))$ is fuzzy closed in (X,T), $f(\operatorname{cl}(\mu_i)) = \operatorname{cl}[f(\operatorname{cl}(\mu_i))]$ and then $\operatorname{int}\operatorname{cl}[f(\operatorname{cl}(\mu_i))] = \operatorname{int}[f(\operatorname{cl}(\mu_i))] = 0$. Since $\operatorname{int}\operatorname{cl}[f(\mu_i)] \leq \operatorname{int}\operatorname{cl}[f(\operatorname{cl}(\mu_i))]$, $\operatorname{int}\operatorname{cl}[f(\mu_i)] = 0$. Then, $[f(\mu_i)]$'s are fuzzy nowhere dense sets in (Y,S). Since f is a one-to –one and onto function, $f(1-\mu) = 1 - f(\mu)$ and $1 - f(\mu) = \bigvee_{i=1}^{\infty} f(\mu_i)$,where $[f(\mu_i)]$'s are fuzzy nowhere dense sets in (Y,S). This implies that $1 - f(\mu)$ is a fuzzy first category set in (Y,S) and hence $f(\mu)$ is a fuzzy residual set in (X,T).

Corollary 3.2 : If $f : (X,T) \rightarrow (Y,S)$ is a one - to- one , fuzzy open function from a fuzzy topological space (X,T) onto a fuzzy topological space (Y,S) in which $f^{-1}(\lambda)$ is a fuzzy somewhere dense set in (X,T) for each non-zero fuzzy open set λ in (Y,S) and μ is a fuzzy first category set in (X,T), then $f(\mu)$ is a fuzzy first category set in (Y,S).

Proof : Let μ be a fuzzy first category set in (Y,S). Then, $1-\mu$ is a fuzzy residual set in (Y,S) and by Proposition **3.2**, $f(1-\mu)$ is a fuzzy residual set in (Y,S). Since f is a one - to- one and onto function, $f(1-\mu) = 1 - f(\mu)$and then $1 - f(\mu)$ is a fuzzy residual set in (X,T) and hence $f^{-1}(\mu)$ is a fuzzy first category set in (X,T).

The following proposition gives a condition for the inverse images of fuzzy Baire sets under fuzzy continuous functions between fuzzy topological spaces to become fuzzy Baire sets.

Proposition 3.3 : If $f : (X,T) \rightarrow (Y,S)$ is a fuzzy continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S) in which $f(\lambda)$ is a fuzzy somewhere dense set in (Y,S) for each non-zero fuzzy open set λ in (X,T) and μ is a fuzzy Baire set in (Y,S), then $f^{-1}(\mu)$ is a fuzzy Baire set in (X,T).

Proof : Let μ be a fuzzy Baire set in the fuzzy topological space (Y,S). Then, $\mu = \delta \wedge \eta$, where δ is a fuzzy open set and η is a fuzzy residual set in (Y,S).Then, $f^{-1}(\mu) = f^{-1}(\delta \wedge \eta) = f^{-1}(\delta) \wedge f^{-1}(\eta)$. Since f is a fuzzy continuous function from (X,T) into (Y,S), $f^{-1}(\delta)$ is a fuzzy open set in (X,T). From the hypothesis, by corollary **3.1**, for the fuzzy residual set η in (Y,S), $f^{-1}(\eta)$ is a fuzzy residual set in (X,T). Then $f^{-1}(\mu) = f^{-1}(\delta) \wedge f^{-1}(\eta)$, where $f^{-1}(\delta)$ is a fuzzy open set and $f^{-1}(\eta)$ is a fuzzy residual set in (X,T) and hence $f^{-1}(\mu)$ is a fuzzy Baire set in (X,T).

Proposition 3.4 : If $f : (X,T) \to (Y,S)$ is a fuzzy continuous and somewhat fuzzy nearly open function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S) and μ is a fuzzy Baire set in (Y,S), then $f^{-1}(\mu)$ is a fuzzy Baire set in (X,T).
Proof : The proof follows from Proposition **3.3** and definition **2.6(ii).**

Proposition 3.5 : If $f : (X,T) \to (Y,S)$ is a one- to- one, fuzzy open function from a fuzzy topological space (X,T) onto a fuzzy topological space (Y,S) in which $f^{-1}(\lambda)$ is a fuzzy somewhere dense set in (X,T), for each non-zero fuzzy open set λ in (Y,S) and μ is a fuzzy Baire set in (X,T), then there exists a fuzzy Baire set θ in (Y,S) such that $f(\mu) \leq \theta$.

Proof : Let μ be a fuzzy Baire set in the fuzzy topological space (X,T). Then, $\mu = \delta \wedge \eta$, where δ is a fuzzy open set and η is a fuzzy residual set in (X,T). Then, $f(\mu) = f(\delta \wedge \eta) \leq f(\delta) \wedge f(\eta)$. Since f is a fuzzy open function from (X,T) into (Y,S), $f(\delta)$ is a fuzzy open set in (Y,S). From the hypothesis, by Proposition **3.2**, for the fuzzy residual set η in (X,T), $f(\eta)$ is a fuzzy residual set in (Y,S). Then, $f(\delta) \wedge f(\eta)$ is a fuzzy Baire set in (Y,S). Let $\theta = f(\delta) \wedge f(\eta)$. Hence, there exists a fuzzy Baire set θ in (Y,S) such that $f(\mu) \leq \theta$.

Proposition 3.6 : If $f : (X,T) \to (Y,S)$ is a fuzzy continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S) in which $f(\lambda)$ is a fuzzy somewhere dense set in (Y,S), for each non-zero fuzzy open set λ in (X,T) and θ is a fuzzy pseudo-open set in (Y,S), then $f^{-1}(\theta)$ is a fuzzy pseudo-open set in (X,T).

Proof : Let θ be a fuzzy pseudo-open set in (Y,S). Then, $\theta = \mu \vee \gamma$, whereμ is a fuzzy open set and γ is a fuzzy first category set in (X,T). Then, $f^{-1}(\theta) = f^{-1}(\mu \vee \gamma) = f^{-1}(\mu) \vee f^{-1}(\gamma)$. Since f is a fuzzy continuous function from (X,T) into (Y,S), $f^{-1}(\mu)$ is a fuzzy open set in (X,T). From the hypothesis, by Theorem **3.1**, for the fuzzy first category set γ in (Y,S), $f^{-1}(\gamma)$ is a fuzzy first category set in (X,T). Thus $f^{-1}(\theta) = f^{-1}(\mu) \vee f^{-1}(\gamma)$, where $f^{-1}(\mu)$ is a fuzzy open set and $f^{-1}(\gamma)$ is a fuzzy first category set in (X,T), implies that $f^{-1}(\mu)$ is a fuzzy pseudo-open set in (X,T).

The following proposition gives a condition for the inverse images of fuzzy pseudo-sets under fuzzy continuous functions between fuzzy topological spaces to become fuzzy pseudo-open sets.

Proposition 3.7 : If $f : (X,T) \to (Y,S)$ is a fuzzy continuous and somewhat fuzzy nearly open function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S) and μ is a fuzzy pseudo-open set in (Y,S), then $f^{-1}(\mu)$ is a fuzzy pseudo-open set in (X,T).

Proof : The proof follows from Proposition **3.6** and definition **2.6 (ii).**

Proposition 3.8 : If $f : (X,T) \to (Y,S)$ is a one - to-one fuzzy continuous and fuzzy open function from a fuzzy topological space (X,T) onto a fuzzy Baire- separated space (Y,S), then (X,T) is a fuzzy Baire- separated space.

Proof : Let μ_1 and μ_2 be fuzzy closed sets in (X,T) such that $\mu_1 \leq 1 - \mu_2$. Then, $f(\mu_1) \leq f(1 - \mu_2)$, in (X, T). Since the function f is one - to-one and onto, $f(1 - \mu_2) = 1 - f(\mu_2)$ and then $f(\mu_1) \leq 1 - f(\mu_2)$. Now $1 - \mu_1$ and $1 - \mu_2$ are fuzzy open sets in (X,T) and f : (X,T)

$\rightarrow$ (Y,S) is a fuzzy open function, imply that $f(1-\mu_1)$ and $f(1-\mu_2)$ are fuzzy open sets in (Y,S).Then, $1-f(\mu_1)$ and$1-f(\mu_2)$ are fuzzy open sets in (Y,S) and thus $f(\mu_1)$ and $f(\mu_2)$ are fuzzy closed sets in (Y,S).Since (Y,S) is a fuzzy Baire - separated space there exists a fuzzy Baire set η in (Y,S) such that $f(\mu_1) \leq \eta \leq 1 - f(\mu_2)$.Then, $f^{-1} f(\mu_1) \leq f^{-1}(\eta) \leq f^{-1}(1 - f(\mu_2)) = 1 - f^{-1} f(\mu_2)$ and $\mu_1 \leq f^{-1} f(\mu_1) \leq f^{-1}(\eta) \leq f^{-1}(1 - f(\mu_2)) = 1 - f^{-1} f(\mu_2) \leq 1 - \mu_2$. Since a fuzzy open function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S) is a somewhat fuzzy nearly open function [**13**], f : (X,T) $\rightarrow$ (Y,S) is a fuzzy continuous and somewhat fuzzy nearly open function from (X,T) onto (Y,S). Then, by Proposition **3.4**, for the fuzzy Baire set η in (Y,S), $f^{-1}(\eta)$ is a fuzzy Baire set in (X,T). Hence, for the fuzzy closed sets μ_1 and μ_2 in (X,T) such that $\mu_1 \leq 1-\mu_2$, there exists a fuzzy Baire set $f^{-1}(\eta)$ in (X,T) such that $\mu_1 \leq f^{-1}(\eta) \leq 1 - \mu_2$, implies that (X,T) is a fuzzy Baire- separated space.

The following proposition gives a condition by means of fuzzy continuous and fuzzy open functions under which fuzzy topological spaces become fuzzy semi-normal spaces.

Proposition 3.9 : If f : (X,T) $\rightarrow$ (Y,S) is a one - to-one fuzzy continuous and fuzzy open function from a fuzzy topological space (X,T) in which fuzzy Baire sets are fuzzy regular open setsonto a fuzzy Baire- separated space (Y,S) , then (X,T) is a fuzzy semi-normal space.

Proof : Let f : (X,T) $\rightarrow$ (Y,S) be a one - to-one fuzzy continuous and fuzzy open function from a fuzzy topological space (X,T) onto a fuzzy Baire- separated space (Y,S). By Proposition **3.8,** (X,T) is a uzzy Baire- separated space. Suppose that λ is a fuzzy closed set and δ is a fuzzy open set in (X,T) such that $\lambda \leq \delta$. Since (X,T) is a fuzzy Baire -separated space, by Theorem **2.1,** there exists a fuzzy Baire set η in (X,T) such that $\lambda \leq \eta \leq \delta$. By hypothesis, the fuzzy Baire set η is a fuzzy regular open set in (X,T) and thus for the fuzzy closed set λ and the fuzzy open set δ such that$\lambda \leq \delta$, there exists a fuzzy regular open set η in (X,T) such that $\lambda \leq \eta \leq \delta$. Hence it follows that (X,T) is a fuzzy semi-normal space.

The following proposition give conditions by means of fuzzy continuous and fuzzy open functions forhyperconnected and fuzzy second category (but not fuzzy Baire) space to become fuzzy non- hereditarily irresolvable spaces.

Proposition 3.10 : If f : (X,T) $\rightarrow$ (Y,S) is a one - to-one fuzzy continuous and fuzzy open function from a fuzzy hyperconnected and fuzzy second category (but not fuzzy Baire) space (X,T) in which $\lambda \leq \delta$ for each fuzzy closed set λ and for each fuzzy open set δ onto a fuzzy Baire- separated space (Y,S), then (X,T) is not a fuzzy hereditarily irresolvable space.

Proof : Let f : (X,T) $\rightarrow$ (Y,S) be a one - to-one fuzzy continuous and fuzzy open function from a fuzzy topological space (X,T) onto a fuzzy Baire- separated space (Y,S). Then, by Proposition **3.8,** (X,T) is a fuzzy Baire- separated space. Suppose that λ is a fuzzy closed set and δ is a fuzzy open set in (X,T) such that $\lambda \leq \delta$. Since (X,T) is a fuzzy Baire -separated space, by Theorem **2.1,** there exists a fuzzy Baire set ηin (X,T) such that $\lambda \leq \eta \leq \delta$. Since (X,T) is a fuzzy hyperconnected and fuzzy second category (but not fuzzy Baire) space, by Theorem **3.2,**(X,T) is not a fuzzy hereditarily irresolvable space.

Proposition 3.11 : If f : (X,T) $\rightarrow$ (Y,S) is a fuzzy continuous function from a fuzzy hyperconnected and fuzzy nodef space (X,T) into a fuzzy topological space (Y,S) and δ is a fuzzy open set in (Y,S), then $f^{-1}(\delta)$ is a fuzzy Baire set in (X,T).

Proof : Let δ be a fuzzy open set in (Y,S). Since f : (X,T) $\rightarrow$ (Y,S) is a fuzzy continuous function, $f^{-1}(\delta)$ is a fuzzy open set in the fuzzy hyperconnected and fuzzy nodef space (X,T). Then, by Theorem **2.3**, $f^{-1}(\delta)$ is a fuzzy Baire set in (X,T).

Proposition 3.12 : If f : (X,T) $\rightarrow$ (Y,S) is a fuzzy continuous function from a fuzzy hyperconnected and fuzzy nodef space (X,T) into a fuzzy topological space (Y,S) , then f is a fuzzy Baire continuous function from (X,T) into (Y,S).

Proof : The proof follows from Proposition **3.11** and definition **2.6(iv).**

Proposition 3.13 : If λ is a fuzzy dense set in a fuzzy Brown and fuzzy nodef space (X,T), then for a fuzzy open set δin (X,T), $\delta \wedge \lambda$ is a fuzzy Baire set in (X,T).

Proof : Let δ be a fuzzy open set in (X,T). Since (X,T) is a fuzzy Brown and fuzzy nodef space, for the dense set λ , by Theorem **2.3**, λ is a fuzzy residual set in (X,T). Then, $\delta \wedge \lambda$ is a fuzzy Baire set in (X,T).

The following proposition shows that fuzzy Baire sets are having zero interior in fuzzy extraresolvable spaces .

Proposition 3.14 : If λ is a fuzzy Baire set in a fuzzy extraresolvable space (X,T), then int (λ) = 0, in (X,T).

Proof : Let λ be a fuzzy Baire set in (X,T). Then, $\lambda = \mu \wedge \eta$, where μ is a fuzzy open set and η is a fuzzy residual set in (X,T). Since (X,T) is a fuzzy extraresolvable space, by Theorem **2.4** , for the fuzzy residual set η, int (η) = 0, in (X,T).Thus ,int (λ) = int ($\mu \wedge \eta$) = int (μ) $\wedge$ int (η) = $\mu \wedge 0$ = 0, in (X,T).

Corollary 3.3 : If λ is a fuzzy Baire set in a fuzzy extraresolvable space (X,T),then cl(1− λ) = 1, in (X,T).

Proposition 3.15 : If f : (X,T) → (Y,S) is a fuzzy continuous function from a fuzzy extraresolvable space (X,T) into a fuzzy topological space (Y,S) in which f (λ) is a fuzzy somewhere dense set in (Y,S) for each non-zero fuzzy open set λ in (Y,S) and if γ is a fuzzy Baire set in (Y,S), int [f^{-1} (γ)] = 0, in (X,T).

Proof : Let γ be a fuzzy Baire set in (Y,S). From the hypothesis, by Proposition **3.3**, f^{-1} (γ) is a fuzzy Baire set in (X,T). Since (X,T) is a fuzzy extraresolvable space, by Proposition **3.14**, int [f^{-1} (γ)] = 0, in (X,T).

Proposition 3.16 : If f : (X,T) → (Y,S) is a fuzzy continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S) in which f (λ) is a fuzzy somewhere dense set in (Y,S)) for each non-zero fuzzy open set λ in (Y,S) and if a fuzzy set γ in Y is having the property of fuzzy Baire in (Y,S), then f^{-1} (γ) is having the property of fuzzy Baire in (X,T).

Proof : Let γ be a fuzzy set λ in Y having the property of fuzzy Baire in (Y,S). Then, $\gamma = [\mu \wedge \eta] \vee \delta$, where μ is a fuzzy open set, η is a fuzzy residual set and δ is a fuzzy first category set in (X,T). Now $f^{-1}(\gamma) = f^{-1}([\mu\wedge\eta] \vee\delta) = f^{-1}([\mu\wedge\eta]) \vee f^{-1}(\delta) = [f^{-1}(\mu) \wedge f^{-1}(\eta)] \vee f^{-1}(\delta)$, in (X,T). Since f is a fuzzy continuous function from (X,T) into (Y,S) , f^{-1} (μ) is a fuzzy open set in (X,T). Also from the hypothesis, by Proposition **3.1**, f^{-1} (δ) is a fuzzy first category set in (X,T) and by Corollary **3.1**, f^{-1} (η) is a fuzzy residual set in (X,T). Hence, f^{-1} (γ) is having the property of fuzzy Baire in (X,T).

Proposition 3.17 : If f : (X,T) → (Y,S) is a fuzzy continuous and somewhat fuzzy nearly open function from a fuzzy hyperconnected space (X,T) into a fuzzy topological space (Y,S) and μ is a fuzzy pseudo-open set in (Y,S), then f^{-1} (μ) is a fuzzy simply open set in (X,T).

Proof : Let μ be a fuzzy pseudo-open set in (Y,S). Since f : (X,T) → (Y,S) is a fuzzy continuous and somewhat fuzzy nearly open function from (X,T) into (Y,S), by Proposition **3.7**, f^{-1} (μ) is a fuzzy pseudo-open open set in (X,T). Also since (X,T) is a fuzzy hyperconnected space ,by Theorem **2.6**, the fuzzy pseudo-open set f^{-1} (μ) is a fuzzy simply open set in (X,T).

Proposition 3.18 : If f : (X,T) → (Y,S) is a fuzzy continuous and somewhat fuzzy nearly open function from a fuzzy hyperconnected space (X,T) into a fuzzy topological space (Y,S) and μ is a fuzzy pseudo-open set in (Y,S), then f^{-1} (μ) is a fuzzy resolvable set in (X,T).

Proof : Let μ be a fuzzy pseudo-open set in (Y,S). Since $f : (X,T) \to (Y,S)$ is a fuzzy continuous and somewhat fuzzy nearly open function from (X,T) into (Y,S), by Proposition **3.7**, $f^{-1}(\mu)$ is a fuzzy pseudo-open open set in (X,T). Also since (X,T) is a fuzzy hyperconnected space , by Theorem **2.7**, the fuzzy pseudo-open set $f^{-1}(\mu)$ is a fuzzy resolvable set in (X,T).

V. Fuzzy Baire Irresolute Functions

Definition 4.1: Let (X,T) and (Y,S) be any two fuzzy topological spaces. A function $f : (X,T) \to (Y,S)$ is called a fuzzy Baire irresolute function, if for every non-zero fuzzy Baire set λ in (Y,S), $f^{-1}(\lambda)$ is a fuzzy Baire set in (X,T).

Example 4.1 : Let $X = \{ a, b, c\}$. Consider the fuzzy sets α, β, λ, μ, δ, η, σ, θ, ω , π and ρ defined on X as follows:

$\alpha : X \to [0, 1]$ is defined as $\alpha(a) = 0.8$; $\alpha(b) = 0.7$; $\alpha(c) = 0.6$,
$\beta : X \to [0, 1]$ is efined as $\beta(a) = 0.6$; $\beta(b) = 0.9$; $\beta(c) = 0.5$,
$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.7$; $\lambda(b) = 0.8$; $\lambda(c) = 0.6$,
$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.9$; $\mu(b) = 0.6$; $\mu(c) = 0.5$,
$\delta : X \to [0, 1]$ is defined as $\delta(a) = 0.3$; $\delta(b) = 0.3$; $\delta(c) = 0.4$,
$\eta: X \to [0, 1]$ is defined as $\eta(a) = 0.4$; $\eta(b) = 0.2$; $\eta(c) = 0.5$,
$\sigma : X \to [0, 1]$ is defined as $\sigma(a) = 0.3$; $\sigma(b) = 0.2$; $\sigma(c) = 0.4$,
$\theta : X \to [0,1]$ is defined as $\theta(a) = 0.2$; $\theta(b) = 0.4$; $\theta(c) = 0.5$,
$\omega : X \to [0,1]$ is defined as $\omega(a) = 0.4$; $\omega(b) = 0.1$; $\omega(c) = 0.5$,
$\pi : X \to [0,1]$ is defined as $\pi(a) = 0.1$; $\pi(b) = 0.4$; $\pi(c) = 0.5$,
$\rho : X \to [0,1]$ is defined as $\rho(a) = 0.2$; $\rho(b) = 0.3$; $\rho(c) = 0.4$.

Then, $T = \{ 0, \alpha, \beta, \alpha \vee \beta, \alpha \wedge \beta, 1\}$ and $S = \{ 0, \lambda, \mu, \lambda \vee \mu, \lambda \wedge \mu, 1\}$ are fuzzy topologies on X. On computation , one can see that the fuzzy nowhere dense sets in (X,T) are $1 - \alpha , 1 - \beta$, $1 - [\alpha \vee \beta]$, $1 - [\alpha \wedge \beta], 1 - \lambda$ and $1 - [\lambda \vee \mu]$. Then, the fuzzy first category sets are $1 - [\alpha \wedge \beta], \delta$, η, σ , ω and the fuzzy residual sets are $\alpha \wedge \beta$, $1 - \delta$, $1 - \eta$, $1 - \sigma$ and $1 - \omega$. The fuzzy Baire sets in (X,T) are $\alpha \wedge \beta$, β , $\alpha \wedge \lambda$, $\beta \wedge \lambda$, and λ.

Also on computation , one can see that the fuzzy nowhere dense sets in (X,S) are $1 - \lambda$, $1 - \mu$, $1 - [\lambda \vee \mu]$, $1 - [\lambda \wedge \mu]$, $1 - \alpha$ and $1 - [\alpha \wedge \beta]$. Then, the fuzzy first category sets are $1 - [\lambda \wedge \mu], \theta$, δ, π , ρ and the fuzzy residual sets are $\lambda \wedge \mu$, $1 - \theta$, $1 - \delta$, $1 - \pi$ and $1 - \rho$. The fuzzy Baire sets in (X,S) are $\lambda \wedge \mu, \alpha$, $\mu \wedge \alpha, \lambda \wedge \alpha$ and μ .

Define a function $f : (X, T) \to (X, S)$ by $f (a) = b$; $f (b) = a$; $f (c) = c$.
On computation, $f^{-1} (\lambda \wedge \mu) = \alpha \wedge \beta$; $f^{-1} (\alpha) = \lambda$; $f^{-1} (\mu \wedge \alpha) = \beta \wedge \lambda$; $f^{-1}(\lambda \wedge \alpha) = \alpha \wedge \lambda$ and $f^{-1} (\mu) = \beta$. Thus, for every fuzzy Baire set γ in (X,S), $f^{-1} (\gamma)$ is a fuzzy Baire set in (X,T). Hence, $f : (X,T) \to (X,S)$ is a fuzzy Baire irresolute function from (X,T) into (X, S).

The following example shows that fuzzy Baire irresolute functions between topological spaces need not be fuzzy Baire continuous functions.

Example 4.2 : Let $X = \{ a,b\ c\}$. Consider the fuzzy sets λ, μ , α and β defined on X as follows :

$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.7$; $\lambda(b) = 0.8$; $\lambda(c) = 0.6$,
$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.9$; $\mu(b) = 0.6$; $\mu(c) = 0.5$,
$\alpha : X \to [0, 1]$ is defined as $\alpha(a) = 0.8$; $\alpha(b) = 0.7$; $\alpha(c) = 0.6$,
$\beta : X \to [0, 1]$ is defined as $\beta(a) = 0.6$; $\beta(b) = 0.7$; $\beta(c) = 0.5$,

Then, $S = \{ 0, \lambda, \mu, \lambda \vee \mu, \lambda \wedge \mu, 1\}$ and $T = \{ 0, \alpha, \beta, 1\}$ are fuzzy topologies on X. On computation , one can see that the fuzzy nowhere dense sets in (X,S) are $1 - \lambda$, $1 - \mu$, $1 - [\lambda \vee \mu]$, $1 - [\lambda \wedge \mu]$. Then, the fuzzy first category set is $1 - [\lambda \wedge \mu]$, and the fuzzy residual set in (X, S) is $\lambda \wedge \mu$. The fuzzy Baire set in (X, S) is $\lambda \wedge \mu$. Also on computation, the fuzzy nowhere dense sets in (X,T) are $1 - \alpha , 1 - \beta$. Then, the fuzzy first category set is $1 - \beta$, and the fuzzy residual set in (X,T) is β .The fuzzy Baire set in (X,T) is β.

Define a function $f : (X, T) \to (X, S)$ by $f (a) = b$; $f (b) = a$; $f (c) = c$.

On computation, $f^{-1} (\lambda \wedge \mu) = \beta$; Thus, the fuzzy Baire set $\lambda \wedge \mu$ in (X,S), $f^{-1} (\lambda \wedge \mu)$ is a fuzzy Baire set in (X,T), implies that $f : (X,T) \to (X,S)$ is a fuzzy Baire irresolute function from (X,T) into (X, S). But for the fuzzy open set μ in (X,S), $f^{-1} (\mu)$ is not a fuzzy Baire set in (X,T), implies that $f : (X,T) \to (X,S)$ is a not fuzzy Baire continuous function from (X,T) into (X, S).

Proposition 4.1 : If $f : (X,T) \to (Y,S)$ is a fuzzy continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S) in which $f(\lambda)$ is a fuzzy somewhere dense set in (Y,S)fo reach non-zero fuzzy open set λ in (X,T), then $f : (X,T) \to (Y,S)$ is a fuzzy Baire irresolute function from (X,T) into (X,S).

Proof : Let μ be a fuzzy Baire set in (Y,S). Since $f : (X,T) \to (Y,S)$ is a fuzzy continuous function from (X,T) into (Y,S) in which $f(\lambda)$ is a fuzzy somewhere dense set in (Y,S), for each non-zero fuzzy open set λ in (X,T), by Proposition **3.3,** $f^{-1}(\mu)$ is a fuzzy Baire set in (X,T) and hence $f : (X,T) \to (Y,S)$ is a fuzzy Baire irresolute function from (X,T) into (Y,S).

Proposition 4.2: If $f : (X,T) \to (Y,S)$ is a fuzzy continuous and somewhat fuzzy nearly open function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S), then $f : (X,T) \to (Y,S)$ is a fuzzy Baire irresolute function from (X,T) into (Y,S).

Proof : Let μ be a fuzzy Baire set in (Y,S). Since $f : (X,T) \to (Y,S)$ is a fuzzy continuous and somewhat fuzzy nearly open function from (X,T) into (Y,S), by Proposition **3.4,** $f^{-1}(\mu)$ is a fuzzy Baire set in (X,T) and hence $f : (X,T) \to (Y,S)$ is a fuzzy Baire irresolute function from (X,T) into (Y,S).

Proposition 4.3 : If $f : (X,T) \to (Y,S)$ is a fuzzy Baire irresolute function from a fuzzy extraresolvablespace (X,T) into a fuzzy topological space (Y,S) and λ is a fuzzy Baire set in (Y,S), then $\operatorname{int}[\, f^{-1}(\lambda)\,] = 0$, in (X,T).

Proof : Let λ be a fuzzy Baire set in (Y,S). Since $f : (X,T) \to (Y,S)$ is a fuzzy Baire irresolute function from (X,T) into (Y,S), $f^{-1}(\lambda)$ is a fuzzy Baire set in (X,T). Since (X,T) is a fuzzy extraresolvable space , by Proposition **3.14 ,** $\operatorname{int}[\, f^{-1}(\lambda)\,] = 0$, in (X,T).

Proposition 4.4 : If $f : (X,T) \to (Y,S)$ is a fuzzy Baire irresolute function from a fuzzy strongly hyperconnectedspace (X,T) into a fuzzy topological space (Y,S) and λ is a fuzzy Baire set in (Y,S), then $f^{-1}(\lambda)$ is a fuzzy open set in (X,T).

Proof : Let λ be a fuzzy Baire set in (Y,S). Since $f : (X,T) \to (Y,S)$ is a fuzzy Baire irresolute function from (X,T) into (Y,S), $f^{-1}(\lambda)$ is a fuzzy Baire set in (X,T). Since (X,T) is a fuzzy strongly hyperconnected space, by Theorem **2.8**, $f^{-1}(\lambda)$ is a fuzzy open set in (X,T).

The following proposition gives a condition for the composition of fuzzy Baire irresolute function and a fuzzy Baire continuous function to be a fuzzy continuous function between topological spaces .

Proposition 4.5 : If $f : (X,T) \to (Y,S)$ is a fuzzy Baire irresolute function from a fuzzy strongly hyperconnected space (X,T) into a fuzzy topological space (Y,S) and $g : (Y,S) \to (Z,W)$ is a fuzzy Baire continuous function from a fuzzy topological space (Y,S) into a fuzzy topological space (Z,W), then the composition $g \bullet f : (X,T) \to (Z,W)$ is a fuzzy continuous function from (X,T) into (Z,W).

Proof : Let λ be a fuzzy Baire set in (Z,W). Since $g : (Y,S) \to (Z,W)$ is a fuzzy Baire continuous function from (Y,S) into (Z,W), $g^{-1}(\lambda)$ is a fuzzy Baire set in (Y,S). Also $f : (X,T) \to (Y,S)$ being a fuzzy Baire irresolute function from a fuzzy strongly hyperconnected space (X,T) into a fuzz topological space (Y,S), by Proposition **4.4**, $f^{-1}(\, g^{-1}(\lambda)\,)$ is a fuzzy open set in (X,T). Since $(\, g \bullet f\,)^{-1}(\lambda) = f^{-1}(\, g^{-1}(\lambda)\,)$, $(\, g \bullet f\,)^{-1}(\lambda)$ is a fuzzy open set in (X,T). Hence $g \bullet f : (X,T) \to (Z,W)$ is a fuzzy continuous function from (X,T) into (Z,W).

Proposition 4.6 : If λ is a fuzzy Baire set in a fuzzy submaximal and fuzzy P -space (X,T) , then for a fuzzy first category set γ in (X,T), $\lambda \vee \gamma$ is a pseudo-open set having the property of fuzzy Baire, in (X,T).

Proof : Let λ be a fuzzy Baire set in (X,T). Then, $\lambda = \mu \wedge \eta$, where μ is a fuzzy open set and η is a fuzzy residual set in (X,T). Since (X,T) is a fuzzy submaximal and fuzzy P -space, by Theorem **2.9,** for a fuzzy first category set γ in (X,T), $\lambda \vee \gamma$ is a fuzzy pseudo-open set in (X,T). Then, $\lambda \vee \mu .= (\mu \wedge \eta) \vee \gamma$ where μ is a fuzzy open set, η is a fuzzy residual set and γ is a fuzzy first category set in X and then $\lambda \vee \gamma$ is having the property of fuzzy Baire , in (X,T).

Proposition 4.7: If λ is a fuzzy Baire set in a fuzzy strongly hyperconnected space (X,T) , then for a fuzzy first category set γ in (X,T), $\lambda \vee \gamma$ is a pseudo-open set having the property of fuzzy Baire, in (X,T).

Proof : Let λ be a fuzzy Baire set in (X,T). Then, $\lambda = \mu \wedge \eta$, where μ is a fuzzy open set and η is a fuzzy residual set in (X,T). Since (X,T) is a fuzzy strongly hyperconnected space, by Theorem **2.10,** for a fuzzy first category set γ in (X,T), $\lambda \vee \gamma$ is a fuzzy pseudo-open set in (X,T). Then, $\lambda \vee \gamma =(\mu \wedge \eta) \vee \gamma$, where μ is a fuzzy open set, η is a fuzzy residual set and γ is a fuzzy first category set in X and then $\lambda \vee \gamma$ is having the property of fuzzy Baire , in (X,T).

Proposition 4.8 : If $f : (X,T) \rightarrow (Y,S)$ is a fuzzy Baire irresolute function from a fuzzy submaximal and fuzzy P -space (X,T) into a fuzzy topological space (Y,S) and λ is a fuzzy Baire set in (Y,S), then for a fuzzy first category set γ in (X,T), $f^{-1}(\lambda) \vee \gamma$ is a pseudo-open set having the property of fuzzy Baire, in (X,T).

Proof : Let λ be a fuzzy Baire set in (Y,S). Since $f : (X,T) \rightarrow (Y,S)$ is a fuzzy Baire irresolute function from (X,T) into (Y,S) , $f^{-1}(\lambda)$is a fuzzy Baire set in (X,T). Now (X,T) being a fuzzy submaximal and fuzzy P -space, by Proposition **4.6,** for a fuzzy first category set γ in (X,T), $f^{-1}(\lambda)\vee \gamma$ is a pseudo-open set having the property of fuzzy Baire, in (X,T).

Proposition 4.9 : If$f : (X,T) \rightarrow (Y,S)$ is a fuzzy Baire irresolute function from a fuzzy strongly hyperconnected space (X,T) into a fuzzy topological space (Y,S) and λ is a fuzzy Baire set in (Y,S), then for a fuzzy first category set γ in (X,T), $f^{-1}(\lambda) \vee \gamma$ is a pseudo-open set having the property of fuzzy Baire, in (X,T).

Proof : Let λ be a fuzzy Baire set in (Y,S). Since $f : (X,T) \rightarrow (Y,S)$ is a fuzzy Baire irresolute function from (X,T) into (Y,S) , $f^{-1}(\lambda)$ is a fuzzy Baire set in (X,T).Now (X,T) being a fuzzy strongly hyperconnected space, by Proposition **4.7,** for a fuzzy first category set γ in (X,T), $f^{-1}(\lambda) \vee \gamma$ is a pseudo-open set having the property of fuzzy Baire , in (X,T).

Remark 4.1 : By Theorem **2.13**, it is clear that each fuzzy subset in a fuzzy hyperconnected space is either fuzzy dense or fuzzy nowhere dense set.

Proposition 4.10 : If (λ_i)'s are fuzzy open sets in a fuzzy hyperconnected space (X,T), then $\lambda_i \wedge [\wedge_{i=1}^{\infty} (\lambda_i)]$ is a fuzzy Baire set in (X,T).

Proof : Let (λ_i)'s (i= 1 to ∞) be fuzzy open sets in (X,T).Since (X,T) is a fuzzy hyperconnected space , (λ_i)'s are fuzzy dense sets in (X,T). and $cl(\lambda_i) = 1$, for each $i \in I$. Now $int\ cl (1 - \lambda_i) = 1 - cl\ int (\lambda_i) = 1 - cl (\lambda_i) = 1 - 1 = 0$ and thus $(1 - \lambda_i)$'s are fuzzy nowhere dense sets in (X,T). Then, $\vee_{i=1}^{\infty}(1 - \lambda_i)$ is a fuzzy first category set in (X,T). Now $1 - \wedge_{i=1}^{\infty}(\lambda_i) = \vee_{i=1}^{\infty}(1 - \lambda_i)$ shows that $1 - \wedge_{i=1}^{\infty}(\lambda_i)$ is a fuzzy first category set in (X,T) and hence $\wedge_{i=1}^{\infty}(\lambda_i)$ is a fuzzy residual set in (X,T). Then, for each fuzzy open set λ_i , $\{ \lambda_i \wedge [\wedge_{i=1}^{\infty} (\lambda_i)] \}$ is a fuzzy Baire set in (X,T).

Proposition 4.11 : If $f : (X,T) \rightarrow (Y,S)$ is a fuzzy continuous function from a fuzzy hyperconnected space (X,T) into a fuzzy hyperconnected space (Y,S), then $f : (X,T) \rightarrow (Y,S)$ is a fuzzy Baire irresolute function from (X,T) into (Y,S).

Proof : Let (λ_i)'s $(i = 1 \text{ to } \infty)$ be fuzzy open sets in (Y,S) .Since (Y,S) is a fuzzy hyperconnected space, by Proposition 4.10, $\{\lambda_i \wedge [\wedge_{i=1}^{\infty} (\lambda_i)]\}$ is a fuzzy Baire set in (Y,S). Let $\lambda = \lambda_i \wedge [\wedge_{i=1}^{\infty} (\lambda_i)]$.Then, $f^{-1}(\lambda) = f^{-1} (\lambda_i \wedge [\wedge_{i=1}^{\infty} (\lambda_i)]) = f^{-1} (\lambda_i) \wedge f^{-1}([\wedge_{i=1}^{\infty} (\lambda_i)]) = f^{-1}(\lambda_i) \wedge [\wedge_{i=1}^{\infty} f^{-1}(\lambda_i)])$. Since f is a fuzzy continuous function from (X,T) into (Y,S), $[f^{-1}(\lambda_i)]$'s are fuzzy open sets in (X,T). Again since (X,T) is a fuzzy hyperconnected space, by Proposition 4.10, $f^{-1}(\lambda_i) \wedge [\wedge_{i=1}^{\infty} f^{-1}(\lambda_i)])$ is a fuzzy Baire set in (X,T) and hence $f^{-1}(\lambda)$ is a fuzzy Baire set in (X,T). Therefore , it follows that $f : (X,T) \to (Y,S)$ is a fuzzy Baire irresolute function from (X,T) into (Y,S).

v. Fuzzy Pseudo Irresolute Functions

Definition 5.1 : Let (X,T) and (Y,S) be any two fuzzy topological spaces. A function $f : (X,T) \to (Y,S)$ is called a fuzzy pseudo-irresolute function if for each fuzzy pseudo- open set λ in (Y,S), $f^{-1}(\lambda)$ is a fuzzy pseudo- open set in (X,T).

Example 5.1 : Let $X = \{a, b\}$. Consider the fuzzy sets λ, μ and α defined on X as follows:

$\lambda: X \to [0, 1]$ is defined as $\lambda(a) = 0.5$; $\lambda(b) = 0.7$;
$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.8$; $\mu(b) = 0.4$;
$\alpha : X \to [0, 1]$ is defined as $\alpha(a) = 0.2$; $\alpha(b) = 0.6$.

Then, $T = \{0, \lambda, \mu, \alpha, \lambda\vee\mu, \mu\vee\alpha, \lambda\wedge\mu, \mu\wedge\alpha, (\alpha\vee[\lambda\wedge\mu]), 1\}$ and $S = \{0, \lambda, \mu, \lambda\vee\mu, \lambda\wedge\mu, 1\}$ are fuzzy topologies on X. On computation, one can see that the fuzzy nowhere dense sets in (X,S) are $1-\lambda, 1-\mu, 1-(\lambda\vee\mu)$ and $1-(\lambda\wedge\mu) = [1-\lambda]\vee[1-\mu]\vee[1-(\lambda\vee\mu)]$ implies that $1-(\lambda\wedge\mu)$ is a fuzzy first category set in (X,S) and the fuzzy pseudo-open sets in (X,S) are $\lambda, \mu\vee\alpha, \lambda\vee\mu$ and $\alpha\vee[\lambda\wedge\mu]$. Also on computation , the fuzzy nowhere dense sets in (X,T) are $1-\lambda, 1-(\lambda\vee\mu)$ and $(1-\lambda)\vee(1-(\lambda\vee\mu)) = 1-\lambda$, implies that $1-\lambda$ is a fuzzy first category set in (X,T) and on computation, one can see that the fuzzy pseudo-open sets in (X,T) are $\lambda, \mu, \mu\vee\alpha, \lambda\vee\mu, \lambda\wedge\mu$ and $\alpha\vee[\lambda\wedge\mu]$.

Define a function $f: (X, T) \to (X, S)$ by $f(a) = a$; $f(b) = b$.

Now $f^{-1}(\lambda) = \lambda$, $f^{-1}(\mu\vee\alpha) = \mu\vee\alpha$, $f^{-1}(\lambda\vee\mu) = \lambda\vee$ and $f^{-1}(\alpha\vee[\lambda\wedge\mu]) = \alpha\vee[\lambda\wedge\mu]and\lambda$, $\mu\vee\alpha$, $\lambda\vee\mu$, and $\alpha\vee[\lambda\wedge\mu]$ are fuzzy pseudo-open sets in (X,T) implies that $f : (X,T) \to (X,S)$ is a fuzzy pseudo-irresolute function from (X,T) into (X,S).

The following example shows that a fuzzy pseudo-irresolute function need not be a pseudo-continuous function between fuzzy topological spaces.

Example 5.2 : Let $X = \{a, b\}$. Consider the fuzzy sets λ, μ and α defined on X as follows:

$\lambda: X \to [0, 1]$ is defined as $\lambda(a) = 0.5$; $\lambda(b) = 0.7$;
$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.8$; $\mu(b) = 0.4$;
$\alpha : X \to [0, 1]$ is defined as $\alpha(a) = 0.2$; $\alpha(b) = 0.6$.

Then, $T = \{0, \lambda, \mu, \alpha, \lambda\vee\mu, \mu\vee\alpha, \lambda\wedge\mu, \mu\wedge\alpha, (\alpha\vee[\lambda\wedge\mu]), 1\}$ and $W = \{0, \lambda, \alpha, 1\}$ are fuzzy topologies on X. On computation, one can see that the fuzzy nowhere dense sets in (X,W) are $1-\lambda, 1-\alpha$ and $1-\alpha = [1-\lambda]\vee[1-\alpha]$ implies that $1-\alpha$ is a fuzzy first category set in (X,W) and the fuzzy pseudo-open sets in (X,W) are $\mu\vee\alpha, \lambda\vee\mu$. Also on computation , the fuzzy pseudo-open sets in (X,T) are $\lambda, \mu, \mu\vee\alpha, \lambda\vee\mu, \lambda\wedge\mu$ and $\alpha\vee[\lambda\wedge\mu]$.

Define a function $f: (X, T) \to (X,W)$ by $f(a) = a$; $f(b) = b$.

Now $f^{-1}(\lambda) = \lambda$, $f^{-1}(\alpha) = \alpha$ and $f^{-1}(\lambda\vee\mu) = \lambda\vee\mu$; $f^{-1}(\mu\vee\alpha) = \mu\vee\alpha$, show that $f : (X,T) \to (X,S)$ is a fuzzy continuous function and fuzzy pseudo-irresolute function from (X,T) into (X,W). Since $f^{-1}(\alpha) = \alpha$ and α is not a fuzzy pseudo-continuous function from (X,T) into (X,W).

Proposition 5.1 : If $f : (X,T) \to (Y,S)$ is a fuzzy continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S) in which $f(\lambda)$ is a fuzzy somewhere dense set in (Y,S) for each non-zero fuzzy open set λ in (X,T), then $f : (X,T) \to (Y,S)$ is a fuzzy pseudo-irresolute function from (X,T) into (Y,S).

Proof : Let θ be a fuzzy pseudo-open set in (Y,S). Since $f : (X,T) \to (Y,S)$ is a fuzzy continuous function from (X,T) into (Y,S) in which $f(\lambda)$ is a fuzzy somewhere dense set in(Y,S), for each non-zero fuzzy open set λ in (X,T), by Proposition **3.6**, $f^{-1}(\theta)$ is a fuzzy pseudo-open set in (X,T) and hence $f : (X,T) \to (Y,S)$ is a fuzzy pseudo-irresolute function from (X,T) into (Y,S).

Proposition 5.2 : If $f : (X,T) \to (Y,S)$ is a fuzzy continuous and somewhat fuzzy nearly open function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S), then $f : (X,T) \to (Y,S)$ is a fuzzy pseudo-irresolute function from (X,T) into (Y,S).

Proof : Let θ be a fuzzy pseudo-open set in (Y,S). Since $f : (X,T) \to (Y,S)$ is a fuzzy continuous and somewhat fuzzy nearly open function from (X,T) into (Y,S), by Proposition **3.7,** $f^{-1}(\theta)$ is a fuzzy pseudo-open set in (X,T). Hence $f : (X,T) \to (Y,S)$ is a fuzzy pseudo-irresolute function from (X,T) into (Y,S).

Proposition 5.3 : If $f : (X,T) \to (Y,S)$ is a fuzzy pseudo-irresolute function from a fuzzy hyperconnectedspace (X,T) into a fuzzy topological space (Y,S) and μ is a fuzzy pseudo-open set in (Y,S), then $f^{-1}(\mu)$ is a fuzzy simply open set in (X,T).

Proof : Let μ be a fuzzy pseudo-open set in (Y,S). Since $f : (X,T) \to (Y,S)$ is a fuzzy pseudo-irresolute function , $f^{-1}(\mu)$ is a fuzzy pseudo-open open set in (X,T). Also since (X,T) is a fuzzy hyperconnected space , by Theorem **2.6**, the fuzzy pseudo-open set $f^{-1}(\mu)$ is a fuzzy simply open set in (X,T).

Proposition 5.4 : If $f : (X,T) \to (Y,S)$ is a fuzzy pseudo-irresolute function from a fuzzy hyperconnected space (X,T) into a fuzzy topological space (Y,S) and μ is a fuzzy pseudo-open set in (Y,S), then $f^{-1}(\mu)$ is a fuzzy resolvable set in (X,T).

Proof : Let μ be a fuzzy pseudo-open set in (Y,S). Since $f : (X,T) \to (Y,S)$ is a fuzzy pseudo-irresolute function , $f^{-1}(\mu)$ is a fuzzy pseudo-open open set in (X,T). Also since (X,T) is a fuzzy hyperconnected space , by Theorem **2.7**, the fuzzy pseudo-open set $f^{-1}(\mu)$ is a fuzzy resolvable set in (X,T).

Proposition 5.5 : If $f : (X,T) \to (Y,S)$ is a fuzzy pseudo-irresolute function from a fuzzy D-Bairespace (X,T) into a fuzzy topological space (Y,S) and μ is a fuzzy pseudo-open set in (Y,S), then $f^{-1}(\mu)$ is a fuzzy simply*-open set in (X,T).

Proof : Let μ be a fuzzy pseudo-open set in (Y,S). Since$f : (X,T) \to (Y,S)$ is a fuzzy pseudo-irresolute function , $f^{-1}(\mu)$ is a fuzzy pseudo-open open set in (X,T). Alsosince (X,T) is a fuzzy D-Bairespace , by Theorem **2.11**, the fuzzy pseudo-open set $f^{-1}(\mu)$ is a fuzzy simply* open set in (X,T).

Proposition 5.6: If $f : (X,T) \to (Y,S)$ is a fuzzy pseudo-continuous function from a fuzzy D-Bairespace (X,T) into a fuzzy topological space (Y,S), then $f : (X,T) \to (Y,S)$ is a fuzzy simply*-continuous function from (X,T) into (Y,S).

Proof : Let μ be a fuzzy open set in (Y,S). Since $f : (X,T) \to (Y,S)$ is a fuzzy pseudo-continuous function from (X,T) into (Y,S), $f^{-1}(\mu)$ is a fuzzy pseudo-open set in (X,T). Also since (X,T) is a fuzzy D-Bairespace , by Theorem **2.11**, the fuzzy pseudo-open set $f^{-1}(\mu)$ is a fuzzy simply* open set in (X,T). Hence $f : (X,T) \to (Y,S)$ is a fuzzy simply*-continuous function from (X,T) into (Y,S).

Proposition 5.7 : If $f : (X,T) \to (Y,S)$ is a fuzzy pseudo-irresolute function from a fuzzy hyperconnectedand fuzzy D-Baire space (X,T) into a fuzzy topological space (Y,S) and μ is a fuzzy pseudo-open set in (Y,S), then $f^{-1}(\mu)$ is a fuzzy simply open set in (X,T).

Proof : Let μ be a fuzzy pseudo-open set in (Y,S). Since $f : (X,T) \to (Y,S)$ is a fuzzy pseudo-irresolute function from (X,T) into (Y,S), $f^{-1}(\mu)$ is a fuzzy pseudo-open set in (X,T).

Also since (X,T) is a fuzzy hyperconnected and fuzzy D-Baire space, by Theorem **2.12**, the fuzzy pseudo-open set $f^{-1}(\mu)$ is a fuzzy simply open set in (X,T).

Proposition 5.8 : If $f : (X,T) \rightarrow (Y,S)$ is a fuzzy pseudo- irresolute function from a fuzzy hyperconnected and fuzzy D-Bairespace (X,T) into a fuzzy topological space (Y,S) and $g : (Y,S) \rightarrow (Z,W)$ is a fuzzy Bairecontinuous function from a fuzzy topological space (Y,S) into a fuzzy topological space (Z,W), then the composition $g \bullet f : (X,T) \rightarrow (Z,W)$ is a fuzzy simply continuous function from (X,T) into (Z,W).

Proof : Let λ be a fuzzy openset in (Z,W). Since $g :(Y,S) \rightarrow (Z,W)$ is a fuzzy pseudo continuous function from (Y,S) into (Z,W), $g^{-1}(\lambda)$ is a fuzzy pseudo-open set in (Y,S). Also $f : (X,T) \rightarrow (Y,S)$ being a fuzzy pseudo- irresolute function from a fuzzy hyperconnected and fuzzy D-Bairespace (X,T) into a fuzzy topological space (Y,S), by Proposition **5.7**, $f^{-1}(g^{-1}(\lambda))$ is a fuzzy simply open set in (X,T). Since $(g \bullet f)^{-1}(\lambda) = f^{-1}(g^{-1}(\lambda))$, $(g \bullet f)^{-1}(\lambda)$ is a fuzzy simply open set in (X,T). Hence $g \bullet f :(X,T) \rightarrow (Z,W)$ is a fuzzy simply continuous function from (X,T) into (Z,W).

The following proposition gives a condition for the composition of a pseudo- continuous and a pseudo- irresolute function between topological spaces to become a fuzzy resolvable function.

Proposition 5.9 : If $f : (X,T) \rightarrow (Y,S)$ is a fuzzy pseudo- irresolute function from a fuzzy hyperconnectedspace (X,T) into a fuzzy topological space (Y,S) and $g : (Y,S) \rightarrow (Z,W)$ is a fuzzy pseudo- continuous function from a fuzzy topological space (Y,S) into a fuzzy topological space (Z,W), then the composition $g \bullet f : (X,T) \rightarrow (Z,W)$ is a fuzzy resolvable function from (X,T) into (Z,W).

Proof : Let λ be a fuzzy open set in (Z,W). Since $g :(Y,S) \rightarrow (Z,W)$ is a fuzzy pseudo continuous function from (Y,S) into (Z,W), $g^{-1}(\lambda)$ is a fuzzy pseudo-open set in (Y,S). Also $f : (X,T) \rightarrow (Y,S)$ being a fuzzy pseudo- irresolute function from a fuzzy hyperconnected space (X,T) into a fuzzy topological space (Y,S), by Proposition **5.4**, $f^{-1}(g^{-1}(\lambda))$ is a fuzzy resolvable set in (X,T). Since $(g \bullet f)^{-1}(\lambda) = f^{-1}(g^{-1}(\lambda))$, $(g \bullet f)^{-1}(\lambda)$ is a fuzzy resolvable set in (X,T). Hence $g \bullet f :(X,T) \rightarrow (Z,W)$ is a fuzzy resolvable function from (X,T) into (Z,W).

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