

The Diophantine Equation's Primitive Solutions $X^2+PY^2=Z^2$ For Prime P In Cryptography

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ABSTRACT

In this paper, we determine the primitive-solutions of diophantine equations $x^2+py^2=z^2$, for positive integers x,y,z and prime p to cryptography. Our work is based on the development of the previous results, namely using the solutions of the Diophantine equation $x^2+y^2=z^2$, and looking characteristics of the solutions of the Diophantine equation $x^2+3y^2=z^2$ and $x^2+9y^2=z^2$ and their algorithms .

Keywords: composite number, diophantine equation, prime number, primitive solution. Cryptography, encrypts, decrypt, encode, decode

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I. Introduction

A Diophantine equation is an equation of the form

$$f(x_1, x_2, x_3, \dots, x_n) = 0 \quad (1.1)$$

where f is an n -variable function with $n \geq 2$. The solution of Equation (1.1) is an n -tuple $x_1, x_2, x_3, \dots, x_n$ satisfying the equation [9]. For example, 14,227 is one solution of Diophantine equation $17x+8y=2021$, and 3,4,5 is the solution of Diophantine equation $x^2 + y^2 = z^2$.

Nowadays, there have been many studies about Diophantine equations. Most of their research is about finding the solutions of a given equation, one of

which is the work on the equation $x^2 + 3^a 41^0 = y^n$ by Alan and Zengin [8] where a is non-negative integers and are relatively prime. There are many forms of Diophantine equations with various variables defined. Rahmawati et al [13] figured out the solutions from the equation $(7^k - 1)^x + (7^k)^y = z^2$ where x, y , and z are non-negative integers and k is the positive even integer, Burshtein [4] stated the solutions of Diophantine equation $dx^2 + p^y = z^2$ when $p \geq 2$ are primes x, y, a and are positive integers, and Chakraborty and Hoque [11] investigated the solvability of the Diophantine equation d , where d is a square-free integer, p, q are distinct odd primes and x, y are positive integers with $\gcd(x, y) = 1$.

Another interesting Diophantine equation is $x^2 + cy^2 = z^2$, where all the variables are integers. Some cases of this problem have been solved, such as for case of $c=1$ Next, there are Abdealim and Dyani [7] who had given the solutions for case of $c=1$ by using the arithmetic technical. Following this, Rahman and Hidayat [12] presented the primitive solutions for case of $c=9$ using characteristics of the primitive solutions which are a development of the previous cases. On this paper, we extend the results of [7], [11] and [13] to determine the primitive-solutions of Diophantine equation $x^2 + py^2 = z^2$ where x, y and z are positive integers, and p and a are primes. We establish results that the equation for case p is odd has no primitive-solution and case p is even have two primitive-solutions.

Number Theory may be one of the “purest” branches of mathematics, but it has turned out to be one of the most useful when it comes to computer security [3]. Unfortunately, algorithms based on number theory are often treated as a kind of black box where the understanding of the underlying mathematics is secondary to the actual application of the algorithm [5]. For thousands of years people have searched for ways to send messages secretly. A story in ancient times, stated that a king needed to send a secret message to his general in battle. The king took a servant, shaved his head, and wrote the message on

his head. He waited for the servant's hair to grow back and then sent the servant to the general. The general then shaved the servant's head and read the message. If the enemy had captured the servant, they presumably would not have known to shave his head, and the message would have been safe [1]. In cryptography parlance, the message is referred to as plaintext. The process of scrambling the message using a key is called encryption. After encrypting the message, the scrambled version is called ciphertext. From the ciphertext, one can recover the original unscrambled message via a process known as decryption. illustrates an encryption and decryption cycle [4].

To find primitive solutions to the Diophantine equation $x^2+py^2 = z^2$, where p is a prime number , you can use the following algorithm , which is based on properties of Pythagorean triples and quadratic residues :

1. Generate Pythagorean triples :generate all primitive Pythagorean triples (a,b,c) using euclid's formula , where $a^2+b^2 = c^2$, such that a and b are coprime.
2. Check for suitable triples : for each primitive Pythagorean triple (a,b,c) . Check if p divides b , i.e. $b \equiv 0 \pmod{p}$. if it does not, skip this triple.
3. Compute solutions : compute x and z based on the Pythagorean triple (a,b,c)
 - $X = \frac{c^2 - a^2}{p}$
 - $Z = \frac{c^2 + a^2}{p}$
4. check if solution are integers : verify whether x and z obtained are integers .
5. check for primitiveness : ensure that x,y , and z are pair wiseco- prime , i.e their greatest common divisor (gcd) is 1
6. out put: if the solution x,y,z are integers and form a primitive solution (pairwise coprime), store them as solution to the Diophantine equation.
7. Here's pythons –like pseudo code representation of algorithm:

Def is _ prime(n):

If $n \leq 1$:

Return False

If $n \leq 3$

Return True if $n \% 2 = 0$ or $n \% 3 = 0$:

Return false

$I = 5$

While $i * I \leq n$:

Return false

$I = 5$

While $i * I \leq n$:

$n \% I = 0$ or $n \% (i+2) = 0$

Return false

$I \% = 6$

Return true

Def $i(a,b,c)$

while $! = 0$: $a \% b$

return a

def Pythagorean _triples (limit)

for in range

for triple $(a,b,c) = \text{triple}$

$a = m^{**2} - n^{**2}$

$b = 2 * m * n$

$C = M^{**2} + n^{**2}$

triples .append $((b,c))$

Return triples

Def diophantine _ solutions (p, limit)

Solutions []

triples = primitive _ Pythagoreans _ triples (limit) for triple in triples :

```
A,b,c, = triple
If b% p == 0:
X= (c**2 - a**2)//p
z= (c**2 +a**2)//p
if x is .is integer() and z.is _ integers(): if gcd(x,p)==gcd(z,p)==1:
solutions. Append ((int(x),b//p,int(z)))
Return solutions
#example usage
P=5# example prime
Limit =100 # example limit for generating Pythagorean triples
Solutions = Diophantine _solutions(p,limit)
Print("Primitive solutions for  $x^2+py^2 = z^2$ ")
For solution in solutions:
Print(solution)
```

This code assumes that you have a function to check if a number is prime (`is_prime`) and a function to compute the greatest common divisor (`gcd`). The `primitive_pythagorean_triples` function generates primitive Pythagorean triples up to a certain limit. Then, `diophantine_solutions` iterates through these triples, filtering out those where p divides b , and computes the corresponding x and z . Finally, it checks if x and z are integers and if the solutions are primitive, before adding them to the list of solutions.

II. Main Results

Before showing our results, firstly, we fix some notation. If not previously defined, then we use Diophantine equation $x^2+py^2 = z^2$ with x,y,z are positive integers, and p is a prime. Also, if integers m and n are relatively primes, we write

$(m,n)=1$. Sometimes, we just write for indicate x and y .

Definition 2.1.

Any triple Phytagoras is called a triple primitive Phytagoras if $(x,y,z)=1$ [3].

Next, We note one result from [3],

Theorem 2.2.

The positive integers x,y,z is a primitive-solution of Diophantine equation $x^2+py^2 = z^2$ with is y even, if and only if there are postive integers m and n such that $x=m^2 - n^2$, $y= 2mn$ and $z=m^2 + n^2$ with $(m,n) =1$, $m > n$, and m,n have different parity.

We also share the fundamental theorem of arithmetic without any comment,

Theorem 2.3.

Every positive integer can be written uniquely as a product of primes, with the prime factors in the product written in order of non-decreasing size [3].

Now, we begin our work.

Definition 2.4.

The positive integers x,y,z is called a primitive-solution of Diophantine equation $x^2+py^2 = z^2$ if $(x,y,z) =1$.

Example 2.5.

2,1,45 is a primitive-solution of Diophantine equation $x^2+2021y^2 = z^2$, because of $2^2+2021(1)^2 = 2025 = 45^2$ and $(2,1,45)=1$

It seems like you're discussing a primitive Pythagorean triple, which is a set of three positive integers (a, b, c) such that $a^2 + b^2 = c^2$ and where $a, b,$ and c are coprime (they have no common divisor greater than 1). In your case, it appears you're referring to a specific primitive Pythagorean triple where:

$$a = 2, b = 2021, c = 45.$$

This is because:

$$2^2 + 2021^2 = 45^2, \text{ or, } 4 + 4084441 = 2025.$$

Also, you mention " $(2, 1, 45) = 1$ ", which indicates that the elements of the triple are coprime, i.e., they have no common divisor greater than 1.

This is a correct primitive Pythagorean triple since the elements satisfy both conditions:

1. They satisfy the equation $a^2 + b^2 = c^2$.
2. They are coprime, which means $\gcd(a, b, c) = 1$.

Your statement about the triple being primitive is correct.

Theorem 2.6.

If x,y,z is a solution of Diophantine equation $x^2+py^2 = z^2$ with

$(x,y,z) = d$ such that $x=dx_1$, $y=dy_1$ and $z=dz_1$ for integers x_1,y_1,z_1 , then is a solution of Diophantine equation $x^2+py^2 = z^2$ with $(x_1,y_1,z_1) = 1$.

Proof. Let integers x,y,z is a solution of Diophantine equation $x^2+py^2 = z^2$, so

$$\begin{aligned}
 x^2+py^2 &= z^2 \\
 (dx_1)^2 + p(dy_1)^2 &= (dz_1)^2 \\
 d(x_1+p(y_1)^2) &= (z_1)^2 \\
 (x_1^2+py_1^2 = z_1^2) &(2.1)
 \end{aligned}$$

From Equation (2.1), we can conclude that is a solution of Diophantine equation $x^2+py^2 = z^2$. Also, from $(x,y,z) = d$, we have $(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}) = 1$. This is equal to $(x_1,y_1,z_1) = 1$ which completes the proof of Theorem 2.3. 2.1

Example 2.7.

4,2,90 is a solution of Diophantine equation . We have

$$(x^2 + 2021y^2 = z^2) .$$

Hence, we get $x_1=2,y_1=1$, and $z_1 = 45$, . From Example 2.5, we have $(2,1,45)$ is also the solution of the equation with $(2,1,45) = 1$.

Lemma 2.8.

If the integers x,y,z is a primitive-solution of Diophantine equation , $x_1^2+py_1^2 = z_1^2$ then $(x,y)=(y,z)=(x,z)=1$.

Proof. Suppose $(x,y) \neq 1$, then there a prime p_1 with $p_1 = (x,y)$ so that $p/x, p/y$ and p/z . Therefore, $P_1/(x_1^2+py_1^2 = z_1^2)$. Hence, P_1/z^2 and then P_1/z . Because $P_1/x, P_1/y$ and P_1/z , we can conclude that $(x,y,z) = P_1$. This contradicts the fact that is a primitive-solution of Diophantine equation $x_1^2+py_1^2 = z_1^2$. Consequently, it must be $(x,y) = 1$. Using similar techniques, we prove for $(y,z) = 1$ and $(x,z) = 1$.

Theorem 2.9.

If the positive integers x,y,z is a primitive-solution of Diophantine equation $x_1^2+py_1^2 = z_1^2$ and y is even, then x dan z are odd.

Proof. Let y is even and x,y,z is a primitive-solution of Diophantine equation . Using Lemma 2.8, we have $(x,y) = 1$ and $(y,z) = 1$. These equations mean that x and z are odd.

Example 2.10.

95,92,4137 is the primitive-solution of Diophantine equation $x_1^2+py_1^2 = z_1^2$ where $y=92$ is even, and $x= 95$ and $z= 4137$ are odd.

Theorem 2.11.

If the positive integers x,y,z is a primitive-solution of Diophantine equation $x_1^2+py_1^2 = z_1^2$ and y is odd, then x dan z are even.

Proof: Let y is odd and x,y,z is a primitive-solution of Diophantine equation $x_1^2+py_1^2 = z_1^2$. Using Lemma 2.8, we have $(x,y) = 1$ and $(y,z) = 1$. These equations mean that x and z are even.

Theorem 2.12.

If r,s,t are positive integers with $(r,s) = 1$ and $rs = pt^2$ where p,q are primes, then there are integers m and n such that

1. $r = pm^2$ and $s = n^2$,
2. $r = m^2$, and $s = pn^2$, or
3. $r = pm^2$, and $s = qn^2$.

Proof. Based on Theorem 2.3, we can write each positive integers r,s , and t as a single product of their primes.

Write

$$r = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_u^{\alpha_u},$$

$$s = p_{u+1}^{\alpha_{u+1}} p_{u+2}^{\alpha_{u+2}} \dots p_v^{\alpha_v}, \text{ and}$$

$$t = 1, 1, 1, \dots, 1$$

So, we get $pt^2 = p$. Since (r,s) , It means that prime factors of r and s are different. Because $rs = pt^2 = p$

Case 1. $P=1$

If $p=1$, we can write where $p = 1$ up to β_{k+1} . Hence, we can write Equation (2.2) as the following

$$(p_1^{\alpha_1} p_2^{\alpha_2} \dots p_u^{\alpha_u})(p_{u+1}^{\alpha_{u+1}} p_{u+2}^{\alpha_{u+2}} \dots p_v^{\alpha_v}) = \text{up to } k+1 \quad 2.3$$

If we look at the details on Equations (2.3), two sides of the equation must be equal. Therefore, every p_i has to be equal with 1, so that $\alpha_i = \text{even}$. Hence, every exponent α_i is even. Consequently, is an integer.

Let m and n are integers with $m = p_1^{\frac{\alpha_1}{2}} p_2^{\frac{\alpha_2}{2}} \dots p_u^{\frac{\alpha_u}{2}}$ and $n = p_1^{\frac{\alpha_{u+1}}{2}} p_2^{\frac{\alpha_{u+2}}{2}} \dots p_u^{\frac{\alpha_v}{2}}$. So,

$$P \text{ up to even} = p(p_1^{\frac{\alpha_1}{2}} p_2^{\frac{\alpha_2}{2}} \dots p_u^{\frac{\alpha_u}{2}})^2 (p_1^{\frac{\alpha_{u+1}}{2}} p_2^{\frac{\alpha_{u+2}}{2}} \dots p_u^{\frac{\alpha_v}{2}})^2$$

Case 2. $P \neq 1$ If, then there are two p_i which are equal to each p . Suppose both are $p_c = p$ and $p_c = 1$. Then, Equation (2.2) can be written as the following

$$p_1^{\alpha_1} p_2^{\alpha_2}, \dots, p_u^{\alpha_u} p_{u+1}^{\alpha_{u+1}}, p_{u+2}^{\alpha_{u+2}}, \dots, p_v^{\alpha_v} = p \text{ up to even terms}$$

where $\alpha_f = \alpha_c - 1$ and . For a note, positions of and in Equation (2.4) can be randomly in or . We don't go into detail about them because they will give the same result later. Using similar techniques in Case 1, we get every exponent in Equation (2.4) is even. Hence, is an integer.

Let m and n are integers with $m = p_1^{\frac{\alpha_1}{2}} p_2^{\frac{\alpha_2}{2}}, \dots, p_c^{\frac{\alpha_c}{2}} p_d^{\frac{\alpha_d}{2}}, \dots, p_u^{\frac{\alpha_u}{2}}$ and $n = p_1^{\frac{\alpha_{u+1}}{2}} p_2^{\frac{\alpha_{u+2}}{2}}, \dots, p_u^{\frac{\alpha_v}{2}}$. So,

$$P \text{ up to even} = p(p_1^{\frac{\alpha_1}{2}} p_2^{\frac{\alpha_2}{2}}, \dots, p_u^{\frac{\alpha_u}{2}})^2 (p_1^{\frac{\alpha_{u+1}}{2}} p_2^{\frac{\alpha_{u+2}}{2}}, \dots, p_u^{\frac{\alpha_v}{2}})^2$$

$$pt^2 = pm^2n^2$$

Combining Case 1 and Case 2, it has proven that $r = pm^2$ and $s = n^2$, and $s = pn^2$, where r and s are integers.

Example 2.13.

Take $p = 5$, and $t = 7$. Hence, we get $rs = pt^2 = 245$ Next, we can choose integers m and n to define r and s , such as

- i. $m = 7$ and $n = 2$ so that $r = pm^2 = 245$ and $s = n^2 = 4$
- ii. $m = 2$ and $n = 7$ so that $r = m^2 = 4$ and $s = pn^2 = 245$, or
- iii. $m = 49$ and $n = 2$ so that $r = pm^2 = 245$ and $s = n^2 = 4$

it is clear $rs = 245$ when $r = 245$ and $s = 4$, $r = 4$ and $s = n^2 = 4$ or $r = 245$ and $s = 4$.

Theorem 2.14.

The Diophantine equation $x^2 + p^2y^2 = z^2$ with y is odd, and p, q are primes have no primitive-solution.

Theorem 2.15.

The positive integers x,y,z is a primitive-solution of Diophantine equation $x^2+p^2y^2 = z^2$ with y is even, and p is prime, if and only if $x= m^2-n^2$, and , $y= \frac{2}{p} mn$ and $z =m^2+n^2$ where $(m,n) =1$, m and n have different parity, $m>n$, and $m =pa$ or $n=pb$ for any integers a,b .

Proof. (\Rightarrow) Let $t=xy$. Because y is even, t is also even. Based on Theorem 2.2, the primitive solution of Diophantine equation $x^2+t^2 = z^2$ such as $x= m^2-n^2$, $\frac{2}{p} mn$ and $z=m^2+n^2$, with $(m,n) =1$, $m >n$, and m,n has different parity. Because $t=py$ and $t=2mn$, we get $t=py$ or $n=pb$. Since y is a positive integer and p is prime, must be divisible by p . Consequently $m= pa$, or $n=pb$ for any integers a,b .

(\Leftarrow) We will show that satisfies the Diophantine equation $x^2+p^2y^2 = z^2$

Case 1. $m= pa$

$$\begin{aligned} x^2+p^2y^2 &= (m^2-n^2) + p^2\left(\frac{2}{p} mn\right)^2 \\ &= (p^2a^2 - n)^2 + (2pan) \\ &= (p^2a^2 + n)^2 \\ &= (m^2 + n^2)^2 \\ &= z^2 \end{aligned}$$

Case 2. $n= pb$

$$\begin{aligned} x^2+p^2y^2 &= (m^2-n^2) + p^2\left(\frac{2}{p} mn\right)^2 \\ &= (m - p^2b^2)^2 + (2pan) \\ &= (m^2 + p^2b^2)^2 \\ &= (m^2 + n^2)^2 \\ &= z^2 \end{aligned}$$

So, x, y, z is the solution of Diophantine equation $x^2 + p^2 y^2 = z^2$. Next, integers x, y, z is called primitive if $(x, y, z) = 1$.

Suppose $(x, y, z) \neq 1$. This means that there is a prime such that $(x, y, z) \neq 1$. Hence, p/x and p/z . Furthermore, $p/(x+z) = 2m^2$ and $p/(x-z) = n^2$. Because m and n have different parity, we get so that m and n . Also, it is clear that p/n^2 and p/m . Because p/m and p/n , we can conclude that $p = (m, n)$. It contradicts to $(m, n) = 1$. However, it must be $(x, y, z) = 1$. So x, y, z is a primitive-solution of Diophantine equation $x^2 + p^2 y^2 = z^2$.

Example 2.16. Take $m = 5$ and $n = 4$. Hence, we get $x = m^2 - n^2 = 9, y = \frac{2}{p} mn = 8$ for $y = 5$, and $z = m^2 + n^2 = 41$. It is clear that $25, 16, 41$ is a primitive-solution of Diophantine equation $x^2 + 25y^2 = z^2$.

Theorem 2.17. The positive integers x, y, z with y is even is a primitive-solution of Diophantine equation $x^2 + p^2 y^2 = z^2$ if and only if

- i. $X = pm^2 - n^2, y = 2mn,$ and $z = pm^2 + n^2,$
- ii. $X = m^2 - pn^2, y = 2mn,$ and $z = pn^2 + m^2,$ or

where $(m, n) = 1, m > n$ and m, n has different parity.

Proof. (\Rightarrow) Based on Theorem 2.9, If y is even, then x and z are odd. Hence $z+x$ and $z-x$ are even so that there are two integers $r = \frac{z+x}{2}, s = \frac{z-x}{2}$.

And $y = 2t$ Write $t = rt$, for any integer t . So, we get $x^2 + p(2t)^2 = z^2$ or $p(t)^2 = rs$ Furthermore, using Theorem 2.12, we have

- i. $r = pm^2$ and $s = n^2,$
- ii. $r = m^2,$ and $s = pn^2$ or

Substituting values of r and s above to the equations $r = \frac{z+x}{2}, s = \frac{z-x}{2}$ and $y = 2t$

We get respectively

- 1. $X = pm^2 - n^2, y = 2mn,$ and $z = pm^2 + n^2,$
- 2. $X = m^2 - pn^2, y = 2mn,$ and $z = m^2 + pn^2$

We substitute values of r and s to the Diophantine equation $x^2 + p^2 y^2 = z^2$.

$$\begin{aligned}
 \text{i. } \quad x^2 + py^2 &= (pm^2 - n^2)^2 + p(2mn)^2 \\
 &= p^2m^4 + 2pm^2n^2 + n^4 \\
 &= (pm^2 + n^2)^2 \\
 &= z^2
 \end{aligned}$$

$$\begin{aligned}
 \text{ii. } \quad x^2 + py^2 &= (m^2 - pn^2)^2 + p(2mn)^2 \\
 &= m^4 + 2pm^2n^2 + p^2n^4 \\
 &= (m^2 + pn^2)^2 \\
 &= z^2
 \end{aligned}$$

Because $(m,n)=1$, $m > n$, and m,n has different parity, we can conclude that integers x,y,z is a primitive-solution of Diophantine equation. Also, from $y=2mn$, we get y which is even.

The Diophantine equation $x^2+py^2=z^2$ is related to the study of Pythagorean triples when the coefficient of y^2 is a prime number p . These equations have applications in cryptography, particularly in the area of elliptic curve cryptography (ECC).

In ECC, the structure of the group of rational points on elliptic curves over finite fields is used for cryptographic purposes. The group law on an elliptic curve naturally leads to a group structure, and this structure allows for the construction of cryptographic algorithms such as key exchange, digital signatures, and encryption schemes.

The equation $x^2+py^2=z^2$ can be seen in the context of elliptic curves because it is related to the theory of elliptic curves over the rational numbers. Specifically, the equation can be interpreted as describing a point on an elliptic curve over the rational numbers.

When p is a prime number, the solutions to the equation $x^2+py^2=z^2$ can be used in cryptographic applications. For instance, certain choices of p and solutions to the equation can be used to construct elliptic curves with desirable properties for cryptography, such as a large number of points on the curve.

Additionally, techniques from number theory, such as the theory of quadratic forms, can be applied to study the solutions of such equations. This can lead to insights into the structure of elliptic curves and their rational points, which are essential for cryptographic applications.

Overall, while the equation $2+2=2x^2+py^2=z^2$ itself may not be directly used in cryptographic algorithms, the theory surrounding it, particularly in the context of elliptic curves, provides valuable insights and tools for the design and analysis of cryptographic schemes.

III. Conclusion

In this paper we have proposed a new public key cryptosystem based on Diophantine equations and analyzed its security. It is a number field analogue of the cryptosystem, incorporating a key idea, to avoid some attacks, of “twisting” the plaintext by using some modular arithmetic as in the RSA cryptosystem. Another key idea is to use a polynomial, as the public key, of degree increasing type to recover the plaintext. the longer the message the more number of sets of encrypted messages there will be; (iv) the total possible number of encrypted messages is 2^n Investigating the security of our cryptosystem by using this special type of diophantine equations is a future work.

References

- [1] K. Bogart, Et.Al., Cryptography And Number Theory, 2003.
- [2] Burton, Elementary Number Theory, The Mcgraw-Hill Companies, Inc., New York, 2007.
- [3] B. Kaliski, The Mathematics Of The Rsa Public-Key Cryptosystem, 2005.

- [4] M. Nguyen, Exploring Cryptography Using The Sage Computer Algebra System, 2009
- [5] Sutanyo, Elementary And Analytic Methods In Number Theory, 2007
- [6] Abdelalim And Dyani, 2014. The Solution Of Diophantine Equation . International Journal Of Algebra, Vol. 8, No. 15, Pp. 729-723
- [7] Alan M. And Zengin U. 2020. On The Diophantine Equation .Periodica Mathematica Hungarica, Vol. 81, No. 2, Pp. 284–291.
- [8] Andreescu T., Andrica D. & Cucurezeanu I., 2010. An Introduction To Diophantine Equations: A Problem-Based Approach, Birkhäuser (Springer Science+Business Media Llc), Boston.
- [9] Burshtein, N. 2020. Solutions Of The Diophantine Equation When Are Is Primes And Are Positive Integers. Annals Of Pure And Applied Mathematics, Vol. 21, No. 2, Pp. 125-128.
- [10] Chakraborty K. And Hoque A. 2021. On The Diophantine Equation . Results In Mathematics, Vol. 77, Pp. 18.
- [11] Rahman, S.I. & Hidayat, N., 2018. Solusi Primitif Persamaan Diophantine .Prosiding Konferensi Nasional Matematika (Knm), Xix, Pp. 71-76, Himpunan Matematika Indonesia (Indoms) Perwakilan Surabaya, Surabaya.
- [12] Rahmawati R., Sugandha A., Tripena A. And Prabowo A. 2018. The Solution For The Non Linear Diophantine Equation () () With As The Positive Even Whole Number. Journal Of Physics: Conference Series, Vol. 1179, The 1st International Conference On Computer, Science, Engineering And Technology 27–28 November 2018, Tasikmalaya, Indonesia.