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On qp*I*-Irresolute Mappings

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Abstract : *In the present paper the concept of qpI-Irresolute mappings have been introduced and studied.*
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I. Preliminaries

Mashhour [6] introduced the concept of preopen sets in topology. A subset A of a topological space (X, τ) is called preopen if $A \subset Int(Cl(A))$. Every open set is preopen but the converse may not be true. In 1961 Kelly [3] introduced the concept of bitopological spaces as an extension of topological spaces. A bitopological space (X, τ_1, τ_2) is a nonempty set X equipped with two topologies τ_1 and τ_2 [3]. The study of quasi open sets in bitopological spaces was initiated by Dutta [1] in 1971. In a bitopological space (X, τ_1, τ_2) a set A of X is said to be quasi open [1] if it is a union of a τ_1-open set and a τ_2-open set**.** Complement of a quasi open set is termed quasi closed. Every τ_1-open (resp. τ_2-open) set is quasi open but the converse may not be true. Any union of quasi open sets of X is quasi open in X. The intersection of all quasi closed sets which contains A is called quasi closure of A. It is denoted by qcl(A) [5]. The union of quasi open subsets of A is called quasi interior of A. It is denoted by qInt(A) [5]. In 1995,Tapi [8] introduced the concept of quasi preopen sets in bitopological spaces. A set A in a bitopological space (X, τ_1, τ_2) is called quasi preopen [8] if it is a union of a τ_1-preopen set and a τ_2-preopen set**.** Complement of a quasi preopen set is called quasi pre closed. Every τ_1-preopen (τ_2-preopen, quasi open) set is quasi preopen but the converse may not be true. Any union of quasi preopen sets of X is a quasi preopen set in X. The intersection of all quasi pre closed sets which contains A is called quasi pre closure of A. It is denoted by qpcl(A). The union of quasi preopen subsets of A is called quasi pre interior of A. It is denoted by qpInt(A).

The concept of ideal topological spaces was initiated Kuratowski [4] and Vaidyanathaswamy [10]. An Ideal I on a topological space (X, τ) is a non empty collection of subsets of X which satisfies: i) $A \in I$ and $B \subset A \Rightarrow B \in I$ and ii) $A \in I$ and $B \in I \Rightarrow A \cup B \in I$ If $\mathscr{P}(X)$ is the set of all subsets of X, in a topological space (X, τ) a set operator $(.)^*: \mathscr{P}(X) \to \mathscr{P}(X)$ called the local function [2] of A with respect to τ and I and is defined as follows:

$$A^*(\tau, I) = \{x \in X \mid U \cap A \notin I, \forall U \in \tau(x)\}, \text{ where } \tau(x) = U \in \tau \mid x \in U\}.$$

Given an ideal bitopological space (X,τ_1,τ_2,I) the quasi local function [3] of A with respect to τ_1, τ_2 and I denoted by A_q^* (τ_1,τ_2,I) (in short A_q^*) is defined as follows:

$$A_q^*(\tau_1, \tau_2, I) = \{x \in X \mid U \cap A \notin I, \forall \text{ quasi open set U containing x}\}.$$

A subset A of an ideal bitopological space (X, τ_1, τ_2) is said to be qI- open[2] if $A \subset qInt\, A_q^*$. A mapping f: $(X,\tau_1,\tau_2,I) \to (Y,\sigma_1,\sigma_2)$ is called qI- continuous[2] if $f^{-1}(V)$ is qI- open in X for every quasi open set V of Y. Recently the authors of this paper[9] defined qpI- open sets and qpI- continuous mappings in ideal bitopological spaces.

*Definition1.1***.** [9] Given an ideal bitopological space (X,τ_1,τ_2,I) the quasi pre local mapping of A with respect to τ_1, τ_2 and I denoted by $A_{qp}^*(\tau_1,\tau_2,I)$ (more generally as A_{qp}^*) is defined as $A_{qp}^*(\tau_1,\tau_2,I) = \{x \in X \mid U \cap A \notin I, \forall \text{ quasi pre-open set U containing x}\}$

*Definition1.2***.** **[**9] A subset A of an ideal bitopological space (X,τ_1,τ_2,I) is qpI- open if $A \subset qpInt(A_{qp}^*)$ and qpI- closed if its complement is qpI- open.

Remark1.1. [9] Every qI- open set is qpI- open but the converse is not true

Remark1.2. [9] The concepts of qpI- open sets and quasi pre open sets are independent.

Definition1.3 [9] A mapping f: $(X,\tau_1,\tau_2,I) \to (Y,\sigma_1,\sigma_2)$ is called a qpI- continuous if $f^{-1}(V)$ is a qpI- open set in X for every quasi open set V of Y

*Remark1.3***.** [9] Every qI- continuous mapping is qpI- continuous but the converse is not true

Definition1.4. [9] In an ideal bitopological space (X,τ_1,τ_2,I) the quasi $*$-pre closure of A of X denoted by $qpcl^*(A)$ is defined by $qpcl^*(A) = A \cup A^*_{qp}$

*Definition1.5***.** [9] A subset A of an ideal bitopological space (X,τ_1,τ_2,I) is said to be a qpI- neighbourhood of a point $x \in X$ if $\exists$ a qpI- open set O such that $x \in O \subset A$

Definition1.6. [9] Let A be a subset of an ideal bitopological space (X,τ_1,τ_2,I) and $x \in X$. Then x is called a qpI-interior point of A if $\exists$ V a qpI- open set in X such that $x \in V \subset A$.
The set of all qpI- interior points of A is called the qpI- interior of A and is denoted by qpIInt(A).

Definition1.7. [9] Let A be a subset of an ideal bitopological space (X,τ_1,τ_2,I) and $x \in X$. Then x is called a qpI-cluster point of A, if $V \cap A \neq \emptyset$. for every qpI- open set V in X. The set of all qpI-cluster points of A denoted by qpIcl(A) is called the qpI-closure of A .

II. qpI- Irresolute Mappings

Definition2.1. A mapping f: $(X,\tau_1,\tau_2,I) \rightarrow (Y,\sigma_1,\sigma_2)$ is called qpI- irresolute if $f^{-1}(V)$ is a qpI- open set in X for every quasi pre open set V of Y.

Remark2.1. Every qpI- irresolute mapping is qpI- continuous but the converse may not true. For,

Example2.1. Let X ={ a, b, c } and I ={ϕ,{a}} be an ideal on X. Let τ_1 ={X, ϕ,{c}}, τ_2 ={X, ϕ, {a, b}}, σ_1={X, ϕ,{b}}and σ_2 ={ ϕ, X} be topologies on X. Then the identity mapping f: $(X,\tau_1,\tau_2,I) \rightarrow (X,\sigma_1,\sigma_2)$ is qpI-continuous but not qpI- irresolute.

Theorem2.1. Let f: $(X,\tau_1,\tau_2,I) \rightarrow (Y,\sigma_1,\sigma_2)$ be a mapping, then the following statements are equivalent:
(a) f is qpI- irresolute.
(b) $f^{-1}(V)$ is qpI- closed set in X for every quasi pre closed set V of Y
(c) for each $x \in X$ and every quasi pre open set V of Y containing f(x), $\exists$ a qpI- open set W of X containing x such that $f(W) \subset V$.
(d) for each $x \in X$ and every quasi pre open set V of Y containing f(x), $f^{-1}(V)^*_{qp}$ is a qpI-neighbourhood of X.
Proof: (a) $\Leftrightarrow$ (b). Obvious.
(a) $\Rightarrow$ (c). Let $x \in X$ and V be a quasi pre open set of Y containing f(x). Since f is qpI- irresolute, $f^{-1}(V)$ is a qpI- open set. Put $W = f^{-1}(V)$, then $x \in W$. Hence $f(W) \subset V$.
(c) $\Rightarrow$ (a). Let A be a quasi pre open set in Y. If $f^{-1}(A) = \emptyset$, then $f^{-1}(A)$ is clearly a qpI- open set. Assume that $f^{-1}(A) \neq \emptyset$ and $x \in f^{-1}(A)$, then $f(x) \in A \Rightarrow \exists$ a qpI- open set W containing x such that $f(W) \subset A$. Thus $W \subset f^{-1}(A)$. Since W is qpI- open, $x \in W \subset qpInt(W^*_{qp}) \subset qpInt(f^{-1}(A)^*_{qp})$ and so $f^{-1}(A) \subset qpInt(f^{-1}(A)^*_{qp})$. Hence $f^{-1}(A)$ is a qpI- open set and so f is qpI- irresolute.
(c) $\Rightarrow$(d). Let $x \in X$ and V be a quasi pre open set of Y containing f(x) then $\exists$ a qpI- open set W containing x such that $f(W) \subset V$. therefore $W \subset f^{-1}(V)$. Since W is a qpI- open set, $x \in W \subset qpInt(W^*_{qp})) \subset qpInt(f^{-1}(V)^*_{qp}) \subset (f^{-1}(V)^*_{qp})$. Hence $f^{-1}(V)^*_{qp})$ is a qpI- neighbourhood of x.
(d) $\Rightarrow$(c). Obvious.

Definition2.2. A mapping f**:** $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2,I)$ is said to be :
(a) qpI- pre open if f(U) is a qpI- open set of Y for every quasi pre open set U of X.
(b) qpI- pre closed if f(U) is a qpI- closed set of Y for every quasi pre closed set U of X.

Theorem2.2. Let f: $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2,I)$ be a mapping. Then the following statements are equivalent:
(a) f is qpI- pre open
(b) $f(qpInt(U)) \subset qpIInt(f(U)$ for each subset U of X.
(c) $qpInt(f^{-1}(V)) \subset f^{-1}(qpIInt(V))$ for each subset V of Y.
Proof: (a) $\Rightarrow$ (b). Let U be any subset of X. Then qpInt(U) is a quasi pre open set of X. Then f(qpInt(U)) is a qpI-open set of Y. Since $f(qpInt(U)) \subset f(U)$, $f(qpInt(U)) = qpIInt(f(qpInt(U)) \subset qpIInt(f(U)$.
(b) $\Rightarrow$ (c). Let V be any subset of Y. Obviously $f^{-1}(V)$ is a subset of X. Therefore by (b), $f(qpInt(f^{-1}(V))) \subset qpIInt(f(f^{-1}(V))) \subset qpIInt(V))$. Hence, $qpInt(f^{-1}(V)) \subset f^{-1}(f(qpInt(f^{-1}(V)))) \subset f^{-1}(qpIInt(V))$

(c) ⇒ (a). Let V be any quasi pre open set of X. Then qpInt(V) = V and f(V) is a subset of Y. So V = qpInt(V) ⊂ qpInt(f^{-1}(f(V))) ⊂ f^{-1}(qpIInt(f(V))). Then f(V) ⊂ f(f^{-1}(qpIInt(f(V))) ⊂ qpIInt(f(V) and qpIInt(f(V) ⊂ f(V). Hence, f(V) is a qpI-open set of Y and f is qpI-open.

Theorem2.3. Let f: $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2,I)$ be a qpI- pre open mapping. If V is a subset of Y and U is a quasi pre closed subset of X containing f^{-1}(V), then there exists a qpI- closed set F of Y containing V such that f^{-1}(F) ⊂ U.
Proof: Let V be any subset of Y and U a quasi pre closed subset of X containing f^{-1}(V), and let F = Y\(f(X\V)). Then f(X\V) ⊂ f(f^{-1}(X\V)) ⊂ (X\V) and X\U is a quasi pre open set of X. Since f is qpI- pre open, f(X\U) is a qpI- open set of Y. Hence F is a quasi pre closed subset of Y and f^{-1}(F) = f^{-1}(Y\(f (X\U)) ⊂ U.

Theorem2.4. A mapping f: $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2,I)$ is qpI- pre closed if and only if qpIcl(f(V) ⊂ f(qpcl(V) for each subset V of X.
Proof: Necessity. Let f be a qpI- pre closed mapping and V be any subset of X. Then f(V) ⊂ f(qpcl(V) and f(qpcl(V) is a qpI- closed set of Y. Thus qpIcl(f(V)) ⊂ qpIcl(f(qpcl(V)) = f(qpcl(V).
Sufficiency. Let V be a quasi pre closed set of X. Then by hypothesis f(V) ⊂ qpIcl(f(V)) ⊂ f(qpcl(V) = f(V) . And so, f(V) is a qpI- closed subset of Y. Hence, f is qpI- pre closed.

Theorem2.5. A mapping f: $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2,I)$ is qpI- pre closed if and only if f^{-1}(qpIcl(V)) ⊂ qpcl(f^{-1}(V)) for each subset V of Y.
Proof: Obvious.

Theorem 2.6. Let f: $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2,I)$ be a qpI- pre closed mapping. If V is a subset of Y and U is a quasi pre open subset of X containing f^{-1}(V), then there exists a qpI- open set F of Y containing V such that f^{-1}(F) ⊂ U.
Proof: Obvious.

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