Effects Of Heat Source And Thermal Diffusion On An Unsteady Free Convection Flow Along A Porous Plate With Constant Heat And Mass Flux In A Rotating System Under Slip Boundary Condition

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Abstract: The present analysis is made to investigate the effects of heat source and thermal diffusion on an unsteady free convection flow along a porous vertical plate in a rotating system. The plate is subjected to constant heat and mass flux also. The problem is solved analytically and expressions for velocity. Energy and temperature profiles, skin friction and Nusselt number are obtained. The effects of different parameter entered in the problem are discussed on the primary and secondary velocities, temperature and concentration distributions, primary and secondary skin frictions and Nusselt number with the help of tables and graphs. Key Words: Diffusion, Heat Source, Porous Medium, Slip Velocity, Unsteady Flow.

I. Introduction

The flow through a porous medium is a common occurrence in industrial environment and so the heat transfer problems of viscous incompressible fluid through a porous medium has attracted the interest of many research workers in view of its applications in geophysics, astrophysics, aerodynamics, boundary layer control and so on.

A number of authors such as Soundalgekar et al. [1], Raptis and Perdikis [2], Mohammad [3], Helmy [4] and recently Jain and Gupta [5] have investigated flow through porous medium on different geometries. The effect of variable permeability on combined free and forced convection in porous media was studied by Chandrasekhara and Namboodiri [6] and Mixed Convection in porous media with uniform heat flux on vertical surface was studied by Joshi and Gebhart [7]. Lai and Kulacki [8] studied the effect of variable viscosity along a vertical surface in a saturated porous medium.

The phenomena of heat and mass transfer is encountered in chemical process industries such as food processing and polymer products. Raptis and Kafousias [9], Bejan and Khair [10], Elbashbeshy [11], Acharya et al. [12], Chand et al. [13] are some of the workers to study the different problems through porous medium with mass transfer effects.

In high attitude flights, situation may arise when the flow becomes unsteady and slip at the boundary take place as well. In such situation of slip flow ordinary continuum approach fails to yield satisfactory result [Tsien [14], Street [15]]. Many authors have solved problems taking slip conditions at the boundary. Recently Jothimani and Anjali Devi [16], Jain et al. [17] and Gupta et al. [18] have considered slip boundary conditions in their problems.

In the present work we study the effects of heat source and permeability on unsteady free convection rotating flow of a viscous fluid past a porous vertical plate with time dependent suction and velocity slip boundary conditions. At the plate there is constant heat flux and mass flux. Perturbation technique is used to obtain the expressions of velocity and temperature fields, concentration profile, skin friction and Nusselt number. Effects of different parameters viz. Grashoff number of heat transfer, Grashoff number for mass transfer, slip parameter, permeability parameter, heat source parameter, Diffusion parameter and Schmidt number are discussed and shown graphically. It is observed that increase in Schmidt number decreases primary skin friction (τ_p) as well as secondary skin friction (τ_s) while increase in Grashoff number increase both the skin fictions. Moreover, increase in Prandtl number decreases Nusselt Number (Nu).

II. Mathematical Formulation

We consider an unsteady free convection flow of an incompressible viscous fluid through a porous medium past an infinite vertical porous plate with constant heat and mass fluxes. Let both the fluid and plate be in a state of rigid rotation with uniform angular velocity Ω about z-axis taken normal to the plane and plate is taken electrically non-conducting. Moreover the plate is assumed to coincide with the plane z = 0. As the plate is infinite in extent and the flow is unsteady, all the physical variables depend on z and t only.

For the governing equations for a free convective flow with heat source along a porous vertical plate in rotating system are given as:

Continuity equation

$$\frac{\partial \mathbf{w}}{\partial \mathbf{z}} = 0 \qquad \dots (2.1)$$

Momentum equations

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{w} \frac{\partial \mathbf{u}}{\partial z} - 2\Omega \mathbf{v} = \mathbf{v} \frac{\partial^2 \mathbf{u}}{\partial z^2} + g\beta(\mathbf{T} - \mathbf{T}_{\infty}) + g\beta'(\mathbf{C} - \mathbf{C}_{\infty}) - \frac{\mathbf{v}}{\mathbf{K}}\mathbf{u} \qquad \dots (2.2)$$

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{w} \frac{\partial \mathbf{v}}{\partial z} + 2\Omega \mathbf{u} = \mathbf{v} \frac{\partial^2 \mathbf{v}}{\partial z^2} - \frac{\mathbf{v}}{\mathbf{K}} \mathbf{v} \qquad \dots (2.3)$$

Energy equation

$$\frac{\partial \mathbf{T}}{\partial \mathbf{t}} + \mathbf{w} \frac{\partial \mathbf{T}}{\partial z} = \frac{\lambda}{\rho C_{p}} \frac{\partial^{2} \mathbf{T}}{\partial z^{2}} + \frac{\mathbf{S}^{*}}{\rho C_{p}} (\mathbf{T} - \mathbf{T}_{\infty}) \qquad \dots (2.4)$$

Concentration equation

$$\frac{\partial \mathbf{C}}{\partial t} + \mathbf{w} \frac{\partial \mathbf{C}}{\partial z} = \mathbf{D} \frac{\partial^2 \mathbf{C}}{\partial z^2} + \mathbf{D}_{\ell} \frac{\partial^2 \mathbf{T}}{\partial z^2} \qquad \dots (2.5)$$

Here ρ is the density, g is the acceleration due to gravity, β is the coefficient of volume expansion, β is the coefficient of concentration expansion, ν is the kinematic viscosity, T_{∞} is the temperature of the fluid in the free stream, C_{∞} is the concentration at infinite, u and v are the velocities in x and y direction respectively, T is the temperature of the fluid, K is the permeability of porous medium, λ is the thermal conductivity, D is the concentration diffusivity, C_p is the specific heat at constant pressure, S^* is the coefficient of heat source, D_{ℓ} is coefficient of diffusivity.

The boundary conditions are:

$$u = L \frac{\partial u}{\partial z}, v = L \frac{\partial v}{\partial z}, \frac{\partial T}{\partial z} = -\frac{q'}{\lambda}, \frac{\partial C}{\partial z} = -\frac{m'}{D} \text{ at } z = 0$$

$$u \to 0, v \to 0, T \to T_{\infty}, C \to C_{\infty} \text{ as } z \to \infty$$

Here q' and m' are uniform heat and concentration flux at the plate respectively, L being Mean free path. Integration of equation (2.1) gives:

$$w = -w_0 (1 + \epsilon e^{-nt})$$
 ...(2.7)

Using equation (2.7) and introducing following non-dimensional quantities

$$u^{*} = \frac{u}{w_{0}}, v^{*} = \frac{v}{w_{0}}, z^{*} = \frac{z w_{0}}{v}, n^{*} = \frac{vn}{w_{0}^{2}}, t^{*} = \frac{w_{0}^{2}t}{v}$$

$$\theta = \frac{(T - T_{\infty})w_{0}\lambda}{q'v}, \phi = \frac{(C - C_{\infty})w_{0}D}{m'v}, K^{*} = \frac{K w_{0}^{2}}{v^{2}} \text{ (Permeability parameter)}$$

$$Pr = \frac{\mu C_{p}}{\lambda} \text{ (Prandtl number)}, Sc = \frac{v}{D} \text{ (Schimdt number)}$$

$$E = \frac{v\Omega}{w_{0}^{2}} \text{ (Rotation velocity parameter)}, S^{*} = \frac{v^{2}S}{\lambda w_{0}^{2}} \text{ (Heat Source parameter)}$$

$$Gr = \frac{g\beta q'\nu^2}{w_0^4\lambda} (Grashoff number for heat transfer),$$

$$Gm = \frac{g\beta'm'\nu^2}{w_0^4D} (Grashoff number for mass transfer)$$

$$A = \frac{q'D_\ell}{\lambda m'} (Diffusion parameter), h = \frac{Lw_0}{\nu} (slip parameter) \dots (2.8)$$

Equations (2.2) to (2.5) reduce to the following form after dropping the asterisks over them:

$$\frac{\partial^2 q}{\partial z^2} + (1 + \epsilon e^{-nt}) \frac{\partial q}{\partial z} - \frac{\partial q}{\partial t} - \left(s + \frac{1}{K}\right) = -Gr \ \theta - Gm \ \phi \qquad \dots (2.9)$$

$$\frac{\partial^2 \theta}{\partial z^2} + \Pr\left(1 + \epsilon e^{-nt}\right) \frac{\partial \theta}{\partial z} - \Pr\left(\frac{\partial \theta}{\partial t}\right) = -S\theta \qquad \dots (2.10)$$

$$\frac{\partial^2 \phi}{\partial z^2} + \operatorname{Sc} \left(1 + \epsilon e^{-nt}\right) \frac{\partial \phi}{\partial z} - \operatorname{Sc} \frac{\partial \phi}{\partial t} = -A \frac{\partial^2 \theta}{\partial z^2} \qquad \dots (2.11)$$

where q = u + iv and s = 2iE.

The corresponding boundary conditions become:

$$q = h \frac{\partial q}{\partial z}, \frac{\partial \theta}{\partial z} = -1, \frac{\partial \phi}{\partial z} = -1 \quad \text{at } z = 0$$

$$q \to 0, \ \theta \to 0, \ \phi \to 0 \qquad \text{as } z \to \infty$$

III. Solution Of The Problem

To solve equations (2.9) to (2.11), we follow the perturbation technique in the form ($\epsilon <<1$) $q = q_1(z) + \epsilon e^{-nt} q_2(z) + 0(\epsilon^2)$...(3.1)

$$\theta = \theta_1(z) + \epsilon^{-nt} \theta_2(z) + 0(\epsilon^2)$$
 ...(3.2)

$$\phi = \phi_1(z) + \epsilon e^{-nt} \phi_2(z) + 0(\epsilon^3) \qquad ...(3.3)$$

Substitution of Equations (3.1) to (3.3) in Equations (2.9) to (2.11) and equating the coefficient of like powers of \in (neglecting \in ² etc.), we obtain the following set of differential equations

$$\frac{d^{2}q_{1}}{dz^{2}} + \frac{dq_{1}}{dz} - \left[s + \frac{1}{k}\right]q_{1} = -Gr \ \theta_{1} - Gm \ \phi_{1} \qquad \dots (3.4)$$

$$\frac{d^{2}q_{2}}{dz^{2}} + \frac{dq_{1}}{dz} - \left[n + s + \frac{1}{K}\right]q_{2} = -Gr \ \theta_{1} - Gm \ \phi_{2} - \frac{dq_{1}}{dz} \qquad \dots (3.5)$$

$$\frac{d^2\theta_1}{dz^2} + \Pr \frac{d\theta_1}{dz} = -S \theta_1 \qquad \dots (3.6)$$

$$\frac{d^2\theta_2}{dz^2} + \Pr \frac{d\theta_2}{dz} = -(n\Pr + S)\theta_2 - \Pr \frac{d\theta_1}{dz} \qquad \dots (3.7)$$

$$\frac{d^{2}\phi_{1}}{dz^{2}} + Sc \frac{d\phi_{1}}{dz} + A \frac{d^{2}\theta_{1}}{dz^{2}} = 0 \qquad \dots (3.8)$$

$$\frac{d^2\phi_2}{dz^2} + Sc \frac{d\phi_2}{dz} = -n Sc \phi_2 - Sc \frac{d\phi_1}{dz} - A \frac{d^2\theta_2}{dz^2} \qquad \dots (3.9)$$

the corresponding boundary conditions are:

$$q_{1} = h \frac{dq_{1}}{dz}, q_{2} = h \frac{dq_{2}}{dz}; \frac{d\theta_{1}}{dz} = -1, \frac{d\theta_{2}}{dz} = 0; \frac{d\phi_{1}}{dz} = -1, \frac{d\phi_{2}}{dz} = 0 \text{ at } z = 0$$

$$q_{1} \rightarrow 0, q_{2} \rightarrow 0; \theta_{1} \rightarrow 0, \theta_{2} \rightarrow 0; \phi_{1} \rightarrow 0, \phi_{2} \rightarrow 0 \text{ as } z \rightarrow \infty$$

On solving equations (3.4) to (3.9) under the transformed boundary conditions (3.10), we get the solution for θ , ϕ and q as follows

$$\theta = \frac{1}{D_1} e^{-D_1 z} + \epsilon e^{-nt} (R_1 e^{-D_1 z} + R_2 e^{-D_2 z}) \qquad \dots (3.11)$$

$$\phi = R_{3} e^{-S_{c}z} - R_{4} e^{-D_{1}z} + \in e^{-nt} (R_{5} e^{-S_{c}z} - R_{6} e^{-D_{1}z} - R_{7} e^{-D_{2}z} + R_{8} e^{-D_{3}z}) \dots (3.12)$$

$$q = -R_{9} e^{-D_{1}z} - R_{10} e^{-S_{c}z} + R_{11} e^{-D_{4}z} + \in e^{-nt} (R_{17} e^{-D_{1}z} + R_{18} e^{-D_{2}z} - R_{19} e^{-D_{3}z} + R_{20} e^{-D_{4}z} - R_{21} e^{-S_{c}z} + R_{22} e^{-D_{5}z}) \dots (3.13)$$

Using q = u+iv, the expressions for the primary and secondary velocities are obtained as follows

$$\begin{aligned} u(z,t) &= [\{ (K_{9} + K_{19} \in e^{-nt}) \cos A_{2}z + (K_{10} + K_{20} \in e^{-nt}) \sin A_{2}z \} e^{-A_{1}z} \\ &+ (K_{25} \cos A_{4}z + K_{26} \sin A_{4}z) \in e^{-nt} e^{-A_{3}z} - (K_{1} - K_{13} \in e^{-nt}) e^{-D_{1}z} \\ &+ K_{15} \in e^{-nt} e^{-D_{2}z} - K_{17} \in e^{-nt} e^{-D_{3}z} - (K_{3} + K_{21} \in e^{-nt}) e^{-Sc z}] \\ &+ (K_{10} + K_{20} \in e^{-nt}) \cos A_{2}z - (K_{9} + K_{19} \in e^{-nt}) \sin A_{2}z \} e^{-A_{1}z} \\ &+ (-K_{25} \sin A_{4}z + K_{26} \cos A_{4}z) \in e^{-nt} e^{-A_{3}z} - (K_{2} - K_{14} \in e^{-nt}) e^{-D_{1}z} \\ &+ K_{16} \in e^{-nt} e^{-D_{2}z} - K_{18} \in e^{-nt} e^{-D_{3}z} - (K_{4} + K_{22} \in e^{-nt}) e^{-Sc z}] \\ &- ...(3.15) \end{aligned}$$

IV. Skin Friction And Heat Transfer

Once the expressions for velocities and temperature are known it is important to calculate the skin-friction and Nusselt number. The skin friction τ_p is due to primary velocity u and skin friction τ_s is due to secondary velocity v in the x and y directions respectively.

The coefficient of skin friction in x and y directions are:

$$\begin{aligned} \tau_{x} = & \left(\frac{\partial u}{\partial z}\right)_{z=0} = [\{-(k_{9} + k_{19} \in e^{-nt})A_{1} + (k_{10} + k_{20} \in e^{-nt})A_{2}\} \\ & -\{k_{25}A_{3} - k_{26}A_{4} + D_{1}k_{13} + D_{2}k_{15} - D_{3}k_{17} - Sc k_{21}\} \in e^{-nt} + D_{1}k_{1} + Sc k_{3}] \\ & \dots (4.1) \\ \tau_{y} = & \left(\frac{\partial v}{\partial z}\right)_{z=0} = [\{-(k_{10} + k_{20} \in e^{-nt})A_{1} - (k_{9} + k_{19} \in e^{-nt})A_{2}\} \end{aligned}$$

$$-\{k_{26}A_3 + k_{25}A_4 + D_1k_{14} + D_2k_{16} - D_3k_{18} - Sc k_{22}\} \in e^{-nt} + D_1k_2 + Sc k_4]$$
...(4.2)

and the rate of heat transfer is given by

$$Nu = \frac{1}{\theta(0, t)} = \frac{D_1}{1 + D_1(R_1 + R_2) \in e^{-nt}}$$
...(4.3)

where

$$\begin{split} A_{1} &= \frac{1+a}{2}, A_{2} = \frac{b}{2}, A_{3} = \frac{1+a'}{2}, A_{4} = \frac{b'}{2} \\ a &= \left[\frac{\{(1+4K^{-1})^{2} + 64E^{2}\}^{1/2} + (1+4K^{-1})}{2} \right]^{1/2} \\ b &= \left[\frac{\{(1+4K^{-1})^{2} + 64E^{2}\}^{1/2} - (1+4K^{-1})}{2} \right]^{1/2} \\ a' &= \left[\frac{\{(1+4K^{-1} + 4n)^{2} + 64E^{2}\}^{1/2} - (1+4K^{-1} + 4n)}{2} \right]^{1/2} \\ b' &= \left[\frac{\{(1+4K^{-1} + 4n)^{2} + 64E^{2}\}^{1/2} - (1+4K^{-1} + 4n)}{2} \right]^{1/2} \\ D_{1} &= \frac{Pr + \sqrt{Pr^{2} - 4S}}{2}, D_{2} &= \frac{Pr + \sqrt{Pr^{2} - 4(n Pr + S)}}{2}, \\ D_{3} &= \frac{Sc + \sqrt{Sc^{2} - 4n Sc}}{2}, D_{4} &= \frac{1 + \sqrt{1 + 4(K^{-1} + 2iE)}}{2}, \\ D_{5} &= \frac{1 + \sqrt{1 + 4(K^{-1} + n + 2iE)}}{2} \\ R_{1} &= \frac{Pr}{D_{1}^{2} - Pr D_{1} + (n Pr + S)}, R_{2} &= -\frac{R_{1}D_{1}}{D_{2}}, \\ R_{3} &= \frac{D_{1}(1 + A) - Sc}{Sc (D_{1} - Sc)}, R_{4} &= \frac{A}{D_{1} - Sc}, \\ R_{5} &= \frac{R_{3}Sc}{n}, R_{6} &= \frac{R_{4}D_{1}Sc + A R_{1}D_{1}^{2}}{D_{1}^{2} - Sc D_{1} + nSc}, \\ R_{7} &= \frac{AR_{2}D_{2}^{2}}{D_{2}^{2} - Sc D_{2} + nSc}, R_{8} &= -\frac{R_{5}Sc + R_{6}D_{1} + R_{7}D_{2}}{D_{3}}, \\ R_{9} &= K_{1} + iK_{2}, R_{10} = K_{3} + i K_{4}, R_{11} = K_{9} + iK_{10}, \\ R_{12} &= K_{5} + iK_{6}, R_{13} = -Gr R_{2} + Gm R_{7}, R_{14} = Gm R_{8}, \\ \end{split}$$

$$\begin{split} & \mathsf{R}_{15} = \mathsf{K}_{11} + \mathsf{i} \mathsf{K}_{12}, \mathsf{R}_{16} = \mathsf{K}_{7} + \mathsf{i} \mathsf{K}_{8}, \mathsf{R}_{17} = \mathsf{K}_{13} + \mathsf{i} \mathsf{K}_{14}, \mathsf{R}_{18} = \mathsf{K}_{15} + \mathsf{i} \mathsf{K}_{16}, \\ & \mathsf{R}_{19} = \mathsf{K}_{17} + \mathsf{i} \mathsf{K}_{18}, \mathsf{R}_{20} = \mathsf{K}_{19} + \mathsf{i} \mathsf{K}_{20}, \mathsf{R}_{21} = \mathsf{K}_{21} + \mathsf{i} \mathsf{K}_{22}, \mathsf{R}_{22} = \mathsf{K}_{25} + \mathsf{i} \mathsf{K}_{26}, \\ & \mathsf{K}_{1} = \frac{(\mathsf{Cr} - \mathsf{D}_{1} \mathsf{Gm} \mathsf{R}_{4})(\mathsf{D}_{1}^{2} - \mathsf{D}_{1} - \mathsf{K}^{-1})^{2} + 4\mathsf{E}^{2}}{\mathsf{D}_{1}^{2} \{(\mathsf{D}_{1}^{2} - \mathsf{D}_{1} - \mathsf{K}^{-1})^{2} + 4\mathsf{E}^{2}\}}, \quad & \mathsf{K}_{2} = \frac{2\mathsf{E} (\mathsf{Gr} - \mathsf{D}_{1} \mathsf{Gm} \mathsf{R}_{4})}{\mathsf{D}_{1}^{2} \{(\mathsf{D}_{1}^{2} - \mathsf{D}_{1} - \mathsf{K}^{-1})^{2} + 4\mathsf{E}^{2}\}}, \\ & \mathsf{K}_{3} = \frac{(\mathsf{Sc}^{2} - \mathsf{Sc} - \mathsf{K}^{-1})\mathsf{Gm} \mathsf{R}_{3}}{(\mathsf{Sc}^{2} - \mathsf{Sc} - \mathsf{K}^{-1})^{2} + 4\mathsf{E}^{2}}, \quad & \mathsf{K}_{4} = \frac{2\mathsf{E} \mathsf{Gm} \mathsf{R}_{3}}{(\mathsf{Sc}^{2} - \mathsf{Sc} - \mathsf{K}^{-1})^{2} + 4\mathsf{E}^{2}}, \\ & \mathsf{K}_{5} = -\mathsf{Gr} \mathsf{R}_{1} + \mathsf{Gm} \mathsf{R}_{5} - \mathsf{K}_{1}\mathsf{D}_{1}, \quad & \mathsf{K}_{6} = -\mathsf{K}_{2}\mathsf{D}_{1}, \\ & \mathsf{K}_{7} = \mathsf{Gm} \mathsf{R}_{5} + \mathsf{Sc} \mathsf{K}_{3}, \quad & \mathsf{K}_{8} = \mathsf{Sc} \mathsf{K}_{4}, \\ & \mathsf{K}_{9} = \left[\frac{(\mathsf{1} + \mathsf{h} \mathsf{A}_{1})[\mathsf{K}_{1}(\mathsf{1} + \mathsf{h} \mathsf{D}_{1}) + \mathsf{K}_{3}(\mathsf{1} + \mathsf{h} \mathsf{Sc})] + \mathsf{h} \mathsf{A}_{2}[\mathsf{K}_{2}(\mathsf{1} + \mathsf{h} \mathsf{D}_{1}) + \mathsf{K}_{3}(\mathsf{1} + \mathsf{h} \mathsf{Sc})]}{(\mathsf{1} + \mathsf{h} \mathsf{A}_{1})^{2} + \mathsf{h}^{2} \mathsf{A}_{2}^{2}}\right], \\ & \mathsf{K}_{10} = \left[\frac{(\mathsf{I} + \mathsf{h} \mathsf{A}_{1})[\mathsf{K}_{2}(\mathsf{1} + \mathsf{h} \mathsf{D}_{1}) + \mathsf{K}_{4}(\mathsf{1} + \mathsf{h} \mathsf{Sc})] - \mathsf{h} \mathsf{A}_{2}[\mathsf{K}_{1}(\mathsf{1} + \mathsf{h} \mathsf{D}_{1}) + \mathsf{K}_{3}(\mathsf{1} + \mathsf{h} \mathsf{Sc})]}{(\mathsf{1} + \mathsf{h} \mathsf{A}_{1})^{2} + \mathsf{h}^{2} \mathsf{A}_{2}^{2}}\right], \\ & \mathsf{K}_{11} = \left\{\frac{\mathsf{K}_{1} \mathsf{K}(\mathsf{D}_{1}^{2} - \mathsf{D}_{1} - \mathsf{n} - \mathsf{K}^{-1}) + \mathsf{K}_{4}(\mathsf{1} + \mathsf{h} \mathsf{Sc})}{(\mathsf{D}_{1}^{2} - \mathsf{D}_{1} - \mathsf{n} - \mathsf{K}^{-1})^{2} + 4\mathsf{E}^{2}}\right\right], \\ & \mathsf{K}_{13} = \left[\frac{\mathsf{K}_{5} \{(\mathsf{D}_{1}^{2} - \mathsf{D}_{1} - \mathsf{n} - \mathsf{K}^{-1}) + \mathsf{E} \mathsf{K}_{5}}{(\mathsf{D}_{1}^{2} - \mathsf{D}_{1} - \mathsf{n} - \mathsf{K}^{-1})^{2} + 4\mathsf{E}^{2}}, \\ & \mathsf{K}_{13} = \left[\frac{\mathsf{K}_{6} \mathsf{K}(\mathsf{D}_{1}^{2} - \mathsf{D}_{1} - \mathsf{n} - \mathsf{K}^{-1}) + \mathsf{E} \mathsf{K}_{5}}{(\mathsf{D}_{2}^{2} - \mathsf{D}_{1} - \mathsf{n} - \mathsf{K}^{-1})^{2} + 4\mathsf{E}^{2}}, \\ & \mathsf{K}_{14} = \left[\frac{\mathsf{K}_{6} \mathsf{K}(\mathsf{D}_{1}^{2} - \mathsf{D}_{1} - \mathsf{D}$$

$$\begin{split} \mathbf{K}_{21} = & \left[\frac{\mathbf{K}_{7}(\mathbf{Sc}^{2} - \mathbf{Sc} - \mathbf{n} - \mathbf{K}^{-1}) - 2\mathbf{E} \, \mathbf{K}_{8}}{(\mathbf{Sc}^{2} - \mathbf{Sc} - \mathbf{n} - \mathbf{K}^{-1}) + 4\mathbf{E}^{2}} \right], \\ & \mathbf{K}_{22} = & \left[\frac{\mathbf{K}_{8}(\mathbf{Sc}^{2} - \mathbf{Sc} - \mathbf{n} - \mathbf{K}^{-1}) + 2\mathbf{E}\mathbf{K}_{7}}{(\mathbf{Sc}^{2} - \mathbf{Sc} - \mathbf{n} - \mathbf{K}^{-1})^{2} + 4\mathbf{E}^{2}} \right], \\ & \mathbf{K}_{23} = \begin{bmatrix} -\mathbf{K}_{13}(1 + \mathbf{h}\mathbf{D}_{1}) - \mathbf{K}_{15}(1 + \mathbf{h}\mathbf{D}_{2}) + \mathbf{K}_{17}(1 + \mathbf{h}\mathbf{D}_{3}) - \mathbf{K}_{19}(1 + \mathbf{h}\mathbf{A}_{1}) \\ & + \mathbf{K}_{20}\mathbf{h}\mathbf{A}_{2} + \mathbf{K}_{21}(1 + \mathbf{h}\mathbf{Sc}) \end{bmatrix}, \\ & \mathbf{K}_{24} = \begin{bmatrix} \mathbf{K}_{14}(1 + \mathbf{h}\mathbf{D}_{1}) + \mathbf{K}_{16}(1 + \mathbf{h}\mathbf{D}_{2}) - \mathbf{K}_{18}(1 + \mathbf{h}\mathbf{D}_{3}) - \mathbf{K}_{20}(1 + \mathbf{h}\mathbf{A}_{1}) \\ & + \mathbf{K}_{19}\mathbf{h}\mathbf{A}_{2} + \mathbf{K}_{22}(1 + \mathbf{h}\mathbf{Sc}) \end{bmatrix}, \\ & \mathbf{K}_{25} = \begin{bmatrix} \frac{\mathbf{K}_{23}(1 + \mathbf{h}\mathbf{A}_{3}) + \mathbf{K}_{24}\mathbf{h}\mathbf{A}_{4}}{(1 + \mathbf{h}\mathbf{A}_{3})^{2} + \mathbf{h}^{2}\mathbf{A}_{4}^{2}} \end{bmatrix}, \qquad \mathbf{K}_{26} = \begin{bmatrix} \frac{-\mathbf{K}_{24}(1 + \mathbf{h}\mathbf{A}_{3}) - \mathbf{K}_{23}\mathbf{h}\mathbf{A}_{4}}{(1 + \mathbf{h}\mathbf{A}_{3})^{2} + \mathbf{h}^{2}\mathbf{A}_{4}^{2}} \end{bmatrix}, \end{split}$$

V. Discussion And Conclusions

In order to get physical insight of the problem, calculations have been made for primary and secondary velocities (u,v), temperature (θ), concentration (ϕ), primary and secondary skin-frictions (τ_p, τ_s) and Nusselt number (Nu) for different parameters viz. Prandtl number (Pr), Schmidt Number (Sc), Permeability parameter (K). Heat source parameter (S), Grashof number for heat transfer (Gr), Grashof number for mass transfer (Gm), Diffusion parameter (A) and Slip parameter (h) fixing $\epsilon = 0.02$, E = 0.4, n = 0.1 and t = 1.0.

From Figure 1, important observation is that increase in h, increases the primary velocity near the plate but after some distance it decreases Physically it due to the fact that effect of slip parameter nullify as we go far from the plate. However, in case of Gm and S velocity increases as both the parameters decreases but Gr has opposite phenomena. From Figure 2, for secondary velocity v (in the figure we have plotted -v) the observations are same as that of primary velocity for the case of Gr, Gm and S but interestingly as h increases secondary velocity decrease near the plate but goes on increasing as we move away from the plate.

In figure 3 and 4 the primary and secondary velocities u and v are plotted for different values of Pr, K and Sc. From the figures we observed that an increase in Pr and Sc decreases primary but increases secondary velocity. It is further observed that for increasing K both velocities (u,v) increases. Physically, we say that increase in K increases the flow space in the porous medium and hence both velocities increase $(K = \infty \text{ is free flow})$.

The non-dimensional temperature θ for different values of S and Pr is shown in figure 5. From this figure it is concluded that an increase in Pr and S temperature decreases for both the basic fluids air (Pr = 0.71) and water (Pr = 7.0). It is interesting to note that temperature is always move for air (Pr = 0.71) as compared to water (Pr = 7.0) for every value of S. Moreover, for both the fluids temperature is move when heat is observed by the fluid (- S).

The concentration profile ϕ is shown in Figure 6 against z for different values of A, Pr and Sc. It is concluded that increase in Pr and Sc decreases concentration but for diffusion parameter A phenomena is opposite in nature.

Primary and secondary skin friction τ_p and τ_s plotted against S are shown in figures 7 and 8 for different values of h, Gr. Gm, Pr, K and Sc. It is noted that the primary skin friction at the plate decreases with the increase in h, Pr and Sc but increases with increase in K, Gr and Gm.

For secondary skin friction at the plate, it is observed that it increases with the increase in Gr, Gm, K while decreases with the increase in Sc, h and Pr. Nusselt number (Nu) is shown in Figure 9, plotted against S for the same fixed values of \in , n

And t. From the figure we observe that increase in Pr decreases the Nusselt number hence Nusselt number is more for air than water.

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Figure 1. : Primary Velocity Distribution U plotted against Z for different values of h, Gr, Gm and S



Figure 2. : Secondary Velocity Distribution -V plotted against Z for different values of h, Gr, Gm and S



Figure 3. : Primary Velocity Distribution U plotted against Z for different values of Pr, K and Sc



Figure 4. : Secondary Velocity Distribution -V plotted against Z for different values of Pr, K and Sc



Figure 5. : Temperature Distribution θ plotted against Z for different values of S and Pr.



Figure 6. : Concentration Profile ϕ plotted against ${\bm z}$ for different values of A, Pr and Sc.





Figure 8. : Skin Friction τ_{s} plotted against S for different values of h, Gr, Gm, Pr, K, and Sc

