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Efficiency of NNBD over NNBIBD using First Order Correlated Models

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Abstract: *Neighbour Balanced Block Designs, permitting the estimation of direct and neighbour effects, are used when the treatment applied to one experimental plot may affect the response on neighbouring plots as well as the response on the plot to which it is applied. The allocation of treatments in these designs is such that every treatment occurs equally often with every other treatment as neighbours. Neighbour Balanced Block Designs for observations correlated within a block have been investigated for the estimation of direct as well as left and right neighbour effects of treatments. It is observed that efficiency for direct as well as neighbour effects is high, in case of Complete block designs i.e.,* $(m=0)$ *for Nearest Neighbour correlation structure with* ρ *in the interval 0.1 to 0.4. In case of incomplete block designs* $(m=1,2,\ldots,v-4)$ *for Nearest Neighbour correlation structure turns out to be more efficient as compared to others models with* ρ *in the interval 0.1 to 0.4. The performance of Nearest neighbour balanced block designs is satisfactory for ARMA(1,1) models. The gain in efficiency of NNBD and NNBIBD over regular block design is high under MA(1) models for direct and neighbour effects of treatments.*

Keywords*: Neighbour Balanced Block Design; Correlated observations; Generalized least squares; AutoRegressive; Moving Average; Nearest neighbour; Efficiency; Regular Block Design.*

I. Introduction:

Experiments conducted in agriculture often show neighbour effects i.e., the response on a given plot is affected by the treatments on the neighbouring plots as well as by the treatment applied to that particular plot. When treatments are varieties, neighbour effects may be caused by differences in height, root vigor, or germination date especially on small plots, which are used in plant breeding experiments. Treatments such as fertilizer, irrigation, or pesticide may spread to adjacent plots causing neighbour effects. Such experiments exhibit neighbour effects, because the effect of having no treatment as a neighbour is different from the neighbour effects of any treatment. Competition or interference between neighbouring units in field experiments can contribute to variability in experimental results and lead to substantial losses in efficiency. In case of block design setup, if each block is a single line of plots and blocks are well separated, extra parameters are needed for the effect of left and right neighbours. An alternative is to have border plots on both ends of every block. Each border plot receives an experimental treatment, but it is not used for measuring the response variable. These border plots do not add too much to the cost of one-dimensional experiments.

Neighbour balanced block designs, where in the allocation of treatments is such that every treatment occurs equally often with every other treatment as neighbours, are used for modeling and controlling interference effects between neighbouring plots. Azais *et al*. (1993) obtained a series of efficient neighbour designs with border plots that are balanced in $v-1$ blocks of size v and v blocks of size $v-1$, where v is the number of treatments. In the analysis of data from comparative experiments, the correlation structure among observation is often unknown. If due care is taken in allocating the treatments by randomization, then the usual analysis assuming independence of observations is approximately valid, Kempthorne (1952). However, in situations where the correlation structure is known it may be advantageous to use this information in designing an experiment and analyzing the data so as to make more precise inference, Williams (1952); Bartlett (1978); Kiefer and Wynn (1981); Martin (1982); Wilkinson et al. (1983). In field experiments block of small sizes are effective in removing heterogeneity due to variation in fertility. In some cases blocks may represent additive effects not necessarily related to local fertility variation. The effect of correlation on the usual two-way analysis of variance and on the power of usual tests has been studied by Box (1954); Anderson, Jenson and Schou (1981) and Aastveit (1983). Highly efficient neighbour balanced designs are introduced by Kunert (1987) for positive correlated structure.

The construction of nearest neighbour balanced designs with partial variance balance was introduced by Morgan and Chakravarti, (1988). Optimal and highly efficient two dimensional designs were constructed for correlated errors on the torus and in the plane by Morgen and Nizamuddin (1991). Druilhet (1999) studied optimality of circular neighbour balanced block designs obtained by Azais *et al*. (1993). Bailey (2003) has given some designs for studying one-sided neighbour effects. These neighbour balanced block designs have been

developed under the assumption that the observations within a block are uncorrelated. In situations where the correlation structure among the observations within a block is known, may be from the data of past similar experiments, it may be advantageous to use this information in designing an experiment and analyzing the data so as to make more precise inference about treatment effects Gill and Shukla (1985). Kunert *et al*. (2003) considered two related models for interference and have shown that optimal designs for one model can be obtained from optimal designs for the other model. Martin and Eccelston (2004) have given variance balanced designs under interference and dependent observations. Tomar and Seema Jaggi (2007) observed that efficiency is quite high, in case of complete block designs for both AR(1) and NN correlation structures. Santharam.C & K.N.Ponnuswamy (1997) observed that the performance of NNBD is quite satisfactory for the remaining models. In this paper, neighbour balanced block designs for observations correlated within a block have been investigated for the estimation of direct as well as left and right neighbour effects of treatments. The performance of these designs for AR(1), MA(1) and ARMA(1,1) and NN error correlation structure is studied when generalized least squares estimation is used. We have also investigated the efficiency of Nearest Neighbour Balanced Block Design (NNBD) and Nearest Neighbour Balanced Incomplete Block Design (NNBIBD) in comparison to regular block design when the error follows first order correlated models.

II. Model Structures And Information Matrix:

Let Δ be a class of binary neighbour balanced block designs with $n = bk$ units that form b blocks each containing k units. Y_{ij} be the response from the i^{th} plot in the j^{th} block $(i = 1,2,\ldots,k; j = 1,2,\ldots,b)$. The layout includes border plots at both ends of every block, i.e. at 0^{th} and $(k+1)^{th}$ position and observations for these units are not modeled. The following fixed effects additive model is considered for analyzing a neighbour balanced block design under correlated observations:

$$Y_{ij} = \mu + \tau_{(i,j)} + l_{(i-1,j)} + \gamma_{(i+1,j)} + \beta_j + e_{ij} \qquad (2.1)$$

where μ is the general mean, $\tau_{(i,j)}$ is the direct effect of the treatment in the i^{th} plot of j^{th} block, β_j is the effect of the j^{th} block. $l_{(i-1,j)}$ is the left neighbour effect due to the treatment in the $(i-1)^{th}$ plot of j^{th} block. $\gamma_{(i+1,j)}$ is the right neighbour effect due to the treatment in the $(i+1)^{th}$ plot in j^{th} block. e_{ij} are error terms distributed with mean zero and a variance-covariance structure $\Omega = I_b \otimes \Lambda$ (I_b is an identity matrix of order b and $\otimes$ denotes the kronecker product). Assuming no correlation among the observations between the blocks and correlation structure between plots within a block to be the same in each block, Λ is the correlation matrix of k observations within a block. The error-in variable model Besag (1977) is closely related to the smooth trend plus error model of Wilkinson *et al*. (1983). This is a general model which gives a better fit in situations where the error structure is non stationary Besag (1977); Wilkinson *et al*. (1983) Patterson (1983). Gill and Shukla (1985) studied universal optimality of NNBD using AR(1) and MA(1) models. The ARMA(1,1) model along with AR(1) and MA(1) and explored the performance of NNBD for $\rho = -0.4(-0.4)0.4$. If the errors within a block follow a **AR(1) structure**, then Λ is a $k \times k$ matrix with $(i,i')^{th}$ entry $(i,i' = 1,2,\ldots,k)$ as $\rho^{|i-i'|}$, $|\rho| < 1$. The **MA(1) structure**, then Λ is a matrix with diagonal entries as 1 and $(i,i')^{th}$ entry $(i,i' = 1,2,\ldots,k)$ as ρ, when $|i-i'| = 1$, otherwise zero Gill and Shukla, (1985). If the errors within a block follow an **ARMA(1,1) model** then $\Omega = I_b \otimes \Lambda$. Where I_b is an identity matrix of order b and $\Lambda = \begin{bmatrix} r_0 & r_1 & r_2 & \ldots & r_{k-1} \\ r_1 & r_0 & r_1 & \ldots & r_{k-2} \\ r_2 & r_1 & r_0 & \ldots & r_{k-3} \\ & & & \vdots & \\ r_{k-1} & r_{k-2} & r_{k-3} & \ldots & r_0 \end{bmatrix}$, where $r_0 = \dfrac{1 + 2\rho_1\rho_2 + \rho_2^2}{1 - \rho_1^2}$,

$r_1 = \dfrac{\rho_1\left(1+\rho_2^2\right)+\rho_2\left(1+\rho_1^2\right)}{1-\rho_1^2}$, $r_k = \rho_1^r\left(k-1\right)$ for $k \geq 2$, Santharam.C & K.N.Ponnuswamy (1997). The **NN correlation structure**, the Λ is a matrix with diagonal entries as 1 and off-diagonal entries as ρ.

Model (2.1) can be rewritten in the matrix notation as follows:

$$Y = \mu 1 + \Delta'\tau + \Delta_1' l + \Delta_2'\gamma + D'\beta + e \tag{2.2}$$

where Y is $n\times 1$ vector of observations, 1 is $n\times 1$ vector of ones, Δ' is an $n\times v$ incidence matrix of observations versus direct treatments, τ is $v\times 1$ vector of direct treatment effects, Δ_1' is a $n\times v$ matrix of observations versus left neighbour treatment, Δ_2' is a $n\times v$ matrix of observations versus right neighbour treatment, l is $v\times 1$ vector of left neighbour effects, γ is $v\times 1$ vector of right neighbour effects, D' is an $n\times b$ incidence matrix of observations versus blocks, β is $b\times 1$ vector of block effects and e is $n\times 1$ vector of errors. The joint information matrix for estimating the direct and neighbour (left and right) effects under correlated observations estimated by generalized least squares is obtained as follows:

$$C = \begin{bmatrix} \Delta(I_b \otimes \wedge^*)\Delta' & \Delta(I_b \otimes \wedge^*)\Delta_1' & \Delta(I_b \otimes \wedge^*)\Delta_2' \\ \Delta_1(I_b \otimes \wedge^*)\Delta' & \Delta_1(I_b \otimes \wedge^*)\Delta_1' & \Delta_1(I_b \otimes \wedge^*)\Delta_2' \\ \Delta_2(I_b \otimes \wedge^*)\Delta' & \Delta_2(I_b \otimes \wedge^*)\Delta_1' & \Delta_2(I_b \otimes \wedge^*)\Delta_2' \end{bmatrix} \tag{2.3}$$

with

$$\wedge^* = \wedge^{-1} - \left(1_k' \wedge^{-1} 1_k\right)^{-1} \wedge^{-1} 1_k 1_k' \wedge^{-1}$$

The above $3v\times 3v$ information matrix (C) for estimating the direct effects and neighbour effects of treatments in a block design setting is symmetric, non-negative definite with row and column sums equal to zero. The information matrix for estimating the direct effects of treatments from (2.3) is as follows:

$$C_\tau = C_{11} - C_{12}C_{22}^{-1}C_{21} \tag{2.4}$$

where

$$C_{11} = \Delta(I_b \otimes \wedge^*)\Delta'$$
$$C_{12} = \begin{bmatrix} \Delta(I_b \otimes \wedge^*)\Delta_1' & \Delta(I_b \otimes \wedge^*)\Delta_2' \end{bmatrix} \quad \text{and}$$
$$C_{22} = \begin{bmatrix} \Delta_1(I_b \otimes \wedge^*)\Delta_1' & \Delta_1(I_b \otimes \wedge^*)\Delta_2' \\ \Delta_2(I_b \otimes \wedge^*)\Delta_1' & \Delta_2(I_b \otimes \wedge^*)\Delta_2' \end{bmatrix}$$

Similarly, the information matrix for estimating the left neighbour effect of treatments (C_l) and right neighbour effect of treatments (C_γ) can be obtained.

2.1 Construction Of Design:

Tomer *et al*. (2005) has constructed neighbour balanced block design with parameters v (prime or prime power), $b = v(v-1), r = (v-1)(v-m)$, $k = (v-m)$, $m = 1,2,\ldots,v-4$ and $\lambda = (v-m)$ using Mutually Orthogonal Latin Squares (MOLS) of order v. This series of design bas been investigated under the correlated error structure. It is seen that the design turns out to be pair-wise uniform with $\alpha = 1$ and also variance balanced for estimating direct (V_1) and neighbour effects $(V_2 = V_3)$.

III. Comparison Of Efficiency:

In this section, a quantitative measure of efficiency of the designs in compared to the universally optimal neighbour balanced design for v treatments in $(v-1)$ complete blocks of Azais *et al*. (1993) considering observations to be correlated within the blocks. We compare the average variance of an elementary treatment contrast $\hat{\tau}_s - \hat{\tau}_{s'}$ in both cases. The average variance of an elementary treatment contrast

Kempthorne (1956) for direct effects of the neighbour balanced design of Azais *et al*. (1993) estimated by generalized least squares methods, is given by

$$V_A = \frac{2\sigma^2}{v-1}\sum_{s=1}^{v-1}\theta_s^{-1}$$

Where θ_s's are the $(v-1)$ non-zero eigen values of C_τ for Azais *et al*. (1993), σ^2 is the variance of an observation. The efficiency factor (E_τ) for direct effects of the neighbour balanced pair-wise uniform block design is thus given as:

$$E_\tau = \frac{(v-1)\sum_{s=1}^{v-1}\theta_s^{-1}}{(v-m)\sum_{s=1}^{v-1}\delta_s^{-1}}$$

δ_s's are the $(v-1)$ non-zero eigen values of C_τ. Similarly the efficiency (E_l) and (E_γ) for neighbour effects (left and right) of treatments is obtained. The ranges of correlation coefficient (ρ) for different correlation structures investigated are $|\rho| \le 0.40$ for AR(1), MA(1), ARMA(1,1) and NN correlation structures. For these ranges, the matrix of correlation coefficients among observations within a block is positive definite. For $\rho = 0$, the efficiency is that of totally balanced designs obtained by Tomer *et al*. (2005). These matrices have been worked out using **R** package.

In Tables 1, 2, 3 & 4, the parameters of neighbour balanced pair-wise uniform block design for $v = 5$ $(m = 0,1)$ and $v = 6$ $(m = 0,1,2)$ along with the efficiency for direct and neighbour effects (left and right) has been shown. The efficiency values have been reported under the AR(1), MA(1), ARMA(1,1) and NN correlation structures with ρ in the interval -0.4 to 0.4. It is seen that efficiency for direct as well as neighbour effects is high, in case of Complete block designs i.e., $(m = 0)$ for NN correlation structure with ρ in the interval 0.1 to 0.4. In case of incomplete block designs $(m = 1,2,\ldots,v-4)$ for NN correlation structure turns out to be more efficient as compared to others models with ρ in the interval 0.1 to 0.4. The performance of Nearest neighbour balanced block designs is satisfactory for ARMA(1,1) models. Moreover, as the block size increases, the efficiency also increases because the plots in a block become more heterogeneous. So we concluded that when block sizes are large and neighbouring plots are highly correlated, generalized least squares for estimation of direct and neighbour effects can be used.

3.1. Efficiency of NNBD and NNBIBD in comparison to Regular Block Design (RBD):

For a regular block design the variance of an elementary treatment contrast, estimated by ordinary least squares methods, is given by

$$V_1 = 2r^{-1}\sigma^2(1-\bar{\rho})$$

where σ^2 is the variance of an observation, $\bar{\rho}$ is the average correlation between observations from any two plots within a block, the average being taken over all possible randomizations; r is the number of replications. If observations within a block follow an errors-in-variable autoregressive model, then Williams (1952)

$$\sigma^2 = \sigma_\varepsilon^2\left(1+\alpha(1-\rho^2)(1-\rho^2)^{-1}\right)$$

and

$$\bar{\rho} = \frac{2\rho}{\{1+\alpha(1-\rho^2)\}(t-1)(1-\rho)}\left\{1-\frac{1-\rho^t}{t(1-\rho)}\right\}$$

so that

$$V_1 = \frac{2\sigma_\varepsilon^2}{r(1-\rho^2)}\left[1+\alpha(1-\rho^2)-\frac{2}{(t-1)(1-\rho)}\left\{1-\frac{(1-\rho^t)}{t(1-\rho)}\right\}\right]$$

When errors follow a first order moving average model, then

$$V_1 = 2r^{-1}\sigma_\varepsilon^2(1+\rho^2)\left(1+\frac{\alpha}{1+\rho^2}-\frac{2}{(1+\rho^2)t}\right)$$

When errors follow first order ARMA model, then

$$V_1 = 2r^{-1}\sigma_\varepsilon^2 r_0\left(1+\frac{\alpha}{r_0}-\frac{2}{t(t-1)}\sum_{s=1}^{(t-1)}(t-s)\gamma_s\right)$$

where $\alpha = \dfrac{\sigma_\eta^2}{\sigma_\varepsilon^2}$

For generalized least squares estimation, the average variance of an elementary treatment contrast, estimated from a design d, is

$$V_2 = \frac{2\sigma^2}{v-1}\sum_{s=1}^{v-1}\theta_s^{-1}$$

where θ_s's are non-zero values of C_d (Kempthorne, 1956).

We define the efficiency of a design d relative to a regular block design as V_1/V_2.

The Tables 5 and 6 shows the efficiencies of NNBD with $t=5, r=20$ and $t=6, r=30$, $\rho=-0.4$ to 0.4 and $\alpha=1$. The Tables 7 and 8 shows the efficiencies of NNBIBD with $t=5, r=16$ and $t=6, r=25$, $\rho=-0.4$ to 0.4 and $\alpha=1$. The values in the tables show that as ρ increases from 0.1 to 0.4 the gain in efficiency also increases under AR(1), MA(1) and ARMA(1,1) models. The gain in efficiency of NNBD and NNBIBD over regular block design is high under MA(1) models for direct and neighbour effects of treatments.

IV. Tables:

Table 1. Efficiency of neighbour balanced pair-wise uniform block designs AR(1) model:

Parameters					Correlation Structure AR(1)			
v	b	m	r	$k=\lambda$	ρ	E_τ	E_l	E_γ
5	20	0	20	5	-0.4	0.68287	0.63750	0.64130
					-0.3	0.70822	0.69768	0.71440
					-0.2	0.77781	0.75809	0.73164
					-0.1	0.77054	0.76368	0.76484
					0.0	0.80000	0.80000	0.80000
					0.1	0.82721	0.82792	0.82486
					0.2	0.88525	0.86802	0.86477
					0.3	0.94868	0.95093	0.96874
					0.4	1.02361	0.98896	0.94436
5	20	1	16	4	-0.4	0.91927	0.77656	0.78475
					-0.3	0.90071	0.89407	0.84008
					-0.2	0.92510	0.87165	0.87467
					-0.1	0.91861	0.95754	0.95331
					0	1.00000	1.00000	1.00000
					0.1	1.06857	1.02668	1.06492
					0.2	1.15298	1.13899	1.13881
					0.3	1.26678	1.27000	1.26607
					0.4	1.43687	1.36980	1.37313
6	30	0	30	6	-0.4	0.75486	0.77745	0.70181
					-0.3	0.76514	0.53448	0.76891
					-0.2	0.77722	0.78902	0.79626
					-0.1	0.79679	0.80511	0.80286
					0.0	0.80000	0.80000	0.80000
					0.1	0.86984	0.86448	0.85899
					0.2	0.90461	0.80481	0.90171
					0.3	0.95909	0.93856	0.94975
					0.4	1.03151	0.99695	1.01706
6	30	1	25	5	-0.4	0.94070	0.90147	0.92474
					-0.3	0.94336	0.93130	0.97118
					-0.2	0.95051	0.94902	0.92849
					-0.1	0.96449	0.92726	0.99197
					0	1.00000	1.00000	1.00000
					0.1	1.04849	1.05117	1.05435

					0.2	1.12566	1.12695	1.11943
					0.3	1.22454	1.20896	1.22333
					0.4	1.36957	1.34255	1.34608
6	30	2	20	4	-0.4	1.28188	1.24096	1.26246
					-0.3	1.24104	1.27744	1.27200
					-0.2	1.14425	1.27770	1.27733
					-0.1	1.25492	1.29384	1.28132
					0	1.25000	1.25000	1.25000
					0.1	1.13327	1.32053	1.35493
					0.2	1.40332	1.42642	1.42774
					0.3	1.50318	1.51162	1.39422
					0.4	1.50767	1.63851	1.66997

Table 2. Efficiency of neighbour balanced pair-wise uniform block designs MA(1) model:

Parameters					Correlation Structure			
					MA(1)			
v	b	m	r	$k=\lambda$	ρ	E_τ	E_l	E_γ
5	20	0	20	5	-0.4	0.66631	0.63644	0.66063
					-0.3	0.71832	0.69436	0.69760
					-0.2	0.76633	0.70641	0.71013
					-0.1	0.76103	0.77614	0.74742
					0.0	0.80000	0.80000	0.80000
					0.1	0.81629	0.84870	0.83885
					0.2	0.86839	0.86158	0.86363
					0.3	0.90889	0.91996	0.88717
					0.4	0.93365	0.88157	0.88499
5	20	1	16	4	-0.4	0.83067	0.79043	0.82266
					-0.3	0.85458	0.89807	0.87499
					-0.2	0.91680	0.90371	0.90170
					-0.1	0.94057	0.91259	0.94392
					0	1.00000	1.00000	1.00000
					0.1	1.01894	1.08297	1.08979
					0.2	1.13295	1.15707	1.12392
					0.3	1.23253	1.17668	1.18523
					0.4	1.35526	1.26243	1.26446
6	30	0	30	6	-0.4	0.72261	0.69922	0.71010
					-0.3	0.73073	0.73851	0.75539
					-0.2	0.77151	0.78147	0.77456
					-0.1	0.79159	0.80102	0.81227
					0.0	0.80000	0.80000	0.80000
					0.1	0.88099	0.85001	0.86476
					0.2	0.89240	0.89178	0.88415
					0.3	0.92133	0.91276	0.90509
					0.4	0.94970	0.87412	0.92125
6	30	1	25	5	-0.4	0.89115	0.86763	0.90908
					-0.3	0.91019	0.91106	0.97813
					-0.2	0.93529	0.93476	0.93494
					-0.1	0.97401	0.97891	0.96524
					0	1.00000	1.00000	1.00000
					0.1	1.05547	1.06322	1.05580
					0.2	1.11470	1.10227	1.12250
					0.3	1.18969	1.17620	1.18041
					0.4	1.29669	1.24530	1.27229
6	30	2	20	4	-0.4	1.16993	1.19977	1.20830
					-0.3	1.19463	1.22385	1.25078
					-0.2	1.21384	1.26023	1.25781

-0.1	1.26544	1.30056	1.31168
0	1.25000	1.25000	1.25000
0.1	1.32778	1.31478	1.33994
0.2	1.38415	1.39339	1.35076
0.3	1.46225	1.23450	1.39011
0.4	1.54402	1.27473	1.40892

Table 3. Efficiency of neighbour balanced pair-wise uniform block designs ARMA(1,1) model:

Parameters					Correlation Structure ARMA(1,1)			
v	b	m	r	$k=\lambda$	ρ	E_τ	E_l	E_γ
5	20	0	20	5	(-0.4,-0.4)	0.32275	0.41998	0.34200
					(-0.3,-0.3)	0.44297	0.25651	0.27193
					(-0.2,-0.2)	0.58318	0.58497	0.55448
					(-0.1,-0.1)	0.69314	0.70919	0.71026
					(0,0)	0.80000	0.80000	0.80000
					(0.1,0.1)	0.83073	0.82819	0.82177
					(0.2,0.2)	0.78885	0.73788	0.75456
					(0.3,0.3)	0.63023	0.34204	0.18718
					(0.4,0.4)	0.58709	0.06418	0.30422
5	20	1	16	4	(-0.4,-0.4)	0.41672	0.26870	0.40328
					(-0.3,-0.3)	0.59656	0.49940	0.54387
					(-0.2,-0.2)	0.63500	0.74506	0.71194
					(-0.1,-0.1)	0.89426	0.86752	0.85693
					(0,0)	1.00000	1.00000	1.00000
					(0.1,0.1)	1.04927	1.09501	1.07778
					(0.2,0.2)	1.10968	1.03478	1.05581
					(0.3,0.3)	0.66680	0.57787	0.87287
					(0.4,0.4)	0.94687	0.81728	0.60362
6	30	0	30	6	(-0.4,-0.4)	0.36038	0.42080	0.62225
					(-0.3,-0.3)	0.49118	0.36038	0.45027
					(-0.2,-0.2)	0.62365	0.60793	0.61215
					(-0.1,-0.1)	0.83020	0.74801	0.74261
					(0,0)	0.80000	0.80000	0.80000
					(0.1,0.1)	0.86001	0.84611	0.84876
					(0.2,0.2)	0.80037	0.75366	0.75237
					(0.3,0.3)	0.65451	0.70426	0.60153
					(0.4,0.4)	0.56920	0.70866	0.43601
6	30	1	25	5	(-0.4,-0.4)	0.41672	0.26870	0.40328
					(-0.3,-0.3)	0.59656	0.49940	0.54387
					(-0.2,-0.2)	0.63500	0.74506	0.71194
					(-0.1,-0.1)	0.89426	0.86752	0.85693
					(0,0)	1.00000	1.00000	1.00000
					(0.1,0.1)	1.04927	1.09501	1.07778
					(0.2,0.2)	1.10968	1.03478	1.05581
					(0.3,0.3)	0.66680	0.57787	0.87287
					(0.4,0.4)	0.94687	0.81728	0.60362
6	30	2	20	4	(-0.4,-0.4)	0.60324	0.55304	0.59972
					(-0.3,-0.3)	0.80020	0.80436	0.80408
					(-0.2,-0.2)	1.00430	1.06373	1.02336
					(-0.1,-0.1)	1.16018	1.20070	1.13169
					(0,0)	1.25000	1.25000	1.25000
					(0.1,0.1)	1.32576	1.28337	1.28064
					(0.2,0.2)	1.27443	1.07083	1.18289
					(0.3,0.3)	1.17668	1.15822	0.81134
					(0.4,0.4)	0.95160	1.40105	1.47460

Table 4. Efficiency of neighbour balanced pair-wise uniform block designs NN model:

Parameters					Correlation Structure			
					NN			
v	b	m	r	$k=\lambda$	ρ	E_τ	E_l	E_γ
5	20	0	20	5	-0.4	0.56287	0.53710	0.56352
					-0.3	0.62530	0.60242	0.58364
					-0.2	0.66174	0.65598	0.66174
					-0.1	0.71680	0.71779	0.72109
					0.0	0.80000	0.80000	0.80000
					0.1	0.89193	0.94829	0.90732
					0.2	0.99359	0.99120	1.02672
					0.3	1.13873	1.17027	1.15689
					0.4	1.37226	1.36867	1.33303
5	20	1	16	4	-0.4	0.71506	0.69636	0.69396
					-0.3	0.75236	0.74652	0.75591
					-0.2	0.82142	0.84158	0.82263
					-0.1	0.91183	0.91525	0.89459
					0	1.00000	1.00000	1.00000
					0.1	1.08703	1.12496	1.12064
					0.2	1.26785	1.26785	1.22112
					0.3	1.46534	1.45527	1.46279
					0.4	1.71428	1.71428	1.71428
6	30	0	30	6	-0.4	0.58753	0.58692	0.58983
					-0.3	0.63785	0.64068	0.62985
					-0.2	0.68384	0.69532	0.65391
					-0.1	0.75234	0.76089	0.77723
					0.0	0.80000	0.80000	0.80000
					0.1	0.93761	0.91002	0.93524
					0.2	1.05008	1.04844	1.04667
					0.3	1.20284	1.20351	1.20192
					0.4	1.40943	1.40854	1.40744
6	30	1	25	5	-0.4	0.71225	0.71208	0.71123
					-0.3	0.77117	0.76562	0.77063
					-0.2	0.83426	0.82881	0.83152
					-0.1	0.91387	0.91031	0.81473
					0	1.00000	1.00000	1.00000
					0.1	1.12770	1.12770	1.11300
					0.2	1.27373	1.27373	1.27373
					0.3	1.45613	1.46291	1.21291
					0.4	1.71922	1.72950	1.50969
6	30	2	20	4	-0.4	0.89534	0.90183	0.88105
					-0.3	0.96847	0.99492	0.98984
					-0.2	1.07332	1.12483	1.06958
					-0.1	1.15528	1.20506	1.19129
					0	1.25000	1.25000	1.25000
					0.1	1.39981	1.39783	1.47445
					0.2	1.61141	1.73902	1.64013
					0.3	1.86870	1.92505	1.93073
					0.4	2.14480	2.26269	2.26082

Table 5. Efficiency of NNBD in comparison to Regular Block Designs:
$t = 5, r = 20 \,\&\, \alpha = 1$

ERROR MODEL		$\rho = -0.4$	$\rho = -0.3$	$\rho = -0.2$	$\rho = -0.1$	$\rho = 0$	$\rho = 0.1$	$\rho = 0.2$	$\rho = 0.3$	$\rho = 0.4$
AR(1)	E_τ	1.47814	1.46146	1.54845	1.49238	1.51678	1.54253	1.62946	1.72863	1.85063
	E_i	1.37991	1.43972	1.50920	1.47910	1.51678	1.54385	1.59775	1.73273	1.78799
	E_γ	1.38815	1.47422	1.45654	1.48135	1.51678	1.53814	1.59176	1.76518	1.76736
MA(1)	E_τ	1.38965	1.43854	1.48927	1.45192	1.51678	1.55734	1.68762	1.82016	1.94719
	E_i	1.32734	1.39055	1.37283	1.48074	1.51678	1.61918	1.67438	1.84234	1.83859
	E_γ	1.37779	1.39703	1.38005	1.42594	1.51678	1.60039	1.67836	1.77666	1.84571
ARMA(1,1)		$\rho_1 = -0.4$ $\rho_2 = -0.4$	$\rho_1 = -0.3$ $\rho_2 = -0.3$	$\rho_1 = -0.2$ $\rho_2 = -0.2$	$\rho_1 = -0.1$ $\rho_2 = -0.1$	$\rho_1 = 0$ $\rho_2 = 0$	$\rho_1 = 0.1$ $\rho_2 = 0.1$	$\rho_1 = 0.2$ $\rho_2 = 0.2$	$\rho_1 = 0.3$ $\rho_2 = 0.3$	$\rho_1 = 0.4$ $\rho_2 = 0.4$
	E_τ	0.98867	1.20145	1.44753	1.62780	1.84604	1.96140	1.97832	1.73609	1.86451
	E_i	1.32456	0.97660	1.45076	1.66676	1.84604	1.95521	1.84707	1.92859	1.72272
	E_γ	1.04873	0.91748	1.37301	1.66932	1.84604	1.94001	1.88997	1.49353	1.94887

Table 6. Efficiency of NNBD in comparison to Regular Block Designs:
$t = 6, r = 30 \,\&\, \alpha = 1$

ERROR MODEL		$\rho = -0.4$	$\rho = -0.3$	$\rho = -0.2$	$\rho = -0.1$	$\rho = 0$	$\rho = 0.1$	$\rho = 0.2$	$\rho = 0.3$	$\rho = 0.4$
AR(1)	E_τ	1.25504	1.26007	1.26649	1.56936	1.61061	1.65862	1.70921	1.80218	1.93826
	E_i	1.21662	1.1182	1.59451	1.58574	1.61061	1.64840	1.52066	1.76360	1.87332
	E_γ	1.53873	1.60865	1.60915	1.58132	1.61061	1.63793	1.70374	1.78462	1.91110
MA(1)	E_τ	1.24456	1.29938	1.35342	1.46817	1.61061	1.82305	2.07825	2.45032	2.98558
	E_i	1.24326	1.30514	1.37614	1.47943	1.61061	1.76939	2.07500	2.45168	2.98370
	E_γ	1.24944	1.28308	1.35356	1.51121	1.61061	1.81843	2.07151	2.44845	2.98136
ARMA(1,1)		$\rho_1 = -0.4$ $\rho_2 = -0.4$	$\rho_1 = -0.3$ $\rho_2 = -0.3$	$\rho_1 = -0.2$ $\rho_2 = -0.2$	$\rho_1 = -0.1$ $\rho_2 = -0.1$	$\rho_1 = 0$ $\rho_2 = 0$	$\rho_1 = 0.1$ $\rho_2 = 0.1$	$\rho_1 = 0.2$ $\rho_2 = 0.2$	$\rho_1 = 0.3$ $\rho_2 = 0.3$	$\rho_1 = 0.4$ $\rho_2 = 0.4$
	E_τ	1.12047	1.32899	1.53475	1.93334	1.89949	2.00399	2.12174	1.78502	2.05476
	E_i	1.30707	0.96501	1.49379	1.73824	1.89949	1.97088	1.86175	1.92420	2.23690
	E_γ	1.93195	1.21224	1.50540	1.72580	1.89949	1.97732	1.85941	1.63881	1.36079

Table 7. Efficiency of NNBIBD in comparison to Regular Block Designs:

$t = 5, r = 16 \,\&\, \alpha = 1$

ERROR MODEL		$\rho=-0.4$	$\rho=-0.3$	$\rho=-0.2$	$\rho=-0.1$	$\rho=0$	$\rho=0.1$	$\rho=0.2$	$\rho=0.3$	$\rho=0.4$
AR(1)	E_z	2.40406	2.24291	1.94392	1.78779	1.75000	1.63277	1.84430	1.92974	2.09111
	E_l	1.60599	2.14795	2.16361	1.67788	1.75000	1.34688	1.80453	1.95517	2.45095
	E_y	1.68570	1.06782	1.55923	1.45009	1.75000	1.96296	2.05801	2.54910	2.76561
MA(1)	E_z	0.66485	1.18572	1.40207	1.56814	1.75000	1.89563	2.05108	2.70763	2.88273
	E_l	0.61608	0.98563	1.25378	1.06007	1.75000	1.99853	2.24473	2.46932	3.04925
	E_y	0.40613	0.61851	1.06635	1.27072	1.75000	2.23253	2.66118	3.93841	4.99749
ARMA(1,1)		$\rho_1=-0.4$ $\rho_2=-0.4$	$\rho_1=-0.3$ $\rho_2=-0.3$	$\rho_1=-0.2$ $\rho_2=-0.2$	$\rho_1=-0.1$ $\rho_2=-0.1$	$\rho_1=0$ $\rho_2=0$	$\rho_1=0.1$ $\rho_2=0.$	$\rho_1=0.2$ $\rho_2=0.2$	$\rho_1=0.3$ $\rho_2=0.3$	$\rho_1=0.4$ $\rho_2=0.4$
	E_z	1.02213	1.21753	1.31032	1.00917	0.93750	0.77900	0.99777	1.20131	1.91861
	E_l	0.33385	0.67311	0.73347	0.89404	0.93750	0.88992	1.12436	1.68039	1.92382
	E_y	0.36534	0.42791	0.69098	0.83773	0.93750	1.20479	1.70126	2.60086	2.70172

Table 8. Efficiency of NNBIBD in comparison to Regular Block Designs:

$t = 6, r = 25 \,\&\, \alpha = 1$

ERROR MODEL		$\rho=-0.4$	$\rho=-0.3$	$\rho=-0.2$	$\rho=-0.1$	$\rho=0$	$\rho=0.1$	$\rho=0.2$	$\rho=0.3$	$\rho=0.4$
AR(1)	E_z	2.47063	2.00443	1.77158	1.55059	1.45713	1.45461	1.43150	1.46008	1.70143
	E_l	1.59125	1.39874	1.43204	1.64688	1.45713	1.53177	1.77508	1.96795	2.25002
	E_y	1.22794	1.81426	1.71675	1.28532	1.45713	1.57240	1.84319	2.00079	2.39380
MA(1)	E_z	0.73380	0.93913	1.15138	1.38203	1.45713	1.59347	1.63743	1.90847	2.34649
	E_l	0.39902	0.67674	0.95515	1.14646	1.45713	1.87677	2.07172	2.62201	3.48893
	E_y	0.50268	0.71305	0.94363	1.18529	1.45713	1.76199	2.25254	3.03607	4.76615
ARMA(1,1)		$\rho_1=-0.4$ $\rho_2=-0.4$	$\rho_1=-0.3$ $\rho_2=-0.3$	$\rho_1=-0.2$ $\rho_2=-0.2$	$\rho_1=-0.1$ $\rho_2=-0.1$	$\rho_1=0$ $\rho_2=0$	$\rho_1=0.1$ $\rho_2=0.1$	$\rho_1=0.2$ $\rho_2=0.2$	$\rho_1=0.3$ $\rho_2=0.3$	$\rho_1=0.4$ $\rho_2=0.4$
	E_z	1.23894	1.08918	0.94535	0.84534	0.76832	0.75204	0.76445	0.90912	1.89671
	E_l	0.52647	0.55659	0.62387	0.65892	0.76823	0.94086	1.07837	2.97753	3.00972
	E_y	0.52408	0.45172	0.67446	0.64942	0.76823	0.95559	1.43839	3.00571	3.11432

V. Conclusion:

We have concluded that the efficiency for direct as well as neighbour effects is high, in case of Complete and Incomplete block designs for NN correlation structure with ρ in the interval 0.1 to 0.4. The performance of Nearest neighbour balanced block designs is satisfactory for ARMA(1,1) models. Moreover, as the block size increases, the efficiency also increases because the plots in a block become more heterogeneous. So we concluded that when block sizes are large and neighbouring plots are highly correlated, generalized least squares for estimation of direct and neighbour effects can be used. The gain in efficiency of NNBD and NNBIBD over regular block design is high under MA(1) models for direct and neighbour effects of treatments.

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