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Stabilization Methods for Two Dis-Similar Biogas Solids Population System with Higher Carrying Capacities

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Abstract: *In this study, we discuss alternative methods of stabilizing theunique positive stable steady-state solutions of interacting biogas solids undersome simplifying assumptions of higher carrying capacities and initial conditions. Numerical results are presented and discussed.*
Key words and phrases: *Stabilization, Biogas Solids, Higher Carrying Capacities*

I. Introduction

Stabilizing a mathematical model of interacting population systems is a fastgrowing research topic in applied mathematics ([1], [2], [3], [4]). So far, most stabilization of population systems are observed at fixed final time ([1]). However, wheneither of the interacting population or a model parameter is controlled, stabilizationoutputs can make meaningful insights which extend previous theory of stabilizing amathematical model of population system ([1]). In this study, we are interested inadopting alternative stabilizing methods for two dis-similar interacting systemsof biogas solids.

Mathematical Formulation

The formulation of two interacting biogas solids reduces to a popular Lotka-Volterra model with the following precise values of its parameters such as $a = 0.1067, b = 0.0099, c = 0.001,$ $d = 0.02, e = 0.001$and $f = 0.0098$. Here, the twocarrying capacities are calculated to be 10.78 ml/gTS and 2.04 ml/gV S where thenotations TS and V S stand for the total solid and volatile solid respectively.

II. Methodology

In this study, we will follow an alternative method of stabilization which isslightly different from the method first proposed in the work of Yan and Ekaka-a([1]). In this method, a controlled model equation is either chosen or a model parameter is controlled. In both cases, the precise controlled parameter Ueandthedefined steady-state solutions are calculated. Following [1], the feedback controlleris designed which is expected to stabilize the interacting population systems. Theresults which we have obtainedusing this alternative method under some simplifications of the final time is presented in the next section of this paper. In order toachieve the primary aim of this paper, we have made the following simplifying assumptions: $B_{1e} = s$ where the precise value of s is a positive constant, $B_{2e} = \frac{a-bB_{1e}}{c}$and the precise value of the controlled parameter can be calculated using the formula $U_e = eB_{1e}B_{2e} + fB_{2e}B_{2e} - dB_{2e}$which was derived from the controlled secondmodel equation involving the growth dynamics of the second biogas solid. Anothermodern method of stabilizing a population system is to control a chosen modelparameter, assume a suitable value for B_{1e} and calculate the precise values of $B_{2e}$and U_e. For the purpose of this challenging study, we have considered the followingcontrolled model equations of the Lotka-Volterra type

$$B_{1e}(aU_e - bB_{1e} - cB_{2e}) = 0$$
$$B_{2e}(d - eB_{1e} - fB_{2e}) = 0$$

Assuming that $B_{1e} = s$where s is a positive constant, we have calculated othervalues such as $B_{2e} = \frac{d-es}{f}$ =provided $s < \frac{d}{e}$and $U_e = \frac{bs+cB_{2e}}{a}$. By choosing thevalues of s as $1.5, 2.00, 2.50, 3.00, 3.50, 4.00, 10, 12$and 18, the appropriatevalues of the steady-state solutions and their corresponding controlled effects can bedetermined. The stabilization of these steady-state solutions under these assumedvalues ofB_{1e}, $B_{2e}$and U_e can be systematically studied.

III. Discussion of Results

A MATLAB program is utilized to study the stabilization of this biogas solids interaction model for a given step length value of 0.01, the number of loops $M = \frac{Tfinal}{k}$, = 4.724, the initial weights of the biogas solids are 0.04 and 0.06 respectively.

Examples	Computation of the stabilized steady-state solution		
No	Tfinal	B_{1e}	B_{2e}
1	10	0.09617457307456	21.814702274046258
2	20	0.219378849540937	21.847871912541073
3	30	0.493981087529847	21.868576003131697
4	40	1.068337198703639	21.912294363131675
5	50	2.133828893904341	21.995153429653527
6	60	3.73147463317342	22.124033280743944
7	70	5.496408899948091	22.273701381198741
8	80	6.892040190451242	22.398312811894446
9	90	7.730813721329238	22.476255676669886
10	100	8.154223145639200	22.516565815956394
11	110	8.349155090241331	22.535355782717517
12	120	8.435077950002022	22.54686021058161
13	130	8.472222745214776	22.547296433733258
14	140	8.488145524741256	22.548845816929799
15	150	8.494946376357317	22.549507895761831

Table 1. Computation of the stabilized steady-state solution fora varying final time

We can observe from this table that the steady-state solution (8.5, 22.55) forU_e= 4.724 is fully stabilized when the value of the final time in days is 150. A fewsimilar calculations have been carried in which we have found that the followingsteady-state solutions can be fully stabilized for varying values of the controlledvalue U_e: N_{1e}= 8.52, N_{2e}= 22.35 for U_e= 4.64; N_{1e}= 8.54, N_{2e}= 22.15 for U_e= 4.556; N_{1e}= 8.56, N_{2e}= 21.96 for U_e= 4.473; N_{1e}= 8.58,N_{2e}= 21.76 forU_e= 4.3909; N_{1e}= 8.60, N_{2e}= 21.56 for U_e= 4.3096.
For the scenario when B_{1e}= 1.50, B_{2e}= 1.887 and U_e= 0.1569, its stabilizationunder changing values of the final time in days is displayed in the table below:

Examples	Computation of the stabilized steady-state solution					
No	Tfinal	B_{1e}	B_{2e}	U_e	$B_1(0)$	$B_2(0)$
1	800	1.499644834765967	1.887737233378940	0.1569	0.04	0.06
2	1000	1.49998207744749	1.88775420857516	0.1569	0.04	0.06
3	1200	1.499999096813964	1.887755056600626	0.1569	0.04	0.06
4	1400	1.499999954459325	1.887755099749620	0.1569	0.04	0.06
5	1600	1.499999997703737	1.887755101925289	0.1569	0.04	0.06
6	1800	1.499999999884218	1.887755102034991	0.1569	0.04	0.06
7	2000	1.499999999994162	1.887755102040523	0.1569	0.04	0.06
8	2200	1.499999999999700	1.887755102040799	0.1569	0.04	0.06
9	2400	1.499999999999926	1.887755102040811	0.1569	0.04	0.06
10	3000	1.499999999999926	1.887755102040811	0.1569	0.04	0.06

Table 2. Stabilization of the steady-state solution if the intrinsicgrowth rate a is controlled when other parameters are fixed

From this simulation, if the intrinsic growth rate a is controlled when other parameters are fixed, $B_{1e}$and B_{2e}will converge approximately to B_{1e}= 1.50 andB_{2e}= 1.887 under the simplifying assumptions of the controlled values and theinitial biogas solids.
Similar stabilizations of other steady-state solutions such as (2, 1.836) for U_e =0.2028, (2.5, 1.786) for U_e= 0.2487, (3.00, 1.7347) for U_e= 0.2946, (3.50, 1.6837)for U_e= 0.3405, (4.00, 1.6327) for U_e=0.3864, (10, 1.0204) for U_e= 0.9374,(12, 0.8163) for U_e= 1.1211 and (15, 0.5102) for U_e= 1.3965 for changing valuesof the final time and using the same initial biogas solids have been systematicallycalculated but their details will not be presented in this present work for obviousreasons.

IV. Conclusion

In this study, we have found alternative stabilizing methods for two controlleddis-similar biogas solids population system with higher carrying capacities. We areyet to use the same technique to find out if it would be possible to fully stabilize twocontrolled similar biogas solids population system with relatively similar carryingcapacities.

Within the environment of biogas production, the inclusion of random noise orsome sort of environmental perturbation may create an impact on the stabilizationof a steady-state solution which has previously been stabilized. But the extent ofthis impact is yet to be quantified. We will attempt to tackle this scientific problemin a future presentation.

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