IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728,p-ISSN: 2319-765X, Volume 7, Issue 6 (Sep. - Oct. 2013), PP 90-99
www.iosrjournals.org

Heat Transfar and Thermal Stress Analysis In Annular Cylinder under Steady State

P.H.Munjankar[1], K.C.Deshmukh[2], V.S.Kulkarni[3]
[1]*Department of Mathematics, Ambedkar Science College, Chandrapur-442401,Maharashtra India.*
[2]*Department of Mathematics, RTM Nagpur University, Nagpur-440010, Maharashtra India.*
[3]*Department of Mathematics, University of Mumbai, Mumbai-400098, Maharashtra India.*

Abstract: *The present manuscript deals with the heat transfer and thermal stress analysis of thick annular cylinder under steady temperature conditions. A annular cylinder is subjected to arbitrary heat flux applied on the upper surface with lower surface is thermally insulated. The fixed circular edges are at zero temperature. The integral transform methods are used for heat transfer analysis to determine temperature change. The theory of linearized thermoelasticity based on solution of Naviers equation in terms of Goodiers thermoelastic displacement potential, Michell's function, and the Boussinesq's function for cylindrical co-ordinate system have been used for thermal stress anaylsis. The results for temperature change, displacement and stresses have been computed numerically and illustrated graphically.*
Key Words: *Heat transfer analysis, Steady state, Thermal Stress analysis.*

I. Introduction

Thermoelasticity is based on temperature changes induced by expansion and compression of the test part. Although this coupling between mechanical deformation and thermal energy has been known for over a century. After world war second, there was very rapid development of thermoelasticity, stimulated by various engineering sciences. Thermoelasticity contains the generalized theory of heat conductions, the generalized theory of the thermal stresses. A considerable progress in the field of air-craft and machine structures, mainly with gas and steam turbines and the emergence of new topics in chemical engineering have given rise to numerous problems in which thermal stresses play an important role and frequently even a primary role.

Singh D.V. [7] solved thermoelastic equations for infinite thick plate with circular hole involving non-dimensional parameters and obtained results in the form of integral equations. Lee C.W.[3] obtained three dimensional series solution for elastic thick plate subjected to general temperature distribution. T. Hata [9] concerned with a method for calculating the thermal-stress distribution in a nonhomogeneous thick elastic plate under steady distribution of the surface temperature whose shear modulus and coefficient of thermal expansion are assumed to be functions of z. Lee Z.Y. et al [4] studied transient response of one dimensional axisymmetric quasi-static coupled thermoelastic problem of multilayered hollow cylinder with orthotropic material properties. Recently Kulkarni et al [1 and 2] determined the temperature changes and thermal stresses due to conduction of heat in the various shaped thick plate under transient and steady-state temperature conditions.

II. Heat Transfar Analysis

2.1 Formulation of the Problem

Consider a thick annular cylinder of thickness *2h* occupying space *D* defined by $a \le r \le b,\ -h \le z \le h$. The arbitrary heat $f(r)$ is applied on the upper surface of cylinder $(z = h)$, lower surface $(z = -h)$ is thermally insulated. The fixed circular edges $(r = a \text{ and } r = b)$ are at zero temperature. Under these prescribed conditions, the temperature changes hence thermal stresses developed within annular cylinder are required to be determined. The steady state temperature of the cylinder satisfies the heat conduction equation,

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}=0 \qquad (2.1.1)$$

with the boundary conditions

$$T=0 \quad \text{at } r = a \quad -h \le z \le h \qquad (2.1.2)$$

$$T=0 \quad \text{at } r = b \quad -h \le z \le h \qquad (2.1.3)$$

$$T=f(r) \quad \text{on } z = h,\ a \le r \le b \qquad (2.1.4)$$

$$\frac{\partial T}{\partial z}=0 \quad \text{at } z=-h \quad a \le r \le b \qquad (2.1.5)$$

2.2 The Solution for Temperature Change

The integral transform techniques is used to find the solution of above heat conduction problem alongwith prescribed boundary conditions.

Introduce the finite Hankal transform over the variable *r* and its inverse transform defined as in [6]

$$\overline{T}(\alpha_n, z) = \int_a^b r K_0(\alpha_n, r) T(r,z) dr \tag{2.2.1}$$

$$T(r,z) = \sum_{n=1}^{\infty} \overline{T}(\alpha_n, z) K_0(\alpha_n, r) \tag{2.2.2}$$

where the kernel of Hankel transform is given by

$$K_0(\alpha_n, r) = \frac{R_0(\alpha_n, r)}{\sqrt{N}} \tag{2.2.3}$$

$$R_0(\alpha_n, r) = \frac{J_0(\alpha_n r)}{J_0(\alpha_n b)} - \frac{Y_0(\alpha_n r)}{Y_0(\alpha_n b)} \tag{2.2.4}$$

The normality constant can be obtained by the orthogonality property of eigen functions as,

$$N = \frac{b^2}{2} \dot{R}_0^{\,2}(\alpha_n, b) - \frac{a^2}{2} \dot{R}_0^{\,2}(\alpha_n, a) \tag{2.2.5}$$

and α_1, α_2 ... are roots of the transcendental equation

$$\frac{J_0(\alpha a)}{J_0(\alpha b)} - \frac{Y_0(\alpha a)}{Y_0(\alpha b)} = 0 \tag{2.2.6}$$

$J_n(x)$ is Bessel function of the first kind of order *n and* $Y_n(x)$ is Bessel function of the second kind of order *n*.

On applying the finite Hankal transform defind in the equation (2.2.1) to the equation (2.1.1),one obtain

$$\frac{d^2 \overline{T}}{dz^2} - \alpha_n^2 \overline{T} = 0 \tag{2.2.7}$$

where $\overline{T}$ is the Hankal transform of *T*.

On solving equation (2.2.7) under condition given in equations (2.1.4) and (2.1.5), one obtain

$$\overline{T}(\alpha_n, z) = \overline{f}(\alpha_n) \left[\frac{\cosh[\alpha_n (z+h)]}{\cosh(2\alpha_n h)} \right]$$

On applying inverse Hankal transform defined in equation (2.2.2),one obtain the expression for temperature as

$$T = \sum_{n=1}^{\infty} \frac{\overline{f}(\alpha_n)}{\sqrt{N}} \left[\frac{J_0(\alpha_n r)}{J_0(\alpha_n b)} - \frac{Y_0(\alpha_n r)}{Y_0(\alpha_n b)} \right] \left[\frac{\cosh[\alpha_n (z+h)]}{\cosh(2\alpha_n h)} \right] \tag{2.2.8}$$

where $\overline{f}(\alpha_n)$ is Hankal transform of $f(r)$.

Since initial temperature T_i =0, the temperature change τ =T (2.2.9)

III. Thermal Stress Analysis

3.1 Development of Thermoelastic Equations

Following Noda et al [4], The Naviers equations for axisymmetric thermoelastic problems can be expressed as

$$\nabla^2 u_r - \frac{u_r}{r^2} + \left(\frac{1}{1-2\nu} \right) \frac{\partial e}{\partial r} - 2\alpha \left(\frac{1+\nu}{1-2\nu} \right) \frac{\partial \tau}{\partial r} + \frac{2(1+\nu)}{E} F_r = 0 \tag{3.1.1}$$

$$\nabla^2 u_z + \left(\frac{1}{1-2\nu} \right) \frac{\partial e}{\partial z} - 2\alpha \left(\frac{1+\nu}{1-2\nu} \right) \frac{\partial \tau}{\partial z} + \frac{2(1+\nu)}{E} F_z = 0 \tag{3.1.2}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} . \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \tag{3.1.3}$$

e - dilatation
E - Young's modulus
α - coefficient of linear thermal expansion
ν - Poisson ratio

The solution of Naviers equations (3.1.1) and (3.1.2) without body forces can be expressed by Goodiers thermoelastic displacement potential ϕ and Boussinesq harmonic functions φ and ψ under the axisymmetric conditions.

The Goodiers thermoelastic displacement potential ϕ must satisfy the governing equations

$$\nabla^2\phi = K\tau$$

i.e. $$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{\partial^2\phi}{\partial z^2} = K\tau \tag{3.1.4}$$

where K *is* Restraient coefficient as

$$K = \frac{\beta}{\lambda + 2\mu} = \left(\frac{1+\nu}{1-\nu}\right)\alpha$$

where β - thermoelastic constant
λ & μ - Lames elastic constants.

Boussinesq harmonic functions φ and ψ must satisfy the governing equations

$$\nabla^2\varphi = \frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} + \frac{\partial^2\varphi}{\partial z^2} = 0 \tag{3.1.5}$$

and

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{\partial^2\psi}{\partial z^2} = 0 \tag{3.1.6}$$

when deformation in the cylindrical coordinate system are discussed, Michells function M instead of Boussinesq harmonic functions φ and ψ is often used.

Taking

$$M = -\int(\varphi + z\psi)dz \tag{3.1.7}$$

The Michell's function M must satisfy

$$\nabla^2\nabla^2 M = 0 \tag{3.1.8}$$

The component of the displacement and stresses are represented by the thermoelastic displacement potential ϕ and Michell's function M as

$$u_r = \frac{\partial\phi}{\partial r} - \frac{\partial^2 M}{\partial r dz} \tag{3.1.9}$$

$$u_z = \frac{\partial\phi}{\partial z} + 2(1-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2} \tag{3.1.10}$$

$$\sigma_{rr} = 2G\left[\frac{\partial^2\phi}{\partial r^2} - K\tau + \frac{\partial}{\partial z}\left(\nu\nabla^2 M - \frac{\partial^2 M}{\partial r^2}\right)\right] \tag{3.1.11}$$

$$\sigma_{\theta\theta} = 2G\left[\frac{1}{r}\frac{\partial\phi}{\partial r} - K\tau + \frac{\partial}{\partial z}\left(\nu\nabla^2 M - \frac{1}{r}\frac{\partial M}{\partial r}\right)\right] \tag{3.1.12}$$

$$\sigma_{zz} = 2G\left[\frac{\partial^2\phi}{\partial z^2} - K\tau + \frac{\partial}{\partial z}\left((2-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right)\right] \tag{3.1.13}$$

and

$$\sigma_{rz} = 2G\left[\frac{\partial^2 \phi}{\partial r \partial z} + \frac{\partial}{\partial r}\left((1-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right)\right] \tag{3.1.14}$$

For traction free surface the stress functions

$$\sigma_{zz} = \sigma_{rz} = 0 \quad \text{at } z = \pm h \tag{3.1.15}$$

The set of equations (3.1.1) to (3.1.15) constitute mathematical formulation for displacement and thermal stresses developed within solid due to temperature change.

3.2 The Solution for Displacement and Thermal Stresses

Assuming displacement function $\phi\ (r, z)$ as

$$\phi(r,z) = \sum_{n=1}^{\infty} D_n \left\{\left[\frac{J_0(\alpha_n r)}{J_0(\alpha_n b)} - \frac{Y_0(\alpha_n r)}{Y_0(\alpha_n b)}\right]\left[\frac{(z+h)\sinh[\alpha_n (z+h)]}{\cosh(2\alpha_n h)}\right]\right\} \tag{3.2.1}$$

Using equation in (3.1.4), one have

$$D_n = \frac{K\overline{f}(\alpha_n)}{2\alpha_n \sqrt{N}}$$

Thus equation (3.2.1) become

$$\phi(r,z) = \frac{K}{2}\sum_{n=1}^{\infty}\left\{\frac{\overline{f}(\alpha_n)}{\sqrt{N}}\left[\frac{J_0(\alpha_n r)}{J_0(\alpha_n b)} - \frac{Y_0(\alpha_n r)}{Y_0(\alpha_n b)}\right]\left[\frac{(z+h)\sinh[\alpha_n (z+h)]}{\alpha_n \cosh(2\alpha_n h)}\right]\right\} \tag{3.2.2}$$

Now suitable form of Michell's function M satisfying (3.1.8) is given by

$$M = K\sum_{n=1}^{\infty}\frac{\overline{f}(\alpha_n)}{\sqrt{N}}\left[\frac{J_0(\alpha_n r)}{J_0(\alpha_n b)} - \frac{Y_0(\alpha_n r)}{Y_0(\alpha_n b)}\right]$$

$$\{B_n \sinh[\alpha_n (z+h)] + C_n\ \alpha_n [z+h]\cosh[\alpha_n (z+h)]\} \tag{3.2.3}$$

where B_n and C_n are arbitrary functions.

Using equations (2.2.8), (2.2.9), (3.2.2) and (3.2.3) in the equations (3.1.9) and (3.3.14), one obtains the expressions for displacement and thermal stress function as

$$u_r = K\sum_{n=1}^{\infty}\frac{\overline{f}(\alpha_n)}{\sqrt{N}}\left[\frac{J_1(\alpha_n r)}{J_0(\alpha_n b)} - \frac{Y_1(\alpha_n r)}{Y_0(\alpha_n b)}\right]\times\left\{\frac{-(z+h)\sinh[\alpha_n (z+h)]}{2\cosh(2\alpha_n h)}\right.$$

$$+ B_n \alpha_n^2 \cosh[\alpha_n (z+h)]$$

$$\left. + C_n \alpha_n^2 [\cosh[\alpha_n (z+h)] + \alpha_n [z+h]\sinh[\alpha_n (z+h)]]\right\} \tag{3.2.4}$$

$$u_z = K\sum_{n=1}^{\infty}\frac{\overline{f}(\alpha_n)}{\sqrt{N}}\left[\frac{J_0(\alpha_n r)}{J_0(\alpha_n b)} - \frac{Y_0(\alpha_n r)}{Y_0(\alpha_n b)}\right]$$

$$\times\left\{\frac{[\sinh[\alpha_n (z+h)] + \alpha_n [z+h]\cosh[\alpha_n (z+h)]]}{2\alpha_n \cosh(2\alpha_n h)}\right.$$

$$- B_n \alpha_n^2 \sinh[\alpha_n (z+h)]$$

$$\left. + C_n \alpha_n^2 [2(1-2\nu)\sinh[\alpha_n (z+h)] - \alpha_n [z+h]\cosh[\alpha_n (z+h)]]\right\} \tag{3.2.5}$$

$$\sigma_{rr} = 2G\,K \sum_{n=1}^{\infty} \frac{\overline{f}(\alpha_n)}{\sqrt{N}} \left\{ \left[\frac{\dot{J}_1(\alpha_n r)}{J_0(\alpha_n b)} - \frac{\dot{Y}_1(\alpha_n r)}{Y_0(\alpha_n b)} \right] \left[\frac{-(z+h)\sinh[\alpha_n(z+h)]}{2\cosh(2\alpha_n h)} \right] \right.$$

$$-\left[\frac{J_0(\alpha_n r)}{J_0(\alpha_n b)} - \frac{Y_0(\alpha_n r)}{Y_0(\alpha_n b)} \right] \left[\frac{\cosh[\alpha_n(z+h)]}{\cosh(2\alpha_n h)} \right]$$

$$+ B_n\alpha_n^2 \left[\frac{\dot{J}_1(\alpha_n r)}{J_0(\alpha_n b)} - \frac{\dot{Y}_1(\alpha_n r)}{Y_0(\alpha_n b)} \right] \cosh[\alpha_n(z+h)]$$

$$\left. + C_n\alpha_n^2 \left[\begin{array}{l} 2v\alpha_n \left[\dfrac{J_0(\alpha_n r)}{J_0(\alpha_n b)} - \dfrac{Y_0(\alpha_n r)}{Y_0(\alpha_n b)} \right] \cosh[\alpha_n(z+h)] \\ + \left[\dfrac{\dot{J}_1(\alpha_n r)}{J_0(\alpha_n b)} - \dfrac{\dot{Y}_1(\alpha_n r)}{Y_0(\alpha_n b)} \right] [\cosh[\alpha_n(z+h)] + \alpha_n[z+h]\sinh[\alpha_n(z+h)]] \end{array} \right] \right\} \tag{3.2.6}$$

where **.** represents differentiation w.r.t. to space variable r.

$$\sigma_{\theta\theta} = 2G\,K \sum_{n=1}^{\infty} \frac{\overline{f}(\alpha_n)}{\sqrt{N}} \left\{ \left(\frac{-1}{r} \right) \left[\frac{J_1(\alpha_n r)}{J_0(\alpha_n b)} - \frac{Y_1(\alpha_n r)}{Y_0(\alpha_n b)} \right] \left[\frac{(z+h)\sinh[\alpha_n(z+h)]}{2\cosh(2\alpha_n h)} \right] \right.$$

$$-\left[\frac{J_0(\alpha_n r)}{J_0(\alpha_n b)} - \frac{Y_0(\alpha_n r)}{Y_0(\alpha_n b)} \right] \left[\frac{\cosh[\alpha_n(z+h)]}{\cosh(2\alpha_n h)} \right]$$

$$+ \frac{B_n\alpha_n^2}{r} \left[\frac{J_1(\alpha_n r)}{J_0(\alpha_n b)} - \frac{Y_1(\alpha_n r)}{Y_0(\alpha_n b)} \right] \cosh[\alpha_n(z+h)]$$

$$\left. + C_n\alpha_n^2 \left[\begin{array}{l} 2v\alpha_n \left[\dfrac{J_0(\alpha_n r)}{J_0(\alpha_n b)} - \dfrac{Y_0(\alpha_n r)}{Y_0(\alpha_n b)} \right] \cosh[\alpha_n(z+h)] + \\ \dfrac{1}{r} \left[\dfrac{J_1(\alpha_n r)}{J_0(\alpha_n b)} - \dfrac{Y_1(\alpha_n r)}{Y_0(\alpha_n b)} \right] [\cosh[\alpha_n(z+h)] + \alpha_n[z+h]\sinh[\alpha_n(z+h)]] \end{array} \right] \right\} \tag{3.2.7}$$

$$\sigma_{zz} = 2GK \sum_{n=1}^{\infty} \frac{\overline{f}(\alpha_n)}{\sqrt{N}} \left[\frac{J_0(\alpha_n r)}{J_0(\alpha_n b)} - \frac{Y_0(\alpha_n r)}{Y_0(\alpha_n b)} \right] \left\{ \frac{(z+h)\alpha_n \sinh[\alpha_n(z+h)]}{2\cosh(2\alpha_n h)} \right.$$

$$- B_n\alpha_n^3 \cosh[\alpha_n(z+h)]$$

$$\left. + C_n\alpha_n^3 \left[(1-2v)\cosh[\alpha_n(z+h)] - \alpha_n(z+h)\sinh[\alpha_n(z+h)] \right] \right\} \tag{3.2.8}$$

$$\sigma_{rz} = 2GK \sum_{n=1}^{\infty} \frac{\overline{f}(\alpha_n)}{\sqrt{N}} \left[\frac{J_1(\alpha_n r)}{J_0(\alpha_n b)} - \frac{Y_1(\alpha_n r)}{Y_0(\alpha_n b)} \right]$$

$$\left\{ -\frac{\sinh[\alpha_n (z+h)] + \alpha_n (z+h) \cosh[\alpha_n (z+h)]}{2\cosh(2\alpha_n h)} \right.$$

$$+ B_n \alpha_n^3 \sinh[\alpha_n (z+h)]$$

$$\left. + C_n \alpha_n^3 [2\nu \sinh[\alpha_n (z+h)] + \alpha_n (z+h) \cosh[\alpha_n (z+h)]] \right\} \tag{3.2.9}$$

Now in order to satisfy above equation (3.1.15), solving equations (3.2.8) and (3.2.9) for B_n and C_n one obtain,

$$B_n = \frac{(1-2\nu)}{2\alpha_n^3 \cosh(2\alpha_n h)} \tag{3.2.10}$$

and

$$C_n = \frac{1}{2\alpha_n^3 \cosh(2\alpha_n h)} \tag{3.2.11}$$

Using these values of B_n and C_n from above equations (3.2.10) and (3.2.11) in equations (3.2.4), to (3.2.9) one obtain the expressions for displacements and stresses as

$$u_r = K \sum_{n=1}^{\infty} \frac{\overline{f}(\alpha_n)}{\sqrt{N}} \left[\frac{J_1(\alpha_n r)}{J_0(\alpha_n b)} - \frac{Y_1(\alpha_n r)}{Y_0(\alpha_n b)} \right] \left[\frac{(1-\nu)\cosh[\alpha_n (z+h)]}{\alpha_n \cosh(2\alpha_n h)} \right] \tag{3.2.12}$$

$$u_z = K \sum_{n=1}^{\infty} \frac{\overline{f}(\alpha_n)}{\sqrt{N}} \left[\frac{J_0(\alpha_n r)}{J_0(\alpha_n b)} - \frac{Y_0(\alpha_n r)}{Y_0(\alpha_n b)} \right] \left[\frac{(1-\nu)\sinh[\alpha_n (z+h)]}{\alpha_n \cosh(2\alpha_n h)} \right] \tag{3.2.13}$$

$$\sigma_{rr} = 2GK \sum_{n=1}^{\infty} \frac{\overline{f}(\alpha_n)}{\sqrt{N}} \left\{ \left[\frac{\dot{J}_1(\alpha_n r)}{J_0(\alpha_n b)} - \frac{\dot{Y}_1(\alpha_n r)}{Y_0(\alpha_n b)} \right] \left[\frac{(1-\nu)\cosh[\alpha_n (z+h)]}{\alpha_n \cosh(2\alpha_n h)} \right] \right.$$

$$\left. - \left[\frac{J_0(\alpha_n r)}{J_0(\alpha_n b)} - \frac{Y_0(\alpha_n r)}{Y_0(\alpha_n b)} \right] \left[\frac{(1-\nu)\cosh[\alpha_n (z+h)]}{\cosh(2\alpha_n h)} \right] \right\} \tag{3.2.14}$$

$$\sigma_{\theta\theta} = 2GK \sum_{n=1}^{\infty} \frac{\overline{f}(\alpha_n)}{\sqrt{N}} \left\{ \left[\frac{J_1(\alpha_n r)}{J_0(\alpha_n b)} - \frac{Y_1(\alpha_n r)}{Y_0(\alpha_n b)} \right] \left[\frac{(1-\nu)\cosh[\alpha_n (z+h)]}{r\alpha_n \cosh(2\alpha_n h)} \right] \right.$$

$$\left. - \left[\frac{J_0(\alpha_n r)}{J_0(\alpha_n b)} - \frac{Y_0(\alpha_n r)}{Y_0(\alpha_n b)} \right] \left[\frac{(1-\nu)\cosh[\alpha_n (z+h)]}{\cosh(2\alpha_n h)} \right] \right\} \tag{3.2.15}$$

$$\sigma_{zz} = 0 \tag{3.2.16}$$

and

$$\sigma_{rz} = 0 \tag{3.2.17}$$

IV. Numerical Calculations

4.1 Special Case

Setting $f(r) = (r^2 - a^2)(r^2 - b^2)$ (4.1.1)

Applying finite Hankal transform as defined in equations (2.2.1) to (2.2.6), one obtain

$$\bar{f}(\alpha_n) = \int_a^b \frac{1}{\sqrt{N}} \left[\frac{J_0(\alpha_n r)}{J_0(\alpha_n b)} - \frac{Y_0(\alpha_n r)}{Y_0(\alpha_n b)} \right] r(r^2 - a^2)(r^2 - b^2) dr$$

$$\bar{f}(\alpha_n) = \frac{8\{(a^2\alpha_n{}^2 - 3b^2\alpha_n{}^2 + 16)J_0(\alpha_n a) - (b^2\alpha_n{}^2 - 3a^2\alpha_n{}^2 + 16)J_0(\alpha_n b)\}}{\pi\sqrt{N}\alpha_n{}^6 J_0(\alpha_n a) J_0(\alpha_n b) Y_0(\alpha_n b)}$$

(4.1.2)

4.2 Dimensions

The inner radius of annular cylinder *a=1m*
The outer radius of annular cylinder *b=2m*
The height of annular cylinder *2h=0.6m*

4.3 Material Properties

The numerical calculation have been carried out for steel (SN 50C) plate
Thermal diffusivity k = 15.9 x 10^{-6} (m^2s^{-1}) and
Poisson ratio $\nu = 0.281$

4.4 Roots of transcendental equation

The roots of transdental equation $\left[\frac{J_0(\alpha a)}{J_0(\alpha b)} - \frac{Y_0(\alpha a)}{Y_0(\alpha b)} \right] = 0$ are given by

$\alpha_1 = 3.120$, $\alpha_2 = 6.2734$, $\alpha_3 = 9.4182$, $\alpha_4 = 12.5614$, $\alpha_5 = 15.7040$.

For convenience setting $A = \frac{K}{\sqrt{N}}$, $B = \frac{2GK}{\sqrt{N}}$. in the expressions (3.2.8) to (3.2.17).The numerical expressions for temperature, displacement and stress components are obtained by equations (2.2.8) and (3.2.8) to (3.2.17).

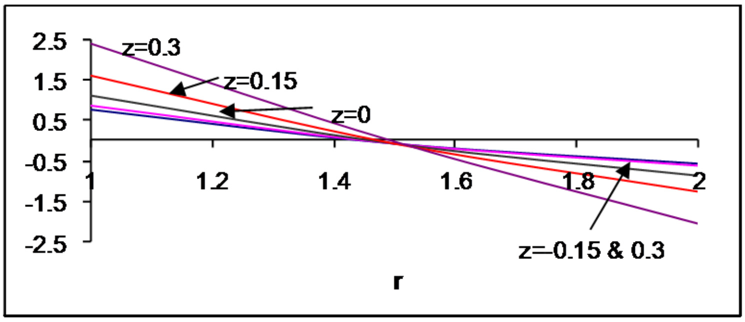

Figure1: The radial displacement function u_r/A in radial direction.

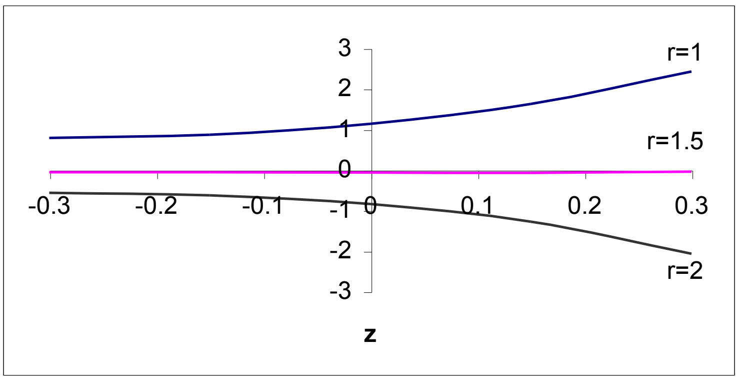

Figure2: The radial displacement function u_r/A in axial direction.

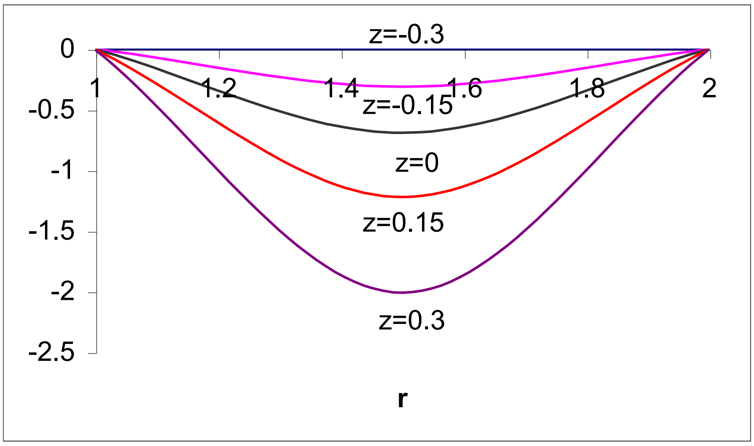

Figure3: The axial displacement function u_z/A in radial direction.

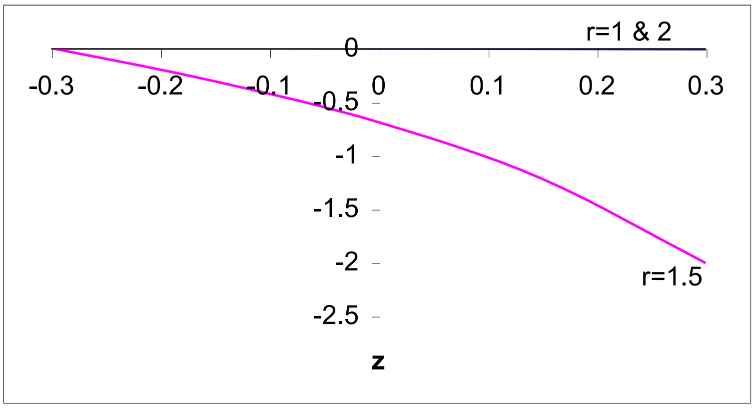

Figure 4 : The axial displacement function u_z/A in axial direction.

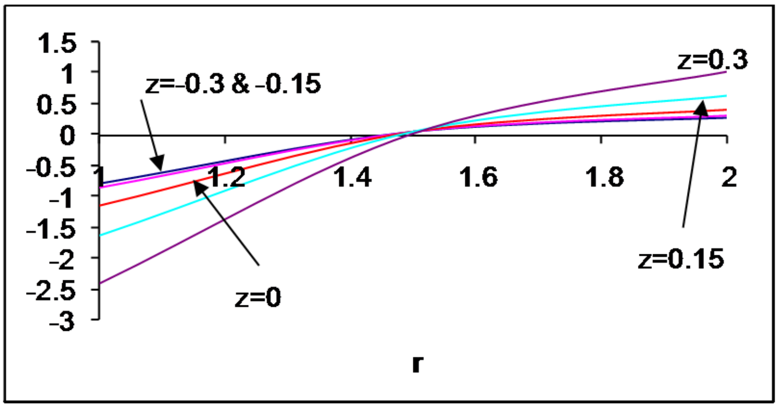

Figure 5 : The radial stress function σ_{rr}/B in radial direction.

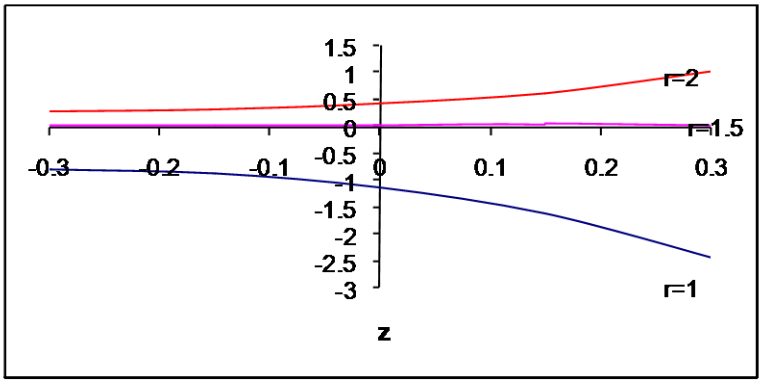

Figure 6 : The radial stress function σ_{rr}/B in axial direction.

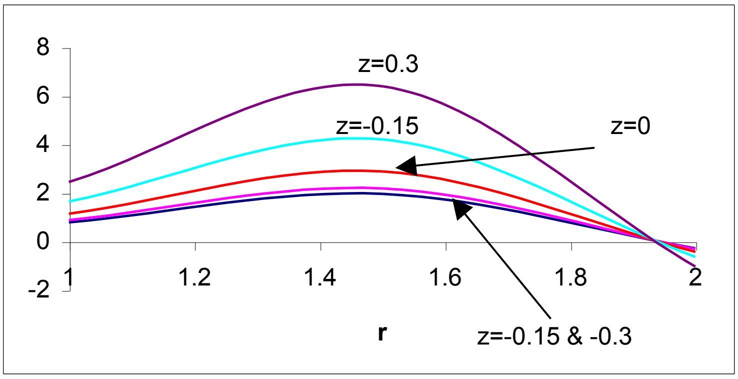

Figure 7 : The stress function $\sigma_{\theta\theta}/B$ in radial direction.

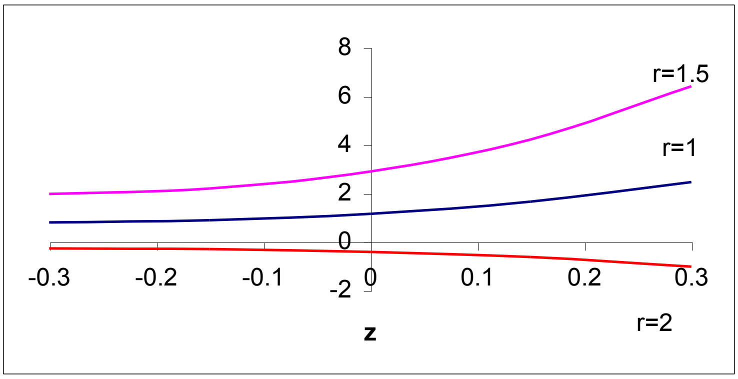

Figure 8 : The stress function $\sigma_{\theta\theta}/B$ in axial direction.

V. Concluding Remarks

In this paper a thick annular cylinder is considered and determined the expressions for temperature, displacement and stress function due to steady state temperature field. As a special case mathematical model is constructed for

$$f(r) = (r^2 - a^2)(r^2 - b^2)$$

and performed numerical calculations. The thermoelastic behaviour is examined such as temperature, displacement and stresses with the help of arbitrary heat applied on the upper surface.

From figure 1 and 2, radial displacement function u_r is decreases from inner circular surface to outer circular surface and it is increases from lower surface to upper surface.

From figure 3 and 4, axial displacement function u_z decreases from lower surface to upper surface. Also it shows variation at r = 1.5.

From figure 5 and 6, stress function σ_{rr} develops compressive stress within annular region $1 \le r \le 1.5$ and tensile stress within annular region $1.5 \le r \le 2$.

From figure 7 and 8, the stress function $\sigma_{\theta\theta}$ develops tensile stress in radial and axial direction except at outer edge of cylinder,it develops compressive tensile stress at the outer edge i.e.r = 2.

Due to applying the arbitrary heat supply on the upper surface on the cylinder, the radial and axial displacements occurs near the heat source and cylinder expands towards the center. The axial stress component and resultant stress component are zero due to exchange of heat through heat transfer in circular boundry. The results presented here will be more useful in engineering problem particularly in the determination of the state of strain in thick circular plate constituting foundations of containers for hot gases or liquids, in the foundations for furnaces etc. Also any particular case of special interest can be derived by assigning suitable values to the parameters and function in the expression (3.2.8) - (3.2.17).

References

[1]. Kulkarni V.S. and Deshmukh K.C., 'Quasi-static thermal stresses in a thick circular plate' Applied Mathematical Modelling, Vol-31, issue 8, pp 1479-1488, 2007.

[2]. Kulkarni V.S. and Deshmukh K.C., 'Thermal stresses in a thick annular disc' Journal of Thermal stresses, Vol-31, issue 4, pp 1-8, 2008.

[3]. Lee C.W., "Thermal stresses in thick plate", Journal of Solids and Structures, Vol.6, issue 5, pp 605-615, 1970.

[4]. Lee Z.Y. Chen C.K. and Hung C.L., "Transient Thermal stress analysis of multilayered hollow cylinder", Acta Mechanica, Vol.151, pp 75-88,2001

[5]. Naotake Noda, Richard B Hetnarski and Yoshinobu Tanigawa. 'Thermal Stresses' *2nd edn. pp 259-261, 2003. Taylor and Francis New York.*

[6]. Ozisik,M.N., 'Boundary value problem of heat conduction' ,International Text book Company,Scranton,Pennsylvania,pp84-87,1968.

[7]. Singh D.V., "Thermal stresses in an infinite thick with circular cylindrical hole" Vol.33, No.1 and 2, 1966, Sneddon I.N., 'The Use of Intergral Transform', McGraw Hill,New York,pp 235-238, 1972.

[8]. Toshiaki Hata, "Thermal stresses in a nonhomogeneous thick plateunder steady distribution of temperature", Journal of Thermal Stresses, Vol.5,v issue 1, pp 1-11, 1982.