IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728,p-ISSN: 2319-765X, Volume 8, Issue 1 (Sep. - Oct. 2013), PP 31-34
www.iosrjournals.org

Pythagorean Equation and Special M-Gonal Numbers

M. A. Gopalan[1], V. Geetha[2]
[1](Department of Mathematics, Shrimathi Indira Gandhi College,Trichirappalli-620 002.)
[2](Department of Mathematics, Cauvery College For Women,Trichirappalli-620 018.)

Abstract: *Employing the solutions of the Pythagorean equation, we obtain the relations between the pairs of special polygonal numbers such that the difference in each pair is a perfect square.*
Key Words: *Pythagorean equation,Polygonal numbers*
MSC Classification Number: 11D09.

NOTATIONS

Pen_P = *Pentagonal number of rank* P

Hex_Q = *Hexagonal number of rank* Q

Hep_H = *Heptagonal number of rank* H

$Dodec_D$ = *Dodecagonal number of rank* D

I. Introduction

In [1,2,4-8,10],employing the integral solutions of special binary quadratic Diophantine equation, special patterns of Pythagorean triangles are generated. In [3],the relations among the pairs of special m-gonal numbers generated through the solutions of the binary quadratic equation $y^2 = 2x^2 - 1$ are determined. In [9], the relations among special figurate numbers through the equation $y^2 = 10x^2 + 1$ are obtained. In[11],employing the solutions of the Pythagorean equation, and obtain the relations between Triangular number and Pentagonal number, Octagonal number, Hexagonal number, Heptagonal number, Decagonal number, Dodecagonal number , Pentagonal number and Hexagonal number, Octagonal number such that the difference in each pair is a perfect square. In this communication, employing the solutions of the Pythagorean equation, we obtain the relations between the pairs of special polygonal numbers which are not mentioned in [11] such that the difference in each pair is a perfect square.

II. Method Of Analysis

Consider the Pythagorean equation

$$x^2 + y^2 = z^2 \qquad (1)$$

whose solutions are

$$x = 2rs, y = r^2 - s^2, z = r^2 + s^2 \qquad (2)$$

where r, s are non – zero distinct integers and $r > s$.

Case.2.1:

The choice

$$10H - 3 = r^2 + s^2, 18P - 3 = r^2 - s^2 \qquad (3)$$

in (1) leads to the relation

$$40Hep_H - 216Pen_P = \alpha^2 \qquad (4)$$

From (3), the values of ranks of the Heptagoanl numbers and Pentagonal numbers are respectively given by

$$P = \frac{r^2 - s^2 + 3}{18}; H = \frac{r^2 + s^2 + 3}{10}$$

It is seen that P, H are integers for the following choices of r and s namely,

(i) $s = 1; r = 14$

(ii) $s = 4; r = 11$

(iii) $s = 15n - 14; r = 15n - 11$

(iv) $s = 15n - 4; r = 15n + 11$

(v) $s = 15n - 4; r = 15n + 29$

For each of the values of r and s the corresponding Pentagonal and Heptagonal numbers satisfying (4) are presented in the Table (1)below.

Table (1)

S.No	P	H	Pen_P	Hep_H	α
1	6	14	51	469	88
2	11	20	176	970	28
3	$5n-4$	$45n^2-75n+32$	$\frac{1}{2}(75n^2-125n+52)$	$\frac{1}{2}(45n^2-75n+32)(225n^2-375n+15$	$(450n^2-750n+308$
4	$25n-6$	$45n^2+21n+14$	$\frac{1}{2}(1875n^2+875n+102)$	$\frac{1}{2}(45n^2+21n+14)(225n^2+105n$	$(450n^2+210n+8$
5	$55n+4$	$45n^2+75n+86$	$\frac{1}{2}(9075n^2+15125n+630$	$\frac{1}{2}(45n^2+75n+86)(225n^2+375n+$	$(450n^2+750n-232$

Case 2.2:

The Choice

$$5D - 2 = r^2 - s^2; 12P - 2 = r^2 + s^2 \qquad (5)$$

in (1) leads to the relation

$$96Pen_P - 5Dodec_D = \alpha^2$$

From (5), the values of ranks of the Pentagonal and Dodecagonal numbers are respectively given by,

$$P = \frac{r^2 + s^2 + 2}{12}, D = \frac{r^2 - s^2 + 2}{5}$$

It is seen that P and D are integers for the following choices of r and s namely,

(i) $s = 1; r = 30n - 27$

(ii) $s = 9; r = 30n \pm 7$

(iii) $s = 9; r = 30n \pm 17$

(iv) $s = 1; r = 30n \pm 3$

(v) $s = 11; r = 30n \pm 3$

(vi) $s = 19; r = 30n \pm 3$

(vii) $s = 30n - 19; r = 60n - 27$

(viii) $s = 30n + 1; r = 60n + 3$

For each of the values of r and s the corresponding values of P and D are presented in the Table (2) below.

Table (2)

S.No	P	D
1	$75n^2 - 135n + 61$	$180n^2 - 324n + 146$
2	$75n^2 \pm 35n + 11$	$180n^2 \pm 84n - 6$
3	$75n^2 \pm 85n + 31$	$180n^2 \pm 204n + 42$
4	$75n^2 \pm 15n + 1$	$180n^2 \pm 36n + 2$
5	$75n^2 \pm 15n + 11$	$180n^2 \pm 36n + 22$
6	$75n^2 \pm 15n + 31$	$180n^2 \pm 36n + 70$

7	$375n^2-365n+91$	$540n^2-420n+74$
8	$375n^2+35n+1$	$540n^2+60n+2$

In Table(3) below represent the corresponding Pentagonal and Dodecagonal numbers are exhibited.

Table(3)

S. No	Pen_P	$Dodec_D$	α
1	$\frac{1}{2}(75n^2-135n+61)(225n^2-405n+182)$	$(180n^2-324n+146)(900n^2-1620n+726)$	$(60n-54)$
2	$\frac{1}{2}(75n^2\pm35n+11)(225n^2\pm105n+32)$	$(180n^2\pm84n-6)(900n^2\pm420n-34)$	$(540n\pm126)$
3	$\frac{1}{2}(75n^2\pm85n+31)(225n^2\pm255n+92)$	$(180n^2\pm204n+42)(900n^2\pm1020n+206)$	$(540n\pm306)$
4	$\frac{1}{2}(75n^2\pm15n+1)(225n^2\pm45n+2)$	$(180n^2\pm36n+2)(900n^2\pm180n+6)$	$(60n\pm6)$
5	$\frac{1}{2}(75n^2\pm15n+11)(225n^2\pm45n+32)$	$(180n^2\pm36n+22)(900n^2\pm180n-114)$	$(660n\pm66)$
6	$\frac{1}{2}(75n^2\pm15n+31)(225n^2\pm45n+92)$	$(180n^2\pm36n+70)(900n^2\pm180n-354)$	$(1140n\pm114)$
7	$\frac{1}{2}(375n^2-365n+91)(1125n^2-1095n+272)$	$(540n^2-420n+74)(2700n^2-2100n+366)$	$(3600n^2-3900n+1026)$
8	$\frac{1}{2}(375n^2+35n+1)(1125n^2+105n+2)$	$(540n^2+60n+2)(2700n^2+300n+6)$	$(3600n^2+300n+6)$

Case2.3:

The Choice

$$12Q-3=r^2+s^2;10H-3=r^2-s^2 \quad (6)$$

in (1) leads to the relation

$$72Hex_Q-40Hep_H=\alpha^2.$$

From (6), the values of ranks of the Hexagonal and Heptagonal numbers are respectively given by

$$Q=\frac{r^2+s^2+3}{12};H=\frac{r^2-s^2+3}{10}$$

which are integers for the following three choices of r and s namely,

(i) $s=15n-3;r=15n+6$

(ii) $s=15m-12;r=30n+15m-21$

(iii) $s=15m-12;r=30n+15m-39$

For each of the values of r and s the values of Q and H are presented in the Table(4) below

Table(4)

S.No	Q	H
1	$\frac{1}{12}(450n^2+90n+48)$	$27n+3$
2	$\frac{1}{12}(900n^2+900nm+450m^2-1260n-990m+588)$	$90n^2+90nm-126n-27m+30$
3	$\frac{1}{12}(900n^2+900nm+450m^2-2340n-1170m+1668)$	$90n^2+90nm-234n-81m+138$

In Table(5) below represent the corresponding Hexagonal and Heptagonal numbers are exhibited.

Table(5)

s.no	Hex_Q	Hep_H	α
1	$\frac{1}{72}(450n^2+90n+48)(450n^2+90n+42)$	$(27n+3)(135n+12)$	$(450n^2+90n-36)$
2	$\frac{1}{72}(900n^2+900nm+450m^2-1260n-990m+588)(900n^2+900nm+450m^2-1260n-990m+582)$	$(90n^2+90nm-126n-27m+30)*(450n^2+450nm-630n-135m+147)$	$450m^2-990m-720n+900nm+504$
3	$\frac{1}{72}(900n^2+900nm+450m^2-2340n-1170m+1688)(900n^2+900nm+450m^2-2340n-1170m+1682)$	$(90n^2+90nm-234n-81m+138)*(450n^2+450nm-1170n-405m+687)$	$450m^2-1170m-720n+900nm+936$

III. Conclusion

One may search for relations among other m-gonal numbers such that the difference in each pair is a perfect square.

References

[1] M.A.Gopalan and G.Janaki,Observations on $y^2=3x^2+1$ Acta Ciencia Indica XXXIVM (2) (2008) 103-106.

[2] M.A.Gopalan and B.Sivakami,Observations on the integral solutions of $y^2=7x^2+1$, Antarctica J. Math. 7(3),(2010) ,291-296

[3] M.A.Gopalan and G.Srividhya,Relations among m-gonal numbers through equation $y^2=2x^2-1$,Antarctica J. Math. 7(3),(2010) ,363-369

[4] M.A.Gopalan and R.Vijayalakshmi,Special Pythagorean triangles generated through the integral solutions of the equation $y^2=(k^2+1)x^2+1$,Antarctica J. Math. 7(5),(2010), 503-507

[5] M.A.Gopalan and R.Vijayalakshmi, Observations on the integral solutions of $y^2=5x^2+1$, Impact J.Sci.Tech. 4(4),(2010), 125-129

[6] M.A.Gopalan and R.S.Yamuna, Remarkable observations on the binary quadratic equation $y^2=(k^2+2)x^2+1$, Impact J.Sci.Tech. 4(4),(2010), 61-65

[7] M.A.Gopalan and G.Sangeetha,A Remarkable observation on the binary quadratic equation $y^2=10x^2+1$, Impact J.Sci.Tech. 4 (2010), 103-106

[8] M.A.Gopalan and R.Palanikumar, Observations on $y^2=12x^2+1$, Antarctica J. Math. 8(2),(2011),149-152

[9] Manju Somanath,G.Sangeetha and M.A.Gopalan, Relations among special figurate numbers through equation $y^2=10x^2+1$, Impact J.Sci.Tech. 5(1),(2011), 57-60

[10] M.A.Gopalan and K.Geetha, Observations on the Hyperbola $y^2=18x^2+1$,RETELL, 13(1), Nov.2012,81-83.

[11]. M.A.Gopalan, V.Sangeetha and Manju Somanath, Pythagorean equation and Special m-gonal numbers, sent to Antartica journal