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On Contra D-Continuous Functions and Strongly D-Colsed Spaces

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Abstract: *In[8], Dontchev introduced and investigated a new notion of continuity called contra-continuity. Recently, Jafari and Noiri ([12], [13], [14]) introduced new generalization of contra-continuity called contra-super-continuity, contra-α-continuity and contra-pre-continuity.It is the objective of this paper to introduce and study a new class of contra-continuous functions via*

I. Introduction

Jafari and Noiri introduced and investigated the notions of contra-pre-continuity [14], contra-α-continuity [13] and contra-super-continuity [12] as a continuation of research done by Dontchev [8], and Dontchev and Noiri [10] on the interesting notions of contra- continuity and contra-semi-continuity, respectively. Caldas and jafari [7] introduced the notion of contra-β-continuous functions in topologiced spaces. The aim of this paper is to introduce and investigate a new class of functions called contra-D-continuous functions.

II. Preliminaries

Throughout this paper (X,τ),(Y,σ) and (Z,η) will always denote topological spaces on which no separation axioms are assumed, unless otherwise mentioned.When A is a subset of (X,τ),
cl(A) and int(A) denote the closure and the interior of A, respectively.
We recall some known definitions needed in this paper.

Definition 2.1. Let (X,τ) be a topological space. A subset A of the space X is said to be
1. Preopen [17] if $A \subseteq int(cl(A))$ and preclosed if $cl(int(A)) \subseteq A$.
2. Semi open [15] if $A \subseteq cl(int(A))$ and semi closed if $int(cl(A)) \subseteq A$.
3. Regular open [26] if A = int(cl(A)) and regular closed if A = cl(int(A)).

Definition 2.2.Let (X,τ) be a topological space . A subset $A \subseteq X$ is said to be
1. g-closed [15] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and Uis open in X.
2. ω-closed[28] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.
3. D-closed[1] if $pcl(A) \subseteq Int(U)$ whenever $A \subseteq U$ and U is ω-open in X.
The complements of above mentioned sets are called their respective open sets.

Definition 2.3. A function $f : (X,\tau) \to (Y,\sigma)$ is called
1. g-continuous [5] if $f^{-1}(V)$ is g-closed in (X,τ) for every closed set V in (Y,σ).
2. ω-continuous [23] if $f^{-1}(V)$ is ω-closed in (X,τ) for every closed set V in (Y,σ).
3. Perfectly continuous [4] if $f^{-1}(V)$ is clopen in (X,τ) for every open set V in (Y,σ).
4. D-continuous [3] if $f^{-1}(V)$ is D-closed in (X,τ) for every closed set V in (Y,σ).
5. D-irresolute [2] if $f^{-1}(V)$ is D-closed in (X,τ) for every D-closed set V in (Y,σ).
6. Strongly D-continuous [3] if $f^{-1}(V)$ is closed in (X,τ) for every D-closed set V in (Y,σ).
7. Pre-D-continuous [3] if $f^{-1}(V)$ is D- closed in (X,τ) for every pre-closed set V in (Y,σ).
8. Perfectly D-continuous[2] if $f^{-1}(V)$ is clopen in (X,τ) for every D-closed set V in (Y,σ).
9. Super continuous [21] if $f^{-1}(V)$ is regular open in (X,τ) for every open set V in (Y,σ).
10. Contra-continuous [8] if $f^{-1}(V)$ is closed in (X,τ) for every open set V in (Y,σ).
11. Contra pre-continuous [14] if $f^{-1}(V)$ is preclosed in (X,τ) for every open set V in (Y,σ).
12. Contra g-continuous [6] if $f^{-1}(V)$ is g-closed in (X,τ) for every open set V in (Y,σ).
13. Contra semi-continuous [10] if $f^{-1}(V)$ is semiclosed in (X,τ) for every open set V in (Y,σ).
14. RC-continuous [10] if $f^{-1}(V)$ is regular closed in (X,τ) for every open set V in (Y,σ).
15. D-open if f(V) is D-open in (Y,σ) for every D-open set V in (X,τ).

Definition 2.4. A space (X,τ) is called
1. A $T_{1/2}$ space [21] if every g-closed set is closed.

2. A T_ω space [23] if every ω-closed set is closed.
3. A D-T_s space [3] if every D-closed set is closed.
4. A D-$T_{1/2}$ space [3] if every D-closed set is preclosed.

Theorem 2.5 [1] Let (X,τ) be a topological space.
1. A subset A of (X,τ) is regular open if and only if A is open and D-closed.
2. A subset A of (X,τ) is open and regular closed then A is D-closed.

Theorem 2.6 [2] Every closed set in a topological space (X,τ) is D-closed.

III. Contra-D-Continuous Functions

Definition 3.1

A function f : (X,τ) → (Y,σ) is called contra-D-continuous if $f^{-1}(V)$ is D-open (resp.D-closed) in (X,τ) for every closed (resp. open) set V in (Y,σ).

Example 3.2

Let X = {a, b, c} = Y, τ = {ϕ, {a}, X} and σ = {ϕ, {b, c}, Y}. Then the identity function f : (X, τ) → (Y, σ) is contra-D-continuous function, since for the closed (resp.open) set V = {a} in (Y,σ), $f^{-1}(V)$ = {a} is D-open (resp. D-closed) in (X, τ).

Definition 3.3

Let A be a subset of a topological space (X, τ). The set ∩{U ∈τ / A ⊂ U} is called the kernel of A [19] and is denoted by Ker(A).

Lemma 3.4 [12]

The following properties hold for subsets A, B of a space X :
1. x∈ Ker(A) if and only if A ∩ F ≠ ϕ for any F∈C(X, x).
2. A ⊂ Ker(A) and A = Ker(A) if A is open in X.
3. If A ⊂ B then Ker(A) ⊂ Ker (B)

Theorem 3.5

Every contra-continuous function is a contra-D-continuous function.

Proof

Let f : (X,τ) → (Y,σ) be a function. Let V be an open set in (Y,σ).Since f is contra- continuous, $f^{-1}(V)$ is closed in (X, τ).Hence by theorem 2.6, $f^{-1}(V)$ is D-closed in (X, τ).
Thus f is a contra-D-continuous function.

Remark 3.6

Converse of this theorem need not be true as seen from the following example.

Example 3.7

Let X = {a, b, c} = Y, τ = {ϕ, {a}, X} and σ = {ϕ, {a}, {b, c}, Y}.
Define f : (X, τ) → (Y, σ) by f(a) = b; f(b) = c and f(c) = a.Then f is contra-D-continuous but not contra-continuous, since for the open (resp.closed) set U = {b,c}, $f^{-1}(U)$ = {a, b} is D-closed (resp. D-open) but it is not closed.

Remark 3.8

Contra-D-continuous and contra-g-continuous (resp. contra-continuous, contra-D-continuous, contra pre-continuous, contra semi-continuous) are independent concepts.

Example 3.9

As in remarks 3.23, 3.15, 3.13 and 3.18 [1], the result follows.

Remark 3.10

The composition of two contra D-continuous functions need not be contra D-continuous and this is shown by the following example.

Example 3.11

Let X = {a, b, c} = Y = Z, τ = {ϕ,{a},X}, σ = {ϕ, {b, c},Y} and η = {ϕ,{a, c},Z}. Define f : (X, τ) → (Y, σ) by f(a) = a; f(b) = b and f(c) = b. Then f is contra-D-continuous, since for the closed set V = {a}, $f^{-1}(V)$ = {a} is D-open in (X, τ).Define g : (Y, σ) → (Z, η) by g(x) = x. Then g is contra-D-continuous, since for the closed set V = {b} in (Z, η), g^{-1}(V) = {b} is D-open in (Y,σ). But their composition is not a contra-D-continuous, since for the closed set V={b}in (Z,η),$f^{-1}(g^{-1}(V))$ =f^{-1}({b})={b, c} is not a D-open in (X, τ).

Theorem 3.12

The following are equivalent for a function f : (X, τ) $\rightarrow$ (Y, σ) : Assume that DO(X) (resp. DC(X)) is closed under any union (resp. intersection)

1. f is contra-D-continuous
2. The inverse image of a closed set V of Y is D-open
3. For each x$\in$X and each V$\in$C(Y, f(x)), there exists U$\in$DO(X, x) such that f(U) $\subseteq$ V.
4. f(D-cl(A)) $\subseteq$ Ker (f(A)) for every subset A of X.
5. D-cl(f^{-1}(B)) $\subseteq$ f^{-1} (Ker (B)) for every subset B of Y.

Proof

The implications (1) $\Rightarrow$ (2), (2) $\Rightarrow$ (3) are obvious.

(3) $\Rightarrow$ (2)

Let V be any closed set of Y and x $\in$ f^{-1}(V). Then f(x) $\in$ V and there exists
$U_x$$\in$DO(X, x) such that f($U_x$) $\subset$V.Hence we obtain f^{-1}(V) = $\cup$\{U_x / x $\in$ f^{-1}(V))and by assumption f^{-1}(V) is D-open.

(2) $\Rightarrow$ (4)

Let A be any subset of X. Suppose that y $\notin$ Ker (f(A)).Then by Lemma 3.4, there exists V $\in$ C (X, x) such that f(A) $\cap$ V = ϕ. Thus we have A $\cap$ f^{-1}(V) = ϕ and D-cl(A) $\cap$ f^{-1}(V) = ϕ. Hence we obtain f(D-cl(A)) $\cap$ V = ϕ and y $\notin$ f(D – cl(A)).Thus f(D-cl(A)) $\subseteq$ Ker(f(A)).

(4) $\Rightarrow$ (5)

Let B be any subset of Y.By (4) and Lemma 3.4, we have
f(D-cl(f^{-1}(B))) $\subset$ Ker(f(f^{-1}(B))) $\subset$ker(B) andD– cl(f^{-1}(B)) $\subset$ f^{-1}(Ker (B)).

(5) $\Rightarrow$ (1)

Let U be any open set of Y.Then by lemma 3.4, we have
D–cl(f^{-1}(U)) $\subset$ f^{-1}(Ker(U)) = f^{-1}(U) and D-cl(f^{-1}(U)) = f^{-1}(U). By assumption, f^{-1}(U) is
D-closed in X.Hence f is contra-D-continuous.

Theorem 3.13

If f : (X, τ) $\rightarrow$ (Y, σ) is D-irresolute (resp. contra-D-continuous) and
g : (Y, σ) $\rightarrow$ (Z, η) in contra-D-continuous (resp. continuous) then their composition
gof : (X,τ) $\rightarrow$ (Z, η) is contra-D-continuous.

Proof

Let U be any open set in (Z, η). Since g is contra-D-continuous (resp. continuous) then g^{-1}(V) is D-closed (resp. open) in (Y, σ) and since f is D-irresolute (resp. contra D-continuous) then $f^{-1}(g^{-1}(V))$ is D-closed in (X, τ). Hence gof is contra-D-continuous.

Theorem 3.14

If f : (X,τ) $\rightarrow$ (Y,σ) is contra-continuous and g : (Y,σ) $\rightarrow$ (Z, η) is continuous then their composition gof : (X, τ) $\rightarrow$ (Z, η) is contra-D-continuous.

Proof

Let U be any open set in (Z, η).Since g is continuous, g^{-1}(U) is open in (Y,σ).
Since f is contra-continuous, $f^{-1}(g^{-1}(U))$ is closed in (X, τ).Hence by theorem 2.6, $(gof)^{-1}$(U) is D-closed in (X, τ).Hence gof is contra-D-continuous.

Theorem 3.15

If f : (X, τ) $\rightarrow$ (Y, σ) is contra-continuous and super-continuous and g : (Y, σ) $\rightarrow$ (Z, η) is contra-continuous then their composition gof : (X, τ) $\rightarrow$ (Z, η) is contra-D-continuous.

Proof

Let U be any open set in (Z, η).Since g is contra-continuous, g^{-1}(U) is closed in (Y, σ) and since f is contra-continuous and super-continuous then $f^{-1}(g^{-1}(U))$ is both open and regular closed in (X, τ). Hence by theorem 2.5(2), $(gof)^{-1}$(U) is D-closed in (X, τ). Hence gof is contra-D-continuous.

Theorem 3.16

Let (X,τ), (Y,σ) be any topological spaces and (Y,σ) be $T_{1/2}$ space (resp.T_{ω}- space).Then the composition gof : (X, τ) $\rightarrow$ (Z, η) of contra-D-continuous function f : (X, τ) $\rightarrow$ (Y, σ) and the g-continuous (resp.ω-continuous) function g : (Y, σ) $\rightarrow$ (Z, η) is contra-D-continuous.

Proof

Let V be any closed set in (Z, η). Since g is g-continuous (resp.ω-continuous), $g^{-1}(V)$ is g-closed (resp.ω-closed) in (Y, σ) and (Y, σ) is $T_{1/2}$ space (resp. $T\omega$-space), hence $g^{-1}(V)$ is closed in (Y, σ). Since f is contra-D-continuous, $f^{-1}(g^{-1}(V))$ is D-open in (X, τ).Hence gof is contra-D-continuous.

Theorem 3.17

If f : (X, τ) $\rightarrow$ (Y, σ) is a surjective D-open function and g : (Y, σ) $\rightarrow$ (Z, η) is a function such that gof : (X, τ) $\rightarrow$ (Z, η) is contra-D-continuous then g is contra-D-continuous.

Proof

Let V be any closed subset of (Z, η). Since gof is contra-D-continuous then $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is D-open in (X, τ) and since f is surjective and D-open, then $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is D-open in (Y, σ). Hence g is contra-D-continuous.

Theorem 3.18

Let $\{X_i / i\in I\}$ be any family of topological spaces. If f : X $\rightarrow \Pi X_i$ is a contra-D- Continuous function. Then $\pi_I \circ f : X \rightarrow X_i$ is contra-D-continuous for each i $\in$I, where π_i is the projection of ΠX_i onto X_i.

Proof

It follows from theorem 3.13 and the fact that the projection is continuous.

Theorem 3.19

If f : (X, τ) $\rightarrow$ (Y, σ) is strongly D-continuous and g : (Y, σ) $\rightarrow$ (Z, η) is contra-D-continuous then gof : (X, τ) $\rightarrow$ (Z, η) is contra-continuous.

Proof

Let U be any open set in (Z, η). Since g is contra-D-continuous, then $g^{-1}(U)$ is D-closed in (Y, σ). Since f is strongly D-continuous, then $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$ is closed in (X, τ). Hence gof is contra-continuous.

Theorem 3.20

If f : (X, τ) $\rightarrow$ (Y, σ) is pre-D-continuous and g : (Y, σ) $\rightarrow$ (Z, η) is contra-pre-continuous then gof : (X, τ) $\rightarrow$ (Z, η) is contra-D-continuous.

Proof

Let U be any open set in (Z, η). Since g is contra-pre-continuous, then $g^{-1}(U)$ is pre-closed in (Y, σ) and since f is pre-D-continuous, then $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$ is D-closed in (X, τ). Hence gof is contra-D-continuous.

Theorem 3.21

If f : (X, τ) $\rightarrow$ (Y, σ) is strongly-D-continuous and g : (Y, σ) $\rightarrow$ (Z, η) is contra-D-continuous then gof : (X, τ) $\rightarrow$ (Z, η) is contra-D-continuous.

Proof

Let U be any open set in (Z, η). Since g is contra-D-continuous, then $g^{-1}(U))$ is D-closed in (Y, σ) and since f is strongly-D-continuous, then $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$ is closed in (X, τ). By theorem 2.6, $(gof)^{-1}(U)$ is D-closed in (X, τ). Hence gof is contra-D-continuous.

Theorem 3.22

Let f : (X, τ) $\rightarrow$ (Y, σ) be surjective D-irresolute and D-open and g : (Y,σ) $\rightarrow$ (Z, η) be any function. Then gof : (X, τ) $\rightarrow$ (Z, η) is contra-D-continuous if and only if g is contra-D-continuous.

Proof

The 'if' part is easy to prove. To prove the 'only if' part, let V be any closed set in (Z, η). Since gof is contra-D-continuous, then $(gof)^{-1}(V)$ is D-open in (X, τ) and since f is D-open surjection, then $f((gof)^{-1}(V)) = g^{-1}(V)$ is D-open in (Y,σ). Hence g is contra-D-continuous.

Theorem 3.23

Let f : (X,τ) $\rightarrow$ (Y,σ) be a contra-D-continuous function and H an open D-closed subset of (X,τ). Assume that DC(X,τ) (the class of all D-closed sets of (X,τ)) is D-closed under finite intersections. Then the restriction $f_H : (H,\tau_H) \rightarrow (Y,\sigma)$ is contra-D-continuous.

Proof

Let U be any open set in (Y,σ). By hypothesis and assumption,$f^{-1}(U)\cap H = H_1$(say) is D-closed in (X,τ).Since $(f_H)^{-1}(U) = H_1$, it is sufficient to show that H_1 is D-closed in H. By hypothesis 4.22 [3], H_1 is D-closed in H.Thus f_H is contra-D-continuous.

Theorem 3.24

Let f : (X,τ) $\rightarrow$ (Y,σ) be a function and g : X $\rightarrow$X $\times$ Y the graph function given by g(x) = (x,f(x)) for every x$\in$X.Then f is contra-D-continuous if g is contra-D-continuous.

Proof

Let V be a closed subset of Y.Then X $\times$ V is a closed subset of X $\times$Y.Since g is contra-D-continuous ,then $g^{-1}(X \times V)$ is a D-open subset of X. Also $g^{-1}(X \times V) = f^{-1}(V)$.Hence f is contra-D-continuous.

Theorem 3.25

If a function $f : (X, \tau) \to (Y, \sigma)$ is contra-D-continuous and Y is regular, then f is D-continuous.

Proof

Let x be an arbitrary point of X and N be an open set of Y containing f(x). Since Y is regular, there exists an open set U in Y containing f(x) such that $cl(U) \subseteq N$. Since f is contra-D-continuous, by theorem 3.12, there exists $W \in DO(X, x)$ such that $f(W) \subseteq cl(U)$. Then $f(W) \subseteq N$. Hence by theorem 4.13 [3], f is D-continuous.

Theorem 3.26

Every continuous and RC-continuous function is contra-D-continuous.

Proof

Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Let U be an open set in (Y, σ). Since f is continuous and RC-continuous, $f^{-1}(U)$ is open and regular closed in (X, τ). Hence by theorem 2.5(1), f is contra-D-continuous.

Theorem 3.27

Every continuous and contra-D-continuous (resp. contra-continuous and D-continuous) function is a super-continuous (resp. RC–continuous) function.

Proof

Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Let U be an open (resp. closed) set in (Y, σ).
Since f is continuous and contra-D-continuous(resp.contra-continuous and D-continuous),$f^{-1}(U)$ is open and D-closed in (X, τ). Hence by theorem 2.5(1), $f^{-1}(U)$ is regular open in (X, τ). This shows that f is a super-continuous (resp. RC-continuous) function.

Theorem 3.28

Let $f : (X, \tau) \to (Y, \sigma)$ be a function and X a D-T_s space. Then the following are equivalent.

1. f is contra-D-continuous.
2. f is contra-continuous

Proof

(1) $\Rightarrow$ (2).

Let U be an open set in (Y, σ). Since f is contra-D-continuous, $f^{-1}(U)$ is D-closed in (X, τ) and since X is D-Ts space, $f^{-1}(U)$ is closed in (X, τ). Hence f is contra-continuous.

(2) $\Rightarrow$ (1).

Let U be an open set in (Y, σ). Since f is contra-continuous, $f^{-1}(U)$ is closed in (X, τ). Hence by theorem 2.6, $f^{-1}(U)$ is D-closed in (X, τ). Hence f is contra-D-continuous.

IV. Contra-D-closed and strongly D-closed

Definition 4.1

The graph G(f) of a function $f : (X, \tau) \to (Y, \sigma)$ is said to be contra-D-closed in $X \times Y$ if for each $(x, y) \in (X \times Y) - G(f)$ there exist $U \in DO(X, x)$ and $V \in C(Y, y)$ such that $(U \times V) \cap G(f) = \phi$.

Lemma 4.2

The graph G(f) of a function $f : (X, \tau) \to (Y, \sigma)$ is contra-D-closed if and only if for each $(x, y) \in (X \times Y) - G(f)$, there exists $U \in DO(X, x)$ and $V \in C(Y, y)$ such that $f(U) \cap V = \phi$.

Theorem 4.3

If $f : (X, \tau) \to (Y, \sigma)$ is contra-D-continuous and Y is Urysohn then G(f) is contra-D-closed in $X \times Y$.

Proof

Let $(x, y) \in X \times Y - G(f)$.Then $y \neq f(x)$ and there exist open sets V,W such that $f(x)\in V$, $y \in W$ and $cl(V) \cap cl(W) = \phi$. Since f is contra-D-continuous and by theorem 3.12 there exists $U\in DO(X, x)$ such that $f(U) \subseteq V$.Hence $f(U) \cap cl(W) = \phi$.Thus by lemma 4.2, G(f) is contra D-closed in $X \times Y$.

Definition 4.4. A topological space (X,τ) is said to be

1. Strongly S-closed [8] if every closed cover of X has a finite subcover.
2. S-closed [29] if every regular closed cover of X has a finite subcover.
3. Strongly compact [18] if every preopen cover of X has a finite subcover.
4. Locally indiscrete [19] if every open set of X is closed in X.
5. Midly Hausdorff [9] if the δ-closed sets form a network for its topology τ,where a δ-closed set is the intersection of regular closed sets.
6. Ultra normal [23] if each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets

7. Nearly compact [24] if every regular open cover of X has a finite subcover.
8. D-compact [3] if every D-open cover of X has a finite subcover.
9. D-connected [3] if X cannot be written as the disjoint union of two non-empty D-open Sets.

Definition 4.5 A topological space (X,τ) is said to be strongly D-closed if every D-closed cover of X has a finite subcover.

Example 4.6

A D-T_s strongly S-closed space is strongly D-closed.

Theorem 4.7

Let (X, τ) be D-Ts space. If $f : (X, \tau) \to (Y, \sigma)$ has a contra-D-closed graph, then the inverse image of a strongly S-closed set K of Y is closed in (X, τ).

Proof

Let K be a strongly S-closed set of Y and $x \in f^{-1}(K)$. For each $k \in K$, $(x, k) \notin G(f)$. By Lemma 4.2, there exist $U_k \in DO(X, x)$ and $V_k \in C(Y, k)$ such that $f(U_k) \cap V_k = \phi$.

Since$\{K \cap V_k / k \in K\}$ is a closed cover of the subspace K, there exists a finite subset $K_0 \subset K$ such that $K \subset \cup\{V_k / k \in K_0\}$.Set $U = \cap \{U_k / k \in K_0\}$. Then U is open, since X is a D-Ts space.Therefore $f(U) \cap K = \phi$ and $U \cap f^{-1}(K) = \phi$.This shows that $f^{-1}(K)$ is closed in (X, τ).

Theorem 4.8

If a space (X,τ) is strongly D-closed then the space is strongly S-closed.

Proof

This proof follows from the definitions of 4.4 and 4.5 and by theorem 2.6.

Theorem 4.9

Let (X,τ) be D-connected and (Y, σ) be a T_1-space. If $f : (X,\tau) \to (Y,\sigma)$ is contra-D-continuous then f is constant.

Proof

Since (Y, σ) is a T_1 space, $\wedge = \{f^{-1}(y) / y \in Y\}$ is a disjoint D-open partition of X.

If $|\wedge| \geq 2$, then X is the union of two non-empty D-open sets. Since (X,τ) is D-connected, $| \wedge | = 1$. Hence f is constant.

Theorem 4.10

Let $f : (X, \tau) \to (Y, \sigma)$ be a contra-D-continuous and pre-closed surjection. If (X, τ) is a D-Ts, then (X, τ) is a locally indiscrete space.

Proof

Let U be any open set in (Y, σ). Since f is contra-D-continuous and (X, τ) is a D-Ts space, $f^{-1}(U)$ is closed in (X, τ). Since f is a pre-closed surjection, then U is pre-closed in (Y, σ). Therefore $cl(U) = cl(Int(U)) \subset U$. Hence U is closed in (Y, σ). Thus (Y, σ) is a locally indiscrete space.

Theorem 4.11

If every closed subset of a space X is D-open then the following are equivalent.

1. X is S-closed
2. X is strongly S-closed

Proof

(1) $\Rightarrow$ (2)

Let $\{V_\alpha / \alpha \in I\}$ be a closed cover of X. Then by hypothesis and by theorem 2.5(1), $\{V_\alpha / \alpha \in I\}$ is a regular closed cover of X. Since X is S-closed, then we have a finite sub cover of X. Hence X is strongly S-closed.

(2) $\Rightarrow$ (1)

Let $\{V_\alpha / \alpha \in I\}$ be a regular closed cover of X. Since every regular closed is closed and X is strongly S-closed, then we have a finite subcover of X. Hence X is S-closed.

Definition 4.12

A topological space (X, τ) is said to be

1. D-Hausdorff if for each pair of distinct points x and y in X there exist disjoint D-open sets U and V of x and y respectively.

2. D-Ultra Hausdorff if for each pair of distinct points x and y in X there exist disjoint D-clopen sets U and V of x and y respectively.

Theorem 4.13

If f : (X, τ) → (Y, σ) is contra-D-continuous injection, where Y is Urysohn then the topological space (X, τ) is a D-Hausdorff.

Proof :

Let x_1 and x_2 be two distinct points of (X, τ). Suppose $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since f is injective and $x_1 \neq x_2$ then $y_1 \neq y_2$. Since the space Y is Urysohn, there exist open sets V and W such that $y_1 \in V, y_2 \in W$ and $cl(V) \cap cl(W) = \phi$. Since f is contra-D-continuous and by theorem 3.12, there exist D-open sets $Ux_1 \in DO(X, x_1)$ and $Ux_2 \in DO(X,x_2)$ such that $f(Ux_1) \subset cl(V)$ and $f(Ux_2) \subset cl(W)$. Thus we have $Ux_1 \cap Ux_2 = \phi$, since $cl(V) \cap cl(W) = \phi$. Hence X is a D-Hausdorff.

Theorem 4.14

If f : (X,τ) → (Y,σ) is a contra-D-continuous injection, where Y is D-ultra Hausdorff then the topological space (X, τ) is D-Hausdorff.

Proof

Let x_1 and x_2 be two distinct points of (X, τ). Since f is injective and Y is D-ultra Hausdorff, then $f(x_1) \neq f(x_2)$ and also there exist clopen sets U and W in Y such that $f(x_1) \in U$ and $f(x_2) \in W$, where $U \cap W = \phi$. Since f is contra-D-continuous, x_1 and x_2 belong to D-open sets $f^{-1}(U)$ and $f^{-1}(W)$ respectively, where $f^{-1}(U) \cap f^{-1}(W) = \phi$.Hence X is D-Hausdorff.

Lemma 4.15 [9]

Every mildly Hausdorff strongly S-closed space is locally indiscrete.

Theorem 4.16

If a function f : (X, τ) → (Y, σ) is continuous and (X, τ) is a locally indiscrete space, then f is contra-D-continuous.

Proof

Let U be any open set in (Y, σ). Since f is continuous, $f^{-1}(U)$ is open in (X, τ) and since (X, τ) is locally indiscrete, $f^{-1}(U)$ is closed in (X, τ). Hence by theorem 2.6, $f^{-1}(U)$ is D-closed in (X, τ). Thus f is contra-D-continuous.

Corollary 4.17

If a function f : (X, τ) → (Y, σ) is continuous and (X, τ) is mildly Hausdorff strongly S-closed space then f is contra-D-continuous.

Proof

It follows from Lemma 4.15 and theorem 4.16.

Theorem 4.18

A contra-D-continuous image of a D-connected space is connected.

Proof

Let f : (X, τ) → (Y, σ) be a contra-D-continuous function of D-connected space onto a topological space Y. If possible, assume that Y is not connected. Then $Y = A \cup B$, $A \neq \phi$, $B \neq \phi$ and $A \cap B = \phi$, where A and B are clopen sets in Y. Since f is contra-D-continuous, $X = f^{-1}(Y) = f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are non-empty D-open sets in X. Also $f^{-1}(A) \cap f^{-1}(B) = \phi$. Hence X is not D-connected, which is a contradiction. Therefore Y is connected.

Definition 4.19

A topological space (X, τ) is said to be D-normal if each pair of non-empty disjoint closed sets can be separated by disjoint D-open sets.

Theorem 4.20

If f : (X, τ) → (Y, σ) is a closed contra-D-continuous injection and Y is ultra-normal, then X is D-normal.

Proof

Let V_1 and V_2 be non-empty disjoint closed subsets of X. Since f is closed and injective, then $f(V_1)$ and $f(V_2)$ are non-empty disjoint closed subsets of Y. Since Y is ultra-normal, then $f(V_1)$ and $f(V_2)$ can be separated by disjoint clopen sets W_1 and W_2 respectively.
Hence$V_1 \subset f^{-1}(W_1)$ and $V_2 \subset f^{-1}(W_1)$.Since f is contra-D-continuous, then $f^{-1}(W_1)$ and $f^{-1}(W_2)$ are D-open subsets of X and $f^{-1}(W_1) \cap f^{-1}(W_2) = \phi$. Hence X is D-normal.

Theorem 4.21

The image of a strongly D-closed space under a contra-D-continuous surjective function is compact.

Proof

Suppose that f : (X, τ) → (Y, σ) is a contra-D-continuous surjection. Let $\{V_\alpha / \alpha \in I\}$ be any open cover of Y. Since f is contra-D-continuous, then $\{f^{-1}(V_\alpha) / \alpha \in I\}$ is a D-closed cover of

X. Since X is strongly D-closed, then there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) / \alpha \in I_0\}$. Thus we have $Y = \cup\{V\alpha / \alpha \in I_0\}$. Hence Y is compact.

Theorem 4.22

Every strongly D-closed space (X, τ) is a compact S-closed space.

Proof

Let $\{V_\alpha / \alpha \in I\}$ be a cover of X such that for every $\alpha \in I$, V_α is open and regular closed due to assumption. Then by theorem 2.5(2), each V_α is D-closed in X. Since X is strongly D-closed, there exists a finite subset I_0 of I such that $X = \cup\{V_\alpha / \alpha \in I_0\}$.Hence (X, τ) is a compact S-closed space.

Theorem 4.23

The image of a D-compact space under a contra-D-continuous surjective function is strongly S-closed.

Proof

Suppose that $f : (X, \tau) \to (Y, \sigma)$ is a contra-D-continuous surjection.Let $\{V_\alpha /\alpha \in I\}$ be any closed cover of Y. Since f is contra-D-continuous , then $\{f^{-1}(V_\alpha) /\alpha \in I\}$ is a D-open cover of X. Since X is D-compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) / \alpha \in I_0\}$. Thus we have $Y = \cup\{V_\alpha / \alpha \in I_0\}$.Hence Y is strongly S-closed.

Theorem 4.24

The image of a D-compact space in any D-Ts space under a contra-D-continuous surjective function is strongly D-closed.

Proof

Suppose that $f : (X, \tau) \to (Y, \sigma)$ is a contra-D-continuous surjection. Let $\{V_\alpha / \alpha \in I\}$ be any D-closed cover of Y. Since Y is D-Ts space, then $\{V_\alpha / \alpha \in I\}$ is a closed cover of Y. Since f is contra-D-continuous, then $\{f^{-1}(V_\alpha) / \alpha \in I\}$ is a D-open cover of X. Since X is D-compact, there exists a finite subset I_0 of I such that $X = \cup \{f^{-1}(V_\alpha) / \alpha \in I_0\}$. Thus we have $Y = \cup\{V_\alpha / \alpha \in I_0\}$. Hence Y is strongly D-closed.

Theorem 4.25

The image of strongly D-closed space under a D-irresolute surjective function is strongly D-closed.

Proof

Suppose that $f : (X, \tau) \to (Y, \sigma)$ is an D-irresolute surjection. Let $\{V_\alpha / \alpha \in I\}$ be any D-closed cover of Y. Since f is D-irresolute then $\{f^{-1}(V_\alpha) / \alpha \in I\}$ is a D-closed cover of X. Since X is strongly D-closed, then there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) / \alpha \in I_0\}$. Thus, we have $Y = \cup\{V_\alpha / \alpha \in I_0\}$. Hence Y is strongly D-closed.

Lemma 4.26

The product of two D-open sets is D-open.

Theorem 4.27

Let $f : (X_1, \tau) \to (Y, \sigma)$ and $g : (X_2, \tau) \to (Y, \sigma)$ be two functions where Y is a Urysohn space and f and g are contra-D-continuous function. Then $\{(x_1,x_2) /f(x_1) = g(x_2)\}$ is D-closed in the product space $X_1 \times X_2$.

Proof

Let V denote the set $\{(x_1,x_2) / f(x_1) = g(x_2)\}$. In order to show that V is D-closed, we show that $(X_1 \times X_2) - V$ is D-open. Let $(x_1,x_2) \notin V$. Then $f(x_1) \neq g(x_2)$.Since Y is Urysohn,there exist open sets U_1 and U_2 of $f(x_1)$ and $g(x_2)$ such that $cl(U_1) \cap cl(U_2) = \phi$. Since f and g are contra-D-continuous, $f^{-1}(cl(U_1))$ and $g^{-1}(cl(U_2))$ are D-open sets containing x_1 and x_2 in X_1 and X_2. Hence by Lemma 4.26, $f^{-1}(cl(U_1)) \times g^{-1}(cl(U_2))$ is D-open. Further $(x_1,x_2) \in f^{-1}(cl(U_1)) \times g^{-1}(cl(U_2)) \subset ((X_1 \times X_2) - V)$. If follows that $(X_1 \times X_2) - V$ is D-open. Thus V is D-closed in the product space $X_1 \times X_2$.

Corollary 4.28

If $f : (X, \tau) \to (Y, \sigma)$ is contra-D-continuous and Y is a Urysohn space, then $V = \{(x_1, x_2) / f(x_1) = f(x_2)\}$ is D-closed in the product space $X_1 \times X_2$.

Theorem 4.29

Let $f : (X, \tau) \to (Y, \sigma)$ be a continuous function. Then f is RC-continuous if and only if it is contra-D-continuous.

Proof

Suppose that f is RC-continuous.

Since every RC-continuous function is contra-continuous,Therefore by Theorem 3.5, f is contra D-continuous.

Conversely,

Let V be any open set in (Y, σ). Since f is continuous and contra-D-continuous, $f^{-1}(V)$ is open and D-closed in (X, τ). By theorem 2.5(1), $f^{-1}(V)$ is regular open in (X, τ). That is, $Int(cl(f^{-1}(V))) = f^{-1}(V)$. Since $f^{-1}(V)$ is open, $Int(cl(f^{-1}(V))) = Int(f^{-1}(V))$ and so $cl(Int(f^{-1}(V))) = f^{-1}(V)$. Therefore V is regular closed in (X, τ). Hence f is RC-continuous.

Theorem 4.30

Let f : (X, τ) $\rightarrow$ (Y, σ) be perfectly D-continuous function, X be locally indiscrete space and connected. Then Y has an indiscrete topology.

Proof

Suppose that there exists a proper open set U of Y. Since Y is locally indiscrete, U is a closed set of Y. Therefore by theorem 2.6, U is a D-closed set of Y. Since f is perfectly D-continuous, $f^{-1}(U)$ is a proper clopen set of X. This shows that X is not connected. Which is a contradiction. Therefore Y has an indiscrete topology.

Theorem 4.31

If f : (X, τ) $\rightarrow$ (Y, σ) is a function and (X, τ) a D-Ts space, then the following statements are equivalent :

1. f is perfectly continuous.
2. f is continuous and contra-continuous
3. f is continuous and contra-D-continuous.
4. f is super-continuous.

Proof

(1) $\Rightarrow$ (2) is obvious.

(2) $\Rightarrow$ (3) by theorem 2.6 , it is clear.

(3) $\Rightarrow$ (4) by theorem 3.27, it is clear

(4) $\Rightarrow$ (1) Let U be any open set in (Y, σ). By assumption, $f^{-1}(U)$ is regular open in (X, τ). By theorem 2.5(1), $f^{-1}(U)$ is open and D-closed in (X, τ). Since (X, τ) is a D-Ts space, $f^{-1}(U)$ is clopen in (X, τ). Hence f is perfectly continuous.

Theorem 4.32

Let f : (X, τ) $\rightarrow$ (Y, σ) be a contra-D-continuous function. Let A be an open D-closed subset of X and let B be an open subset of Y. Assume that DC(X, τ) (the class of all D-closed sets of (X, τ)) be D-closed under finite intersections. Then, the restriction f | A : (A, τ_A) $\rightarrow$ (B, σ_B) is a contra-D-continuous function.

Proof

Let V be an open set in (B, σ_B). Then V = B$\cap$K for some open set K in (Y, σ). Since B is an open set of Y, V is an open set in (Y, σ).By hypothesis and assumption,$f^{-1}(V) \cap A = H_1$ (say) is a D-closed set in (X, τ). Since $(f|A)^{-1}(V) = H_1$, it is sufficient to show that H_1 is a D-closed set in (A,τ_A). Let G_1 be ω-open in (A,τ_A) such that $H_1 \subseteq G_1$. Then by hypothesis and by Lemma 4.21[3], G_1 is ω-open in (X, τ). Since H_1 is a D-closed set in (X, τ), we have $pcl_X(H_1) \subseteq Int(G_1)$. Since A is open and Lemma 2.10[11], $pcl_A(H_1) = pcl_X(H_1) \cap A \subseteq Int(G_1) \cap Int(A) = Int(G_1 \cap A) \subseteq Int(G_1)$ and so $H_1 = (f|A)^{-1}(V)$ is a D-closed set in (A, τ_A). Hence f | A is contra-D-continuous function.

Theorem 4.33

A topological space (X, τ) is nearly compact if and only if it is compact and strongly D-closed .

Proof

Obvious by theorem 2.5(1).

Theorem 4.34

If a topological space (X, τ) is locally indiscrete space then compactness and strongly D-closedness are the same.

Proof

Let (X, τ) be a compact space. Since (X, τ) is a locally indiscrete space, then every open set is closed and by theorem 2.6, compactness and strongly D-compactness are the same in a locally indiscrete topological space.

Theorem 4.35

A topological space (X, τ) is S-closed if and only if it is strongly S-closed and D-compact.

Proof

It follows from theorem 2.5(1).

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