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On Bihermitian Matrices

G.Ramesh*, N.Anbarasi**
**Associate Professor of Mathematics, Govt. Arts College(Autonomous), Kumbakonam.*
***Assistant Professor of Mathematics, Arasu Engineering College, Kumbakonam.*

***Abstract**: Bihermitian matrices are studied as a generalization of hermitian matrices. Some of the properties of hermitian matrices are extended to bihermitian matrices. Some important results of hermitian matrices are generalized to bihermitian matrices.*
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I. Introduction

Matrices provide a very powerful tool for dealing with linear models. Bimatrices are still a powerful and an advanced tool which can handle over one linear model at a time. Bimatrices are useful when time bound comparisons are needed in the analysis of a model. Bimatrices are of several types. Here we consider all matrices belongs to C_{nxn}. For any matrix A, A^H denotes the conjugate transpose of A. In this paper we study bihermitian matrices as a generalization of hermitian matrices. Some of the properties of hermitian matrices are extended to bihermitian matrices. Some important results of hermitian matrices are generalized to bihermitian matrices.

II. Preliminaries

Definition 2.1[3]

A bimatrix A_B is defined as the union of two rectangular array of numbers A_1 and A_2 arranged into rows and columns. It is written as $A_B = A_1 \cup A_2$ with $A_1 \neq A_2$ (except zero and unit bimatrices) where,

$$\mathrm{A}_1 = \begin{bmatrix} a_{11}^1 & a_{12}^1 & \cdots & a_{1n}^1 \\ a_{21}^1 & a_{22}^1 & \cdots & a_{2n}^1 \\ \vdots & & & \\ a_{m1}^1 & a_{m2}^1 & \cdots & a_{mn}^1 \end{bmatrix} \quad \text{and} \quad \mathrm{A}_2 = \begin{bmatrix} a_{11}^2 & a_{12}^2 & \cdots & a_{1n}^2 \\ a_{21}^2 & a_{22}^2 & \cdots & a_{2n}^2 \\ \vdots & & & \\ a_{m1}^2 & a_{m2}^2 & \cdots & a_{mn}^2 \end{bmatrix}$$

$'\cup'$ is just the notational convenience (symbol) only.

Definition 2.2[3]

Let $A_B = A_1 \cup \mathrm{A}_2$ and $C_B = C_1 \cup C_2$ be any two mxn bimatrices. The sum D_B of the bimatrices A_B and C_B is defined as

$D_B = A_B + C_B = (A_1 \cup \mathrm{A}_2) + (C_1 \cup C_2) = (\mathrm{A}_1 + C_1) \cup (A_2 + C_2)$, where $A_1 + A_2$ and $C_1 + C_2$ are the usual addition of matrices.

Definition 2.3[4]

If $A_B = A_1 \cup A_2$ and $C_B = C_1 \cup C_2$ be two bimatrices ,then A_B and C_B are said to be equal (written as $A_B = C_B$) if and only if A_1 and C_1 are identical and A_2 and C_2 are identical. (That is, $A_1 = C_1$ *and* $A_2 = C_2$).

Definition 2.4[4]

Given a bimatrix $A_B = A_1 \cup \mathrm{A}_2$ and a scalar λ, the product of λ and A_B written as λA_B is defined to be

$$\lambda A_B = \begin{bmatrix} \lambda a_{11}^1 & \lambda a_{12}^1 & \cdots & \lambda a_{1n}^1 \\ \lambda a_{21}^1 & \lambda a_{22}^1 & \cdots & \lambda a_{2n}^1 \\ \vdots & & & \\ \lambda a_{m1}^1 & \lambda a_{m2}^1 & \cdots & \lambda a_{mn}^1 \end{bmatrix} \cup \begin{bmatrix} \lambda a_{11}^2 & \lambda a_{12}^2 & \cdots & \lambda a_{1n}^2 \\ \lambda a_{21}^2 & \lambda a_{22}^2 & \cdots & \lambda a_{2n}^2 \\ \vdots & & & \\ \lambda a_{m1}^2 & \lambda a_{m2}^2 & \cdots & \lambda a_{mn}^2 \end{bmatrix}$$

That is, each element of A_1 and A_2 are multiplied by λ.

Remark 2.5[4]

If $A_B = A_1 \cup A_2$ be a bimatrix, then we call A_1 and A_2 as the component matrices of the bimatrix A_B.

Definition 2.6[3]

If $A_B = A_1 \cup \mathrm{A}_2$ and $C_B = C_1 \cup C_2$ are both nxn square bimatrices then, the bimatrix multiplication is defined as,

$$\mathrm{A}_B \times C_B = (A_1C_1) \cup (A_2C_2).$$

III. Hermitian Bimatrices

Definition 3.1

A bihermitian matrix is a bimatrix $A_B = A_1 \cup \mathrm{A}_2$ for which $A_B = A_B^H$. That is, the component matrices of A_B are hermitian matrices. (That is, $A_B = A_B^H = A_1^H \cup A_2^H$).

Remark 3.2

Let $A_B = A_1 \cup \mathrm{A}_2$ be a bihermitian matrix. If A_1 and A_2 are square and posses the same order then A_B is called square bihermitian matrix, and if A_1 and A_2 are of different orders then A_B is called mixed square bihermitian matrix.

Example 3.3

$$(i)\, A_B = \begin{bmatrix} 1 & 1-i & -3+2i \\ 1+i & 2 & -i \\ -3-2i & i & 0 \end{bmatrix} \cup \begin{bmatrix} 1 & 1+2i & 2-3i \\ 1-2i & 5 & -4-2i \\ 2+3i & -4+2i & 13 \end{bmatrix}$$

is a square bihermitian.

$$(ii)\, A_B = \begin{bmatrix} 3 & 2-i & -3i \\ 2+i & 0 & 1-i \\ 3i & 1+i & 0 \end{bmatrix} \cup \begin{bmatrix} 2 & 2+i \\ 2-i & 3 \end{bmatrix}$$

is a mixed square bihermitian.

Remark 3.4

Any complex matrix can be represented as $A = A_R + iA_I$, where A_R is symmetric matrix and A_I is skew symmetric matrix. Now, any bihermitian matrix $A_B = A_1 \cup \mathrm{A}_2$ can be represented as,

$$A_B = (A_{1R} + iA_{1I}) \cup (A_{2R} + iA_{2I}) = (A_{1R}^T - iA_{1I}^T) \cup (A_{2R}^T - iA_{2I}^T) = A_1^H \cup A_2^H = A_B^H$$

Definition 3.5

Let $\mathrm{A}_B = \mathrm{A}_1 \cup A_2$ be a nxn square bimatrix. That is, A_1 and A_2 are nxn square matrices. A skew hermitian bimatrix is a bimatrix A_B for which $A_B = -A_B^H$,where $-A_B^H = -A_1^H \cup -A_2^H$. That is, the component matrices A_1 and A_2 of A_B are skew hermitian matrices.

Theorem 3.6

If A_B and B_B are bihermitian matrices then $(A_B + B_B)$ is also bihermitian.

Proof

Given $A_B = A_1 \cup \mathrm{A}_2$ and $B_B = B_1 \cup \mathrm{B}_2$ are bihermitian.

$$\left(A_B + B_B\right)^H = A_B^H + B_B^H$$
$$= \left(A_1 \cup A_2\right)^H + \left(B_1 \cup B_2\right)^H$$
$$= \left(A_1^H \cup A_2^H\right) + \left(B_1^H \cup B_2^H\right)$$
$$= \left(A_1 \cup A_2\right) + \left(B_1 \cup B_2\right)$$
$$\left(\text{since } A_1, A_2, B_1 \text{ and } B_2 \text{ are Hermitian matrices}\right)$$
$$\left(A_B + B_B\right)^H = A_B + B_B$$

Hence, $\left(A_B + B_B\right)$ is bihermitian.

Example 3.7

Let $A_B = \begin{bmatrix} 1 & 1-i & -3+2i \\ 1+i & 2 & -i \\ -3-2i & i & 0 \end{bmatrix} \cup \begin{bmatrix} 1 & 1+2i & 2-3i \\ 1-2i & 5 & -4-2i \\ 2+3i & -4+2i & 13 \end{bmatrix}$

and $B_B = \begin{bmatrix} 1 & -2i & 0 \\ 2i & -1 & 2i \\ 0 & -2i & 2 \end{bmatrix} \cup \begin{bmatrix} 1 & 2i & -i \\ -2i & -1 & 1 \\ i & 1 & 2 \end{bmatrix}$

$$A_B + B_B = \begin{bmatrix} 2 & 1-3i & -3+2i \\ 1+3i & 1 & i \\ -3-2i & i & 2 \end{bmatrix} \cup \begin{bmatrix} 1 & 2i & -i \\ -2i & -1 & 1 \\ -i & 1 & 2 \end{bmatrix}$$

Hence, $A_B + B_B$ is bihermitian.

Theorem 3.8

If A_B is bihermitian, for any scalar k, $\left(A_B - kI_B\right)$ is bihermitian.

Proof

Given $A_B = A_1 \cup A_2 = A_1^H \cup A_2^H$

That is, $A_B = \left(A_{1R} + iA_{1I}\right) \cup \left(A_{2R} + iA_{2I}\right) = \left(A_{1R}^T - iA_{1I}^T\right) \cup \left(A_{2R}^T - iA_{2I}^T\right)$

$$\left(A_B - kI_B\right)^H = \left[\left(A_1 \cup A_2\right) - k\left(I_1 \cup I_2\right)\right]^H$$
$$= \left[\left(A_1 \cup A_2\right) - \left(kI_1 \cup kI_2\right)\right]^H$$
$$= \left[\left(A_1 - kI_1\right) \cup \left(A_2 - kI_2\right)\right]^H$$
$$= \left[\left(A_{1R} + iA_{1I}\right) - kI_1\right]^H \cup \left[\left(A_{2R} + iA_{2I}\right) - kI_2\right]^H$$
$$= \left[\left(A_{1R} - kI_1\right) + iA_{1I}\right]^H \cup \left[\left(A_{2R} - kI_2\right) + iA_{2I}\right]^H$$
$$= \left[\left(A_{1R} - kI_1\right)^T - iA_{1I}^T\right] \cup \left[\left(A_{2R} - kI_2\right)^T - iA_{2I}^T\right]$$
$$= \left(A_{1R}^T - kI_1^T - iA_{1I}^T\right) \cup \left(A_{2R}^T - kI_2^T - iA_{2I}^T\right)$$
$$= \left(A_{1R}^T - iA_{1I}^T\right) - kI_1^T \cup \left(A_{2R}^T - iA_{2I}^T\right) - kI_2^T$$
$$= \left(A_{1R} + iA_{1I}\right) - kI_1 \cup \left(A_{2R} + iA_{2I}\right) - kI_2$$
$$= \left(A_1 - kI_1\right) \cup \left(A_2 - kI_2\right)$$
$$= \left(A_1 \cup A_2\right) - k\left(I_1 \cup I_2\right)$$

$= A_B - kI_B$

Hence, $A_B - kI_B$ is bihermitian.

Example 3.9

Consider the bihermitian matrix,

$$A_B = \begin{bmatrix} 1 & 1-i & -3+2i \\ 1+i & 2 & -i \\ -3-2i & i & 0 \end{bmatrix} \cup \begin{bmatrix} 1 & 1+2i & 2-3i \\ 1-2i & 5 & -4-2i \\ 2+3i & -4+2i & 13 \end{bmatrix}$$

$$I_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cup \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$kI_B = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} \cup \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$$

$$A_B - kI_B = \begin{bmatrix} 1 & 1-i & -3+2i \\ 1+i & 2 & -i \\ -3-2i & i & 0 \end{bmatrix} \cup \begin{bmatrix} 1 & 1+2i & 2-3i \\ 1-2i & 5 & -4-2i \\ 2+3i & -4+2i & 13 \end{bmatrix} - \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} \cup \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$$

$$A_B - kI_B = \begin{bmatrix} 1-k & 1-i & -3+2i \\ 1+i & 2-k & -i \\ -3-2i & i & -k \end{bmatrix} \cup \begin{bmatrix} 1-k & 1+2i & 2-3i \\ 1-2i & 5-k & -4-2i \\ 2+3i & -4+2i & 13-k \end{bmatrix}$$

Which is bihermitian.

Theorem 3.10

Any integral power of a bihermitian matrix is also bihermitian.

Proof

Let $A_B = A_1 \cup A_2$ be bihermitian.

$$A_B^H = A_1^H \cup A_2^H = A_1 \cup A_2 = A_B \quad \text{---------------}\rangle \ (1)$$

$$\left(A_B^2\right)^H = \left(A_B A_B\right)^H$$

$$= A_B^H \ A_B^H$$

$$= \left(A_1 \cup A_2\right)^H \left(A_1 \cup A_2\right)^H$$

$$= \left(A_1^H \cup A_2^H\right)\left(A_1^H \cup A_2^H\right)$$

$$= \left(A_1^H\right)^2 \cup \left(A_2^H\right)^2$$

$$= A_1^2 \cup A_2^2 \ \left(\sin ce\, A_1 \ and \ A_2 \ are\ hermitian\ matrices\right)$$

$$\left(A_B^2\right)^H = A_B^2$$

Hence, A_B^2 is bihermitian.

Assume that A_B^k is bihermitian, that is $\left(A_B^k\right)^H = A_B^k$ ---------------› (2)

To prove A_B^{k+1} is bihermitian.

$$\left(\mathrm{A}_B^{k+1}\right)^{\mathrm{H}} = \left(A_B^k\, A_B\right)^{\mathrm{H}}$$
$$= A_B^H \left(A_B^k\right)^H$$
$$= A_B\, A_B^k \quad \text{(since by (1) and (2))}$$
$$= A_B^{1+k}$$
$$\left(A_B^{k+1}\right)^{\mathrm{H}} = A_B^{k+1}$$

Hence, any integral power of a bihermitian matrix is also bihermitian.

Example 3.11

$$\text{Let } A_B = \begin{bmatrix} 1 & 1-i & -3+2i \\ 1+i & 2 & -i \\ -3-2i & i & 0 \end{bmatrix} \cup \begin{bmatrix} 1 & 1+2i & 2-3i \\ 1-2i & 5 & -4-2i \\ 2+3i & -4+2i & 13 \end{bmatrix}$$

$$A_B{}^2 = \begin{bmatrix} 1 & 1-i & -3+2i \\ 1+i & 2 & -i \\ -3-2i & i & 0 \end{bmatrix} \cup \begin{bmatrix} 1 & 1+2i & 2-3i \\ 1-2i & 5 & -4-2i \\ 2+3i & -4+2i & 13 \end{bmatrix} \times$$

$$\begin{bmatrix} 1 & 1-i & -3+2i \\ 1+i & 2 & -i \\ -3-2i & i & 0 \end{bmatrix} \cup \begin{bmatrix} 1 & 1+2i & 2-3i \\ 1-2i & 5 & -4-2i \\ 2+3i & -4+2i & 13 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 1-6i & -4+i \\ 1+6i & 3 & -5-3i \\ -4-i & -5+3i & 14 \end{bmatrix} \cup \begin{bmatrix} 19 & 4+28i & 28-52i \\ 4-28i & 48 & -76-43i \\ 28+52i & -76+43i & 202 \end{bmatrix}$$

Hence, A_B^2 is bihermitian.

Theorem 3.12

For any square matrix A, $\mathrm{A} + \mathrm{A}^{\mathrm{H}}$ is bihermitian.

Proof

Let A_B be any square bimatrix. That is, $A_B = A_1 \cup A_2$.

$$\left(A_B + A_B^H\right)^H = \left[\left(A_1 \cup A_2\right) + \left(A_1 \cup A_2\right)^H\right]^{\mathrm{H}}$$
$$= \left[\left(A_1 \cup A_2\right) + \left(A_1^H \cup A_2^H\right)\right]^H$$
$$= \left[\left(A_1 + A_1^H\right) \cup \left(A_2 + A_2^H\right)\right]^H$$
$$= \left(A_1 + A_1^H\right)^H \cup \left(A_2 + A_2^H\right)^H$$
$$= \left(A_1^H + A_1\right) \cup \left(A_2^H + A_2\right)$$
$$= \left(A_1^H \cup A_2^H\right) + \left(A_1 \cup A_2\right)$$
$$= \left(A_1 \cup A_2\right)^H + \left(A_1 \cup A_2\right)$$

$$= A_B^H + A_B$$

$$\left(A_B + A_B^H\right)^H = A_B + A_B^H$$

Hence, $A_B + {A_B}^H$ is bihermitian.

Example 3.13

Let $A_B = \begin{bmatrix} 1 & 2i & i \\ 0 & 2 & -i \\ 0 & i & 3 \end{bmatrix} \cup \begin{bmatrix} 3 & 2-i & -3i \\ 1+i & 1 & -i \\ 2+i & -i & 1 \end{bmatrix}$

$$A_B^H = \begin{bmatrix} 1 & 0 & 0 \\ -2i & 2 & -i \\ -i & i & 3 \end{bmatrix} \cup \begin{bmatrix} 3 & 1-i & 2-i \\ 2+i & 1 & i \\ 3i & i & 1 \end{bmatrix}$$

$A_B + A_B^H = \begin{bmatrix} 2 & 2i & i \\ -2i & 4 & -2i \\ -i & 2i & 6 \end{bmatrix} \cup \begin{bmatrix} 6 & 3-2i & 2-4i \\ 3+2i & 2 & 0 \\ 2+4i & 0 & 2 \end{bmatrix}$,which is bihermitian .

Theorem 3.14

For any square matrix A_B, $A_B - A_B^H$ is skew bihermitian.

Proof

Let $A_B = A_1 \cup A_2$ be a square bimatrix.

$$A_B - A_B^H = \left(A_1 \cup A_2\right) - \left(A_1 \cup A_2\right)^H$$

$$= \left(A_1 \cup A_2\right) - \left(A_1^H \cup A_2^H\right)$$

$$A_B - A_B^H = \left(A_1 - A_1^H\right) \cup \left(A_2 - A_2^H\right)$$

$$\left(A_B - A_B^H\right)^{\mathrm{H}} = \left[\left(A_1 - A_1^H\right) \cup \left(A_2 - A_2^H\right)\right]^H$$

$$= \left(A_1 - A_1^H\right)^H \cup \left(A_2 - A_2^H\right)^H$$

$$= \left(A_1^H - A_1\right) \cup \left(A_2^H - A_2\right)$$

$$= \left(A_1^H \cup A_2^H\right) - \left(A_1 \cup A_2\right)$$

$$= \left(A_1 \cup A_2\right)^H - \left(A_1 \cup A_2\right)$$

$$= A_B^H - A_B$$

$$\left(A_B - A_B^H\right)^{\mathrm{H}} = -\left(A_B - A_B^H\right)$$

Hence, $A_B - A_B^H$ is skew bihermitian.

Theorem 3.15

If A_B is bihermitian then iA_B is skew bihermitian.

Proof

Given A_B is bihermitian. That is, $A_B^H = A_1^H \cup A_2^H = A_B$

Now, $\left(iA_B\right)^H = \left[i\left(A_1 \cup A_2\right)\right]^H$

$$= -i\left(A_1 \cup A_2\right)^H$$

$$= -i\left(A_1^H \cup A_2^H\right)$$

$$= -i\left(A_1 \cup A_2\right) \quad (\text{since } A_1 \text{ and } A_2 \text{ are hermitian})$$

$$\left(iA_B\right)^H = -i\,A_B$$

Hence, iA_B is skew bihermitian.

Example 3.16

Let $A_B = \begin{bmatrix} 1 & 1-i & -3+2i \\ 1+i & 2 & -i \\ -3-2i & i & 0 \end{bmatrix} \cup \begin{bmatrix} 1 & 1+2i & 2-3i \\ 1-2i & 5 & -4-2i \\ 2+3i & -4+2i & 13 \end{bmatrix}$

$$iA_B = \begin{bmatrix} i & 1+i & -2-3i \\ -1+i & 2i & 1 \\ 2-3i & -1 & 0 \end{bmatrix} \cup \begin{bmatrix} i & -2+i & 3+2i \\ 2+i & 5i & 2-4i \\ -3+2i & -2-4i & 13i \end{bmatrix}$$

$$\left(iA_B\right)^H = \begin{bmatrix} -i & -1-i & 2+3i \\ 1-i & -2i & -1 \\ -2+3i & 1 & 0 \end{bmatrix} \cup \begin{bmatrix} -i & 2-i & 2i+3 \\ -2-i & -5i & -2+4i \\ 3-2i & 2+4i & -13i \end{bmatrix}$$

$$= -\left\{\begin{bmatrix} i & 1+i & -2-3i \\ -1+i & 2i & 1 \\ 2-3i & -1 & 0 \end{bmatrix} \cup \begin{bmatrix} i & -2+i & 3+2i \\ 2+i & 5i & -2+4i \\ -3+2i & -2-4i & 13i \end{bmatrix}\right\}$$

$$\left(iA_B\right)^H = -\left(iA_B\right)$$

Hence, iA_B is skew bihermitian.

Theorem 3.17

If A_B is skew bihermitian then iA_B is bihermitian.

Proof

Given A_B is skew bihermitian, that is $A_B = -A_B^H$

$$\left(iA_B\right)^H = \left[i\left(A_B\right)\right]^H$$

$$= -i\,A_B^{\,H}$$

$$= -i\left(A_1 \cup A_2\right)^H$$

$$= -i\left(A_1^H \cup A_2^H\right)$$

$$= i\left(-A_1^H \cup -A_2^H\right)$$

$$= i\left(A_1 \cup A_2\right), \quad (\text{since } A_1 \text{ and } A_2 \text{ are skew hermitian})$$

$$= iA_B$$

Hence, iA_B is bihermitian.

Theorem 3.18

Any bimatrix A_B can be uniquely written in the form, $A_B = H_B + S_B$, where H_B is bihermitian and S_B is skew bihermitian.

Proof

Let A be any square matrix.

We know that any square matrix A can be represented as,

$$A=\left(\frac{A+A^H}{2}\right)+\left(\frac{A-A^H}{2}\right), \quad \text{---------------}\rangle\ (1)$$

where $\frac{A+A^H}{2}$ is hermitian and $\frac{A-A^H}{2}$ is skew hermitian.

Let $A_B = A_1 \cup A_2$ be any square bimatrix, where A_1 and A_2 are any two square matrices.

$$\begin{aligned}
\text{Let } A_B &= \frac{A_B + A_B^H}{2} + \frac{A_B - A_B^H}{2} \\
&= \frac{(A_1 \cup A_2) + (A_1 \cup A_2)^H}{2} + \frac{(A_1 \cup A_2) - (A_1 \cup A_2)^H}{2} \\
&= \frac{(A_1 \cup A_2) + (A_1^H \cup A_2^H)}{2} + \frac{(A_1 \cup A_2) - (A_1^H \cup A_2^H)}{2} \\
&= \frac{(A_1 + A_1^H) \cup (A_2 + A_2^H)}{2} + \frac{(A_1 - A_1^H) \cup (A_2 - A_2^H)}{2} \\
&= \left[\left(\frac{A_1 + A_1^H}{2}\right) \cup \left(\frac{A_2 + A_2^H}{2}\right)\right] + \left[\left(\frac{A_1 - A_1^H}{2}\right) \cup \left(\frac{A_2 - A_2^H}{2}\right)\right] \\
&= \left[\left(\frac{A_1 + A_1^H}{2}\right) + \left(\frac{A_1 - A_1^H}{2}\right)\right] \cup \left[\left(\frac{A_2 + A_2^H}{2}\right) + \left(\frac{A_2 - A_2^H}{2}\right)\right] \\
&= (H_1 + S_1) \cup (H_2 + S_2),
\end{aligned}$$

where H_1, H_2 are hermitian and S_1, S_2 are skew hermitian(since by (1))

Hence, $A_B = H_B + S_B$, Where H_B is bihermitian and S_B is skew bihermitian.

Example 3.18

$$\text{Let } A_B = \begin{bmatrix} 2+i & 2-2i & -5-i \\ 4i & 1+2i & 1+i \\ -1-5i & -1-i & 2 \end{bmatrix} \cup \begin{bmatrix} 1+i & -2+3i & 3+i \\ 2-i & -1+5i & 3-4i \\ -3+3i & -1-4i & 2+13i \end{bmatrix} = A_1 \cup A_2\ (say)$$

$$\text{where } A_1 = \begin{bmatrix} 2+i & 2-2i & -5-i \\ 4i & 1+2i & 1+i \\ -1-5i & -1-i & 2 \end{bmatrix} ; \quad A_1^H = \begin{bmatrix} 2-i & -4i & -1+5i \\ 2+2i & 1-2i & -1+i \\ -5+i & 1-i & 2 \end{bmatrix}$$

$$\text{and } A_2 = \begin{bmatrix} 1+i & -2+3i & 3+i \\ 2-i & -1+5i & 3-4i \\ -3+3i & -1-4i & 2+13i \end{bmatrix} ; \quad A_2^H = \begin{bmatrix} 1-i & 2+i & -3-3i \\ -2-3i & -1-5i & -1+4i \\ 3-i & 3+4i & 2-13i \end{bmatrix}$$

$$\frac{A_1 + A_1^H}{2} = \frac{1}{2}\begin{bmatrix} 4 & 2-6i & -6+4i \\ 2+6i & 2 & 2i \\ -6-4i & -2i & 4 \end{bmatrix} = \begin{bmatrix} 2 & 1-3i & -3+2i \\ 1+3i & 1 & i \\ -3-2i & -i & 2 \end{bmatrix} = H_1\ (say)$$

$$\frac{A_1 - A_1^H}{2} = \frac{1}{2}\begin{bmatrix} 2i & 2+2i & -4-6i \\ -2+2i & 4i & 2 \\ 4-6i & -2 & 0 \end{bmatrix} = \begin{bmatrix} i & 1+i & -2-3i \\ -1+i & 2i & 1 \\ 2-3i & -1 & 0 \end{bmatrix} = S_1\ (say)$$

$$\frac{A_2 + A_2^H}{2} = \frac{1}{2}\begin{bmatrix} 2 & 4i & -2i \\ -4i & -2 & 2 \\ 2i & 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2i & -i \\ -2i & -1 & 1 \\ i & 1 & 2 \end{bmatrix} = H_2\left(say\right)$$

$$\frac{A_2 - A_2^H}{2} = \frac{1}{2}\begin{bmatrix} 2i & -4+2i & 6+4i \\ 4+2i & 10i & 4-8i \\ -6+4i & -4-8i & 0+26i \end{bmatrix} = \begin{bmatrix} i & -2+i & 3+2i \\ 2+i & 5i & 2-4i \\ -3+2i & -2-4i & 13i \end{bmatrix} = S_2\left(say\right)$$

$$A_B = \begin{bmatrix} 2 & 1-3i & -3+2i \\ 1+3i & 1 & i \\ -3-2i & -i & 2 \end{bmatrix} \cup \begin{bmatrix} 1 & 2i & -i \\ -2i & -1 & 1 \\ i & 1 & 2 \end{bmatrix} +$$

$$\begin{bmatrix} i & 1+i & -2-3i \\ -1+i & 2i & 1 \\ 2-3i & -1 & 0 \end{bmatrix} \cup \begin{bmatrix} i & -2+i & 3+2i \\ 2+i & 5i & 2-4i \\ -3+2i & -2-4i & 13i \end{bmatrix}$$

Hence, $A_B = H_B + S_B$ (Say),where H_B is bihermitian and S_B is skew bihermitian matrices.

IV. Conclusion

Some of the properties of hermitian matrices are proved for bihermitian matrices. In a similar way all the properties of hermitian matrices can be verified for bihermitian matrices.

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