# MHD Three Dimensional Couette Flow past a Porous Plate with Heat Transfer 

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#### Abstract

In this paper a steady three dimensional flow of a viscous - incompressible fluid through highly porous medium bounded by a vertical infinite porous plate under the influence of a transverse applied magnetic field, with periodic suction velocity is considered. Governing equations are solved by using perturbation technique and obtained the expressions for velocity and temperature. The above flow quantities are discussed through the graphs for different physical parameters. Also the expressions for shear stress and the rate of heat transfer co-efficient are derived and discussed.


Keywords: MHD, Laminar flow, porous medium, suction velocity, shear stress \& rate of heat transfer.

## I. Introduction

The Problem of MHD laminar flow through a porous medium has become very important in recent years because of its possible applications in many branches of Science and Technology, particularly in the field of Agricultural Engineering to study the underground water resources, seepage of water in river beds; In Chemical Engineering for filtration and purification process; In Petroleum Technology to study the movement of natural gas, oil and water through the oil reservoirs. Feike et al. [1] first discussed analytical solutions for solute transport in three dimensional semi-infinite porous media. Ling et al. [2] discussed steady mixed convection boundary layer flow over a vertical flat plate in a porous medium filled with water at $4^{0}$ c: case of variable wall temperature. Later Ling et al. [3] discussed steady mixed convection boundary-layer flow over a vertical flat surface in a porous medium filled with water at $4^{0} \mathrm{c}$ variable surface heat flux. Gupta and Rajesh [4] discussed MHD Three dimensional flow past a porous plate. Wang and Leung [5] discussed Numerical solutions for flow in porous media. Alam and Rahman [6] discussed numerical study of the combined freeforced convection and mass transfer flow past a vertical porous plate in a porous medium with heat generation and thermal diffusion. Bejan and Khair [7] discussed heat and mass transfer by natural convection in a porous medium. Lai and Kulacki [8] discussed the effect of variable Viscosity on convective heat transfer along a vertical surface in a saturated porous medium. Singh and Sharma [9] discussed three dimensional coquette flows through a porous medium with heat transfer. Raju et al. [10] discussed soret effects due to natural convection between heat inclined plates with magnetic field. Reddy et al. [11] discussed the effect of aligned magnetic field on unsteady flow between a stretching sheet and oscillation porous plate with constant suction. Reddy et al. [12] discussed unsteady free convective MHD Non Newtonian flow through a porous medium bounded by an infinite inclined porous plate. Magnetic field effect on a three dimensional mixed convection flow with mass transfer along an infinite vertical porous plate is studied by Ahmed [13]. Balamurugan et al. [14] studied the effects of chemical reaction and thermo diffusion on MHD three dimensional free convection flows with heat absorption. Later they extended this problem with the effect of Joules dissipation [15].

The purpose of the present paper is to study the hydro magnetic field effects of electrically conducting three dimensional flow of viscous incompressible fluid through a porous medium which is bounded by infinite vertical porous plates with periodic suction at constant temperature.

## II. Formulation of the Problem

We consider a three-dimensional flow of viscous incompressible fluid through a highly porous medium which is bounded by a vertical infinite porous plate. We choose a coordinate system with plates lying vertically on $\mathrm{x}^{*}-\mathrm{z}^{*}$ plane such that $\mathrm{x}^{*}$ - axis is taken along the plate in the direction of the flow and $\mathrm{y}^{*}$ - axis is perpendicular to the plane of the plate and directed into the fluid which is flowing with free stream velocity U . All physical quantities will be independent of $\mathrm{x}^{*}$, however, the flow remains three dimensional due to the variation of the suction velocity distribution of the form: $v^{*}\left(z^{*}\right)=-V\left(1+\varepsilon \cos \frac{\pi z^{*}}{L}\right)$, the negative sign in the above equation indicates that the suction is towards the plate. With the above assumptions and under the usual boundary layer, the equations governing the flow are given below.

## Continuity Equation

$\frac{\partial v^{*}}{\partial y^{*}}+\frac{\partial w^{*}}{\partial z^{*}}=0$
Momentum equation
$v^{*} \frac{\partial u^{*}}{\partial y^{*}}+w^{*} \frac{\partial u^{*}}{\partial z^{*}}=v\left(\frac{\partial^{2} u^{*}}{\partial y^{* 2}}+\frac{\partial^{2} u^{*}}{\partial z^{* 2}}\right)-\frac{\sigma B_{0}{ }^{2} u^{*}}{\rho}-\frac{v}{k} u^{*}$
$v^{*} \frac{\partial v^{*}}{\partial y^{*}}+w^{*} \frac{\partial v^{*}}{\partial z^{*}}=-\frac{1}{\rho} \frac{\partial p^{*}}{\partial y^{*}}+v\left(\frac{\partial^{2} v^{*}}{\partial y^{* 2}}+\frac{\partial^{2} v^{*}}{\partial z^{* 2}}\right)-\frac{v}{k^{*}} u^{*}$
$v^{*} \frac{\partial w^{*}}{\partial y^{*}}+w^{*} \frac{\partial v^{*}}{\partial z^{*}}=-\frac{1}{\rho} \frac{\partial p^{*}}{\partial z^{*}}+v\left(\frac{\partial^{2} w^{*}}{\partial y^{* 2}}+\frac{\partial^{2} w^{*}}{\partial z^{2}}\right)-\frac{\sigma B_{0}{ }^{2} w^{*}}{\rho}-\frac{v}{k^{*}} w^{*}$

## Energy Equation

$$
\begin{equation*}
\rho C_{p}\left(v^{*} \frac{\partial T^{*}}{\partial y^{*}}+w^{*} \frac{\partial T^{*}}{\partial z^{*}}\right)=\kappa\left(\frac{\partial^{2} T^{*}}{\partial y^{* 2}}+\frac{\partial^{2} T^{*}}{\partial z^{* 2}}\right)+\mu \phi^{*} \tag{5}
\end{equation*}
$$

$$
\begin{equation*}
\text { where } \phi^{*}=2\left\{\left(\frac{\partial v^{*}}{\partial y^{*}}\right)^{2}+\left(\frac{\partial w^{*}}{\partial z^{*}}\right)^{2}\right\}+\left\{\left(\frac{\partial u^{*}}{\partial y^{*}}\right)^{2}+\left(\frac{\partial w^{*}}{\partial y^{*}}+\frac{\partial w^{*}}{\partial z^{*}}\right)+\left(\frac{\partial u^{*}}{\partial z^{*}}\right)^{2}\right\} \tag{6}
\end{equation*}
$$

The boundary conditions are
$y^{*}=0, u^{*}=0, v^{*}=-v\left(1+\varepsilon \cos \frac{\pi z^{*}}{L}\right), w^{*}=0, T^{*}=T_{w}{ }^{*} \quad y^{*} \rightarrow \infty, u^{*}=U, p^{*}=p^{*}, T^{*}=T_{\infty}{ }^{*}$

## III. Method of Solution

We now introduce the following non-dimensional variables
$y=\frac{y^{*}}{L}, z=\frac{z^{*}}{L}, u=\frac{u^{*}}{U}, v=\frac{v^{*}}{U}, w=\frac{w^{*}}{U}, p=\frac{p^{*}}{\rho U^{2}}, \theta=\frac{T^{*}-T^{*}}{T^{*}{ }_{w}-T_{\infty}^{*}}, \operatorname{Re}=\frac{U L}{v}$,
$M=\frac{\sigma B_{0}{ }^{2} L}{U \rho}, \quad \operatorname{Pr}=\frac{\mu C_{p}}{k}, \quad E=\frac{U^{2}}{C_{p}\left(T^{*}{ }_{w}-T^{*}{ }_{\infty}\right)}, \quad k=\frac{k^{*} U^{2}}{v^{2}}, \alpha=\frac{V}{U}$
Hence by using (8) in (1) to (7), we get
$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$
$v \frac{\partial u}{\partial y}+w \frac{\partial u}{\partial z}=\frac{1}{\operatorname{Re}}\left(\frac{\partial^{2} u}{\partial y^{2}}+\frac{\partial^{2} u}{\partial z^{2}}\right)-M u-K_{1} u$
$v \frac{\partial v}{\partial y}+w \frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}+\frac{1}{\operatorname{Re}}\left(\frac{\partial^{2} v}{\partial y^{2}}+\frac{\partial^{2} v}{\partial z^{2}}\right)-K_{1} v$
$v \frac{\partial w}{\partial y}+w \frac{\partial w}{\partial z}=-\frac{1}{\rho} \frac{\partial p}{\partial z}+\frac{1}{\operatorname{Re}}\left(\frac{\partial^{2} w}{\partial y^{2}}+\frac{\partial^{2} w}{\partial z^{2}}\right)-M w-K_{1} w$
$v \frac{\partial \theta}{\partial y}+w \frac{\partial \theta}{\partial z}=\frac{1}{\operatorname{PrRe}}\left(\frac{\partial^{2} \theta}{\partial y^{2}}+\frac{\partial^{2} \theta}{\partial z^{2}}\right)+\frac{E}{\operatorname{Re}} \phi \quad$ Where $K_{1}=\frac{k}{\operatorname{Re}}$
The corresponding boundary conditions in non-dimensional form are given by
$y=0, u=0, v=-\alpha(1+\varepsilon \cos (\pi z)), \theta=1, w=0$
$y \rightarrow \infty, u=1, p=p_{\infty}, \theta=0, w=0$

## Iv. Solution of the problem

To solve the equations (9) to (13) we assume that
$\left.\begin{array}{l}u(y, z)=u_{0}(y)+\varepsilon u_{1}(y, z) \\ v(y, z)=v_{0}(y)+\varepsilon v_{1}(y, z) \\ w(y, z)=w_{0}(y)+\varepsilon w_{1}(y, z) \\ \theta(y, z)=\theta_{0}(y)+\varepsilon \theta_{1}(y, z) \\ p(y, z)=p_{0}(y)+\varepsilon p_{1}(y, z)\end{array}\right\}$

Substituting (15) in the equations (9) to (13) and equating the coefficient of $0, \varepsilon^{1}$ and neglecting $\varepsilon^{2}$, we get the following sets of differential equations.
Zeroth-order equations:

$$
\begin{align*}
& v_{0}^{\prime}=0  \tag{16}\\
& u_{0}^{\prime \prime}+\alpha \operatorname{Re} u_{0}^{\prime}-\left(\operatorname{Re} M+\operatorname{Re} K_{1}\right) u_{0}=0  \tag{17}\\
& v_{0}^{\prime \prime}+(\alpha \operatorname{Re}) v_{0}^{\prime}-\left(K_{1} \operatorname{Re}\right) v_{0}=p_{0}^{\prime} \operatorname{Re}  \tag{18}\\
& w_{0}^{\prime \prime}+(\alpha \operatorname{Re}) w_{0}^{\prime}-w_{0}\left(M \operatorname{Re}+\operatorname{Re} K_{1}\right)=0  \tag{19}\\
& \theta_{0}^{\prime \prime}+(\alpha \operatorname{PrRe}) \theta_{0}^{\prime}=-E \operatorname{Pr} U_{0}^{\prime 2} \tag{20}
\end{align*}
$$

## First order equations:

$$
\begin{align*}
& \frac{\partial v_{1}}{\partial y}+\frac{\partial w_{1}}{\partial z}=0  \tag{21}\\
& v_{1} \frac{\partial u_{0}}{\partial z}-\alpha \frac{\partial u_{1}}{\partial y}=\frac{1}{\operatorname{Re}}\left(\frac{\partial^{2} u_{1}}{\partial y^{2}}+\frac{\partial^{2} u_{1}}{\partial z^{2}}\right)-M u_{1}(y, z)-K_{1} u_{1}(y, z)  \tag{22}\\
& -\alpha \frac{\partial v_{1}}{\partial y}=-\frac{\partial p_{1}}{\partial y}+\frac{1}{\operatorname{Re}}\left(\frac{\partial^{2} v_{1}}{\partial y^{2}}+\frac{\partial^{2} v_{1}}{\partial z^{2}}\right)-K_{1} v_{1}  \tag{23}\\
& -\alpha \frac{\partial w_{1}}{\partial y}=-\frac{\partial p_{1}}{\partial z}+\frac{1}{\operatorname{Re}}\left(\frac{\partial^{2} w_{1}}{\partial y^{2}}+\frac{\partial^{2} w_{1}}{\partial z^{2}}\right)-M w_{1}-K_{1} w_{1}  \tag{24}\\
& v_{1} \frac{\partial \theta_{0}}{\partial y}-\alpha \frac{\partial \theta_{1}}{\partial y}=\frac{1}{\operatorname{Pr} \operatorname{Re}}\left(\frac{\partial^{2} \theta_{1}}{\partial y^{2}}+\frac{\partial^{2} \theta_{1}}{\partial z^{2}}\right)+2 \frac{E}{\operatorname{Re}} \frac{\partial u_{0}}{\partial y} \frac{\partial u_{1}}{\partial y} \tag{25}
\end{align*}
$$

And the corresponding boundary conditions are

$$
\begin{align*}
& y=0, u_{0}=0, v_{0}=-\alpha, w_{0}=0, \theta_{0}=1 \\
& u_{1}=0, v_{1}=-\alpha \cos \pi z, w_{1}=0, \theta_{1}=1 \\
& y \rightarrow \infty, u_{0}=1, p_{0}=p_{\infty}, w_{0}=0, \theta_{0}=0 \\
& u_{1}=0, p_{1}=0, w_{1}=0, \theta_{0}=0 \tag{26}
\end{align*}
$$

The solutions of the equations (16) to (20) subject to the boundary conditions (26) are respectively

$$
\begin{align*}
& v_{0}=-\alpha  \tag{27}\\
& u_{0}=1-e^{-m y}  \tag{28}\\
& w_{0}=0  \tag{29}\\
& \theta_{0}=e^{-\alpha p R y}+E_{1}\left(e^{-\alpha p R y}-e^{-2 m y}\right) \tag{30}
\end{align*}
$$

In order to solve equations (21) to (25) under the above boundary conditions, we assume

$$
\begin{align*}
& u_{1}(y, z)=u_{11}(y) \cos (\pi z)  \tag{31}\\
& v_{1}(y, z)=v_{11}(y) \cos (\pi, z)  \tag{32}\\
& w_{1}(y, z)=-\frac{1}{\pi} v_{11}^{\prime}(y) \sin (\pi, z)  \tag{33}\\
& \theta_{1}(y, z)=\theta_{11}(y) \cos (\pi z)  \tag{34}\\
& p_{1}(y, z)=p_{11}(y) \cos (\pi, z) \tag{35}
\end{align*}
$$

By substituting equations (31) to (35) in equations (21) to (25), we have
$v_{11}^{\prime \prime}+(\alpha \operatorname{Re}) v_{11}^{\prime}-\left(\pi^{2}+K_{1} \operatorname{Re}\right) v_{11}=p_{11}^{\prime} \operatorname{Re}$
$-v_{11}^{\prime \prime \prime}-(\operatorname{Re} \alpha) v_{11}^{\prime \prime}+\left(\pi^{2}+M \operatorname{Re}+K_{1} \operatorname{Re}\right) v_{11}^{\prime}=-\operatorname{Re} \pi^{2} p_{11}$
$\operatorname{Re} v_{11} u_{0}^{\prime}=u_{11}^{\prime \prime}+(\operatorname{Re} \alpha) u_{11}^{\prime}-\left(\pi^{2}+M \operatorname{Re}+K_{1} \mathrm{Re}\right) u_{11}$
$\operatorname{PrRe} v_{11} \theta_{0}^{\prime}=\theta_{11}^{\prime \prime}+(\operatorname{PrRe} \alpha) \theta_{11}^{\prime}-\pi^{2} \theta_{11}+2 E \operatorname{Pr} u_{0}^{\prime} u_{11}^{\prime}$
Subject to the boundary conditions:
$y=0, \quad v_{11}=-\alpha, v_{11}^{\prime}=0, u_{11}=0, \quad \theta_{11}=0$
$y \rightarrow \infty, v_{11}=0, v_{11}^{\prime}=0, p_{11}=0, u_{11}=0, \theta_{11}=0$
Using boundary conditions (40), and solving equations (36) to (39) we get

$$
\begin{align*}
& u_{11}=\frac{\operatorname{Re} m \alpha}{\left(\beta_{1}-\beta_{3}\right)}\left[\begin{array}{l}
A_{1} e^{-\left(\beta_{1}+m\right) y}-A_{2} e^{-\left(\beta_{3}+m\right) y} \\
-\left(A_{1}-A_{2}\right) e^{-r_{4} y}
\end{array}\right]  \tag{41}\\
& v_{11}=\frac{\alpha}{\beta_{1}-\beta_{3}}\left[\beta_{3} e^{-\beta_{1} y}-\beta_{1} e^{-\beta_{3} y}\right]  \tag{42}\\
& \theta_{11}(y, z)=c e^{-\left(\xi_{y} y\right.}+\frac{\operatorname{Pr}^{2} \operatorname{Re}^{2} \alpha^{2}}{\left(\beta_{1}-\beta_{3}\right)}\left[-B_{1} e^{-\left(\alpha \mathrm{PrRe}+\beta_{1}\right) y}+B_{2} e^{-\left(\alpha \mathrm{PRRe}+\beta_{3}\right) y}-B_{3} e^{-\left(\alpha \mathrm{PRRe}+\beta_{1}\right) y}+B_{4} e^{-\left(\alpha \mathrm{PrRe}+\beta_{3}\right) y}\right]+  \tag{43}\\
& \frac{\alpha \operatorname{Pr} \operatorname{Re} 2 m}{\left(\beta_{1}-\beta_{3}\right)}\left[B_{5} e^{-\left(\beta_{1}+2 m\right) y}-B_{6} e^{-\left(\beta_{3}+2 m\right) y}-B_{7} e^{-\left(\left(r_{4}+m\right) y\right.}\right]
\end{align*}
$$

From the equations (31) to (35), we get
$u_{1}(y, z)=\frac{\operatorname{Re} m \alpha}{\left(\beta_{1}-\beta_{3}\right)}\left[A_{1} e^{-\left(\beta_{1}+m\right) y}-A_{2} e^{-\left(\beta_{3}+m\right) y}-\left(A_{1}-A_{2}\right) e^{-r_{4} y}\right] \cos (\pi z)$
$v_{1}(y, z)=\frac{\alpha}{\beta_{1}-\beta_{3}}\left[\beta_{3} e^{-\beta_{1} y}-\beta_{1} e^{-\beta_{3} y}\right] \cos (\pi z)$
$\theta_{1}(y, z)=c e^{-r_{5} y}+\frac{\operatorname{Pr}^{2} \operatorname{Re}^{2} \alpha^{2}}{\left(\beta_{1}-\beta_{3}\right)}\left[-B_{1} e^{-\left(\alpha \operatorname{PrRe}+\beta_{1}\right) y}+B_{2} e^{-\left(\alpha \operatorname{PrRe}+\beta_{3}\right) y}-B_{3} e^{-\left(\alpha \operatorname{PrRe}+\beta_{1}\right) y}+\right.$
$\left.B_{4} e^{-\left(\alpha \operatorname{PrRe}+\beta_{3}\right) y}\right]+\frac{\alpha \operatorname{Pr} \operatorname{Re} 2 m}{\left(\beta_{1}-\beta_{3}\right)}\left[B_{5} e^{-\left(\beta_{1}+2 m\right) y}-B_{6} e^{-\left(\beta_{3}+2 m\right) y}-B_{7} e^{-\left(r_{4}+m\right) y}\right] \cos (\pi z)$
$w_{1}(y, z)=\frac{\alpha \beta_{1} \beta_{3}}{\pi\left(\beta_{1}-\beta_{3}\right)}\left[-e^{-\beta_{1} y}+e^{-\beta_{3} y}\right] \sin (\pi z)$
Therefore the solutions of $\mathrm{u}, \mathrm{v}, \mathrm{w}$ and $\theta$ are given by
$u(y, z)=\left(1-e^{-m y}\right)+\varepsilon \frac{\operatorname{Re} m \alpha}{\left(\beta_{1}-\beta_{3}\right)}\left[A_{1} e^{-\left(\beta_{1}+m\right) y}-A_{2} e^{-\left(\beta_{3}+m\right) y}-\left(A_{1}-A_{2}\right) e^{-r_{4} y}\right] \cos (\pi z)$
$\theta(y, z)=e^{-\alpha p r \operatorname{Re} y}+E_{1}\left[e^{-\alpha p r \operatorname{Re} y}-e^{-2 m y}\right]+\varepsilon c e^{-r_{5} y}+\varepsilon \frac{\operatorname{Pr}^{2} \operatorname{Re}^{2} \alpha^{2}}{\left(\beta_{1}-\beta_{3}\right)}\left[-B_{1} e^{-\left(\alpha \mathrm{PrRe}+\beta_{1}\right) y}+B_{2} e^{-\left(\alpha \mathrm{PrRe}+\beta_{3}\right) y}-\right.$
$\left.B_{3} e^{-\left(\alpha \operatorname{PrRe}+\beta_{1}\right) y}+B_{4} e^{-\left(\alpha \operatorname{PrRe}+\beta_{3}\right) y}\right]+\varepsilon \frac{\alpha \operatorname{Pr} \operatorname{Re} 2 m}{\left(\beta_{1}-\beta_{3}\right)}\left[B_{5} e^{-\left(\beta_{1}+2 m\right) y}-B_{6} e^{-\left(\beta_{3}+2 m\right) y}-B_{7} e^{-\left(r_{4}+m\right) y}\right] \cos (\pi z)$
$v(y, z)=-\alpha+\varepsilon \frac{\alpha}{\left(\beta_{1}-\beta_{3}\right)}\left[\beta_{3} e^{-\beta_{1} y}-\beta_{1} e^{-\beta_{3} y}\right] \cos (\pi z)$
$w(y, z)=\varepsilon \frac{\alpha \beta_{1} \beta_{3}}{-\pi\left(\beta_{1}-\beta_{3}\right)}\left[-e^{-\beta_{1} y}+e^{-\beta_{3} y}\right] \sin (\pi z)$
Knowing the velocity field, we can obtain the expressions for the shear stress components in the $X^{*}$ and $Z^{*}$ directions in the non-dimensional from:
$\tau_{x}=\frac{\tau_{x}^{*}}{\rho U V}=\frac{v}{V L}\left(\frac{\partial u}{\partial y}\right)_{0}=\frac{v}{U L} \cdot \frac{U}{V}\left(\frac{\partial u}{\partial y}\right)_{0}=\frac{1}{\mathrm{Re} \alpha}\left(\frac{\partial u}{\partial y}\right)_{0}$
$\Rightarrow \tau_{x}=\frac{m}{\operatorname{Re} \alpha}+\varepsilon\left(1-F_{1}(\alpha, \operatorname{Re}, M)\right] \cos (\pi z)$

Similarly

$$
\tau_{z}=\frac{\tau_{z}^{*}}{\mu V / L}=\frac{U}{V}\left(\frac{\partial w}{\partial y}\right)_{0}
$$

$\Rightarrow \tau_{z}=-\varepsilon F_{2}(\alpha, \mathrm{Re}, M) \sin (\pi z)$
where $F_{2}(\alpha, \mathrm{Re}, M)=\frac{\alpha \beta_{1} \beta_{3}}{\pi\left(\beta_{1}-\beta_{3}\right)}\left(\beta_{1}-\beta_{3}\right)$

## V. Results And Discussions

The effects of various physical parameters viz., Hartmann number (M), permeability of the porous medium (k) suction parameter ( $\alpha$ ), Prandtl number ( $\operatorname{Pr}$ ) on the main flow velocity $u$ are displayed in figures 1 and 2. Figure 1 depicts the velocity profiles for different values of suction parameter $\alpha$ and Reynolds number Re. From this figure it is noticed that velocity increases with the increase in suction parameter $\alpha$ and Reynolds number Re. Effects of M, k and Pr are shown in figure 2. From this figure it is observed that velocity increases as both M and k increase and the effect of Pr on velocity is observed to be very small. In figure 3 temperature profiles are presented for fixed values of $\mathrm{M}=2, \mathrm{k}=0.2, \mathrm{z}=0.5$ and $\mathrm{E}=0.5$ with the variations in $\alpha$ and Re . From this figure it is noticed that temperature decrease with the increasing values of Re and Pr.

Cross flow velocity profiles for different values of $\alpha, \mathrm{Re}, \mathrm{M}, \mathrm{k}$ and $\operatorname{Pr}$ are displayed in figure 4. From this figure it is noticed that cross flow velocity increase with the increase in $\alpha$ when $\operatorname{Re}=1, \mathrm{M}=2, \mathrm{k}=0.2$ and $\operatorname{Pr}=0.71$. When $\alpha=2, \mathrm{M}=2$, and $\mathrm{k}=0.2$ velocity is observed to increase with the increasing values of $\operatorname{Re}$ and Prwhere as cross flow velocity is observed to decrease with the increase in $M$ and $k$. The main flow components of skin friction $\tau_{x}$ and transverse flow components of skin friction $\tau_{z}$ are presented through tables 1 and 2 .
From these tables it is noticed that $\tau_{x}$ increases with the increase in $M$ and decreases with the increase in $\alpha$ and
k . But $\tau_{z}$ decreases with the increase in M and $\alpha$ and reverse effect is noticed in the case of k .

## VI. Conclusions

In this paper we have studied three dimensional flow of a viscous -incompressible fluid through porous medium bounded by a vertical infinite porous plate under the influence of a transverse applied magnetic field with periodic suction velocity. In this analysis the following conclusions are made
a. Velocity increases with the increase of Hartmann number M, suction parameter $\alpha$, Reynolds number Re and permeability of the porous medium K . It is also observed that the velocity increases up to $\mathrm{y}=5$ and then it becomes linear thereafter.
b. Temperature decreases as $\alpha$ and Re increase.
c. Cross flow velocity increases with the increase in $\alpha, \mathrm{Re}, \operatorname{Pr}$ and it decreases as $\mathrm{M} \& \mathrm{k}$ increase.
d. The main flow components of skin friction $\tau_{x}$ increase with the increase in M and decrease with increase in $\alpha$ and k .
e. The transfer flow components of skin friction $\tau_{z}$ increase with the increase in $\mathrm{k} \&$ decrease with the increase M \& $\alpha$.


Fig1. Velocity profiles for variations of $\alpha$ and $\operatorname{Re}$ for fixed values of $\mathrm{z}=0.5, \mathrm{E}=0.5, \mathrm{M}=2, \mathrm{k}=0.2$,
$\operatorname{Pr}=0.71$ Table1: Main flow components of skin friction $\boldsymbol{\tau}_{\boldsymbol{X}}$ at $\operatorname{Re}=1, \operatorname{Pr}=0.71, \mathrm{z}=0.5, \boldsymbol{\epsilon}=\mathbf{0} .2$

| $\alpha$ | $\mathrm{M}=0$ | $\mathrm{M}=2$ | $\mathrm{M}=4$ | $\mathrm{~K}=0.2$ | $\mathrm{~K}=0.8$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.5 | 9.5000 | 12.5416 | 14.9568 | 12.5416 | 9.7195 |
| 1.0 | 9.5000 | 11.1301 | 12.5416 | 6.5828 | 5.1904 |
| 1.5 | 3.6447 | 4.6231 | 5.4103 | 4.6231 | 3.7146 |



Fig2. Velocity profiles for variations of $M, K$ and $\operatorname{Pr}$ for fixed values of $z=0.5, E=0.5, \alpha=0.5, \operatorname{Re}=1$


Fig3. Temperature profiles for the variations of $\alpha$ and $\operatorname{Re}$ for fixed values of $\mathrm{M}=2, \mathrm{k}=0.2, \mathrm{z}=0.5, \mathrm{E}=0.5$.


Fig 4. Cross flow velocity profiles for the variations of $\alpha, \operatorname{Re}, \mathrm{M}, \mathrm{k}$ and $\operatorname{Pr}$ for fixed values of $\mathrm{z}=0.5, \mathrm{E}=0.5$

Table2: Transverse flow components of skin friction $\boldsymbol{\tau}_{\boldsymbol{z}}$ at $\operatorname{Re}=1, \operatorname{Pr}=0.71, \mathrm{z}=0.5, \boldsymbol{\epsilon}=\mathbf{0} .2$

| $\alpha$ | $\mathrm{M}=0$ | $\mathrm{M}=2$ | $\mathrm{M}=4$ | $\mathrm{~K}=0.2$ | $\mathrm{~K}=0.8$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.5 | -0.1458 | -0.1773 | -0.1777 | -0.2092 | -0.1773 |
| 1.0 | -0.5203 | -0.7454 | -0.8197 | -0.8644 | -0.7454 |
| 1.5 | -1.1108 | -1.7731 | -2.0282 | -2.0410 | -1.7731 |

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VIII. Nomenclature
$B_{0}$ : Intensity of the applied Magnetic field;
g : Acceleration due to gravity
$\mathrm{K}^{*}$ : Permeability of the porous medium;
$\mathrm{P}^{*}$ : Pressure;
Re : Reynolds number;
U : Free stream velocity;
V : Basic steady distribution;
$C_{p}$ : Specific heat of the fluid at constant pressure;
$T_{\infty}^{*}$ : Temperature of the fluid far away from the Plate;
L : Half-Wave length of the periodic suction velocity;
$U^{*}, V^{*}, W^{*}$ : Velocity components in $x^{*}, y^{*}, z^{*}$ direction;
$\mathrm{u}, \mathrm{v}, \mathrm{w} \quad:$ Dimensionless velocity components;

Greek Symbols
$\rho:$ Density of the fluid;
$\mu \quad:$ Viscosity;
$\kappa$ : Thermal conductivity;

E : Eckert number;
K :Permeability parameter;
M
Pr : Prandtl number;
T* : Temperature of the Plate;
$v_{0} \quad:$ Constant suction velocity;
$\mathrm{x}^{*}, \mathrm{y}^{*}, \mathrm{z}^{*}$ : Co-ordinate system;
$\mathrm{x}, \mathrm{y}, \mathrm{z} \quad$ : Dimensional co-ordinates;

| $v$ | $:$ Kinematic viscosity; |
| :--- | :--- |
| $\alpha$ | $:$ Suction Parameter; |

