

A Marketing Decision Problem in a Periodic Review Model with Exponential Demand and Shortages

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Abstract: The paper considers a marketing decision problem in a periodic review model. The demand distribution is assumed to be exponential, with the mean demand being a concave increasing function of the level of marketing effort. The optimum ordering policy and the optimum level of marketing are determined so as to minimize the total expected cost over a re-order interval. Optimum decision rules are given for some particular functional forms of the mean demand.

I. Introduction :

Marketing management is one of the major components of business management. It is a business discipline which focuses on the practical application of marketing techniques and the management of a firm's marketing resources and activities. Marketing technique includes advertising, distribution, communication, selling, etc. and, owing to the growing competition, a rapid growth in it is observed in various business organizations.

A marketing policy is expected to induce increase in demand and sale. As such, it is likely that the inventory maintained by a business will be affected by the marketing policy adopted. Till date, few researchers have investigated inventory models with demand dependent on the marketing strategy. See, for example, Ladany and Sternlieb (1974), Lal and Staelin (1984), Jucker and Rosenblatt (1985), Shah and Jha (1991), Eliashberg and Steinberg (1993), Bhunia and Maiti (1997), Pal and Manna (2003).

Advertisement is a common marketing strategy. It is generally observed that as the frequency of advertisement m increases, the demand for a product also increases, but at a decreasing rate. Pal and Manna (2003) incorporated this feature of the marketing effort in their study of the classical single period stochastic inventory model. However, they assumed m to be continuous, which is not true. In this paper, we consider a dynamic risk inventory model allowing shortage, where mean demand is assumed to be an increasing concave function of the frequency of advertisement, but with a diminishing rate. Demand is assumed to have a one-parameter exponential distribution. The paper is organized as follows. The problem is formulated in section 2. In section 3, the main results are given. In section 4, different functional forms of demand in terms of marketing effort are considered, and section 5. Gives the concluding remarks.

II. The Problem And Its Formulation :

Consider a dynamic risk inventory model. The policy is to place an order at each reorder point, the order quantity being sufficient to bring up the stock height to a level S . The model is governed by the following assumptions:

- 1) Reorder points are equidistant on the planning horizon.
- 2) Demand (X) during a reorder interval is a non-negative, continuous random variable With mean demand dependent on the level of marketing effort.
- 3) The marketing effort is measured by the number of units of advertisements m .
- 4) μ_m Denotes the mean demand during a reorder period.
- 5) μ_m is an increasing integer concave function of m , that is, μ_m increases with m , but the rate of increase is non-increasing. Mathematically, the assumption means
$$\mu_{m+1} - \mu_m > 0, \quad \mu_{m+2} - 2\mu_{m+1} + \mu_m \leq 0.$$
- 6) The pdf and cdf of X are $f(x; \mu_m)$ and $F(x; \mu_m)$, respectively, $x \geq 0$.
- 7) Supply is instantaneous on ordering.
- 8) The costs are
 - (i) C_1 = Carrying cost per unit per unit time
 - (ii) C_2 = Shortage cost per unit short per unit time
 - (iii) r = cost of an advertisement

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The problem is to find S and m so as to minimize the total expected cost over a reorder interval.

The cost function over a re-order interval is given by

$$C(S, m | x) = cx + rm + C_1(S - x/2), \text{ if } x \leq S$$

$$= cx + rm + C_1S^2 / 2x + C_1(x - S)^2 / 2x, \text{ if } x \geq S$$

Hence, the expected cost over a reorder interval comes out to be

$$C(S, m) = c\mu_m + rm + C_1 \int_0^S (S - x/2) f(x; \mu_m) dx + C_1 S^2 \int_S^\infty (1/2x) f(x; \mu_m) dx$$

$$+ C_2 \int_S^\infty \{(x - S)^2 / 2x\} f(x; \mu_m) dx \quad (2.1)$$

The problem is to find values of m and S that minimize $C(S, m)$.

III. Main Result :

For given m , the optimal value of S , which minimizes (2.1), satisfies $\frac{\partial}{\partial S} C(S, m) = 0$, which gives

$$F(S; \mu_m) + S \int_S^\infty \frac{1}{x} f(x; \mu_m) dx = \frac{C_2}{C_1 + C_2} \quad \dots\dots\dots (2.2)$$

As left hand side of (2.2) is an increasing function of S for given m , (2.2) gives a unique solution of S in terms of m .

Now, differentiating (2.2) with respect to μ_m , we get

$$\frac{\partial S}{\partial \mu_m} = -[1 + Sf(S; \mu_m) + \int_0^S \frac{1}{2x} f(x; \mu_m) dx]^{-1} \int_0^S (1 + \frac{S}{2x}) \frac{\partial}{\partial \mu_m} f(x; \mu_m) dx \quad \dots\dots\dots (2.3)$$

(2.3) means that the optimal value of S is a decreasing function of μ_m if $\frac{\partial}{\partial \mu_m} f(x; \mu_m) > 0$ for all $x \geq 0$, and is an increasing function of μ_m if $\frac{\partial}{\partial \mu_m} f(x; \mu_m) < 0$ for all $x \geq 0$. Then, using assumption (5), we have the following theorem:

Theorem 2.1: If S_m be the optimum value of S minimizing $C(S, m)$ for given m , then S_m is a decreasing function of m if $\frac{\partial}{\partial \mu_m} f(x; \mu_m) > 0$ for all $x \geq 0$, and S_m is an increasing function of m if $\frac{\partial}{\partial \mu_m} f(x; \mu_m) < 0$ for all $x \geq 0$.

Let S_m be the optimum value of S minimizing $C(S, m)$ for given m . Then, $C(S_m, m)$ is the minimum value of $C(S, m)$, for given m . We, therefore, obtain the optimum value of m by minimizing $C(S_m, m)$ with respect to m . This means that optimum m is the smallest value of m for which $C(S_{m+1}, m+1) - C(S_m, m) \geq 0$.

Numerical Example:

1. Let us consider the mean demand function, depend on m , and be $\mu = m^\alpha$, $0 < \alpha < 1$. Then the minimum cost is given from the table:

C_1	C_2	m	α	$\mu = m^\alpha$	$s = 0.3 * \mu$	$c(s, \mu)$
5	6	1	0.3	1	0.3	0.559106
5	6	2	0.3	1.231144	0.32	0.523883
5	6	3	0.3	1.390389	0.34	0.508899
5	6	4	0.3	1.515717	0.36	0.502055
<u>5</u>	<u>6</u>	<u>5</u>	<u>0.3</u>	<u>1.620657</u>	<u>0.38</u>	<u>0.499874</u>
5	6	6	0.3	1.711177	0.4	0.500867
5	6	7	0.3	1.79279	0.42	0.504249
5	6	8	0.3	1.866066	0.44	0.509546
5	6	9	0.3	1.933182	0.46	0.516445
5	6	10	0.3	1.995262	0.48	0.524725
5	6	11	0.3	2.053136	0.5	0.534219
5	6	12	0.3	2.107436	0.52	0.544802
5	6	13	0.3	2.158654	0.54	0.556372

5	6	14	0.3	2.207183	0.56	0.568846
5	6	15	0.3	2.253343	0.58	0.582155

Therefore the minimized cost $C(S, m) = 0.499874$ at optimized $m=5$, for $c_1=5$, $c_2=6$ and $\alpha=0.3$, $s=0.38$.

2. Let us consider the mean demand function, depend on m , be $\mu = A - B * (m^\alpha)$. Then the minimized cost is given from the table:

m	A	B	α	$\mu = A - B * (m^\alpha)$	$S = 0.29 * \mu$	$c(s, \mu)$
1	1	1	0.23	0.77	0.2233	0.593159
<u>2</u>	<u>2</u>	<u>2</u>	<u>0.23</u>	<u>1.8942</u>	<u>0.549318</u>	<u>0.591214</u>
3	3	2	0.23	2.975666	0.862943	0.819411
4	4	3	0.23	3.991605	1.157565	1.135291
5	5	3.5	0.23	4.997747	1.449347	1.519608
6	6	4.25	0.23	5.999371	1.739818	1.962337
7	7	4.875	0.23	6.999834	2.029952	2.458924
8	8	5.5625	0.23	7.999956	2.319987	3.006369
9	9	6.21875	0.23	8.999989	2.609997	3.602659

Therefore

The minimized cost

$C(S, m) = 0.5912144$, at optimized $m=2$.

IV. Different Function Form In Terms Of Marketing Effort :

In this portion we consider different functional form of demand in terms of marketing effort. Here our aim to find out the variation of cost function with different values of one parameter, when the other parameters are remains fixed.

Firstly, consider the mean demand function as $\mu = m^\alpha$. Then,

For fixed α and C_2 :

C_1	Optimized m	Optimized S	$\mu = m^\alpha$	Min C(s,m)
2	5	0.38	1.620657	0.295329
5	5	0.38	1.620657	0.499874
10	5	0.38	1.620657	0.670329
15	5	0.38	1.620657	0.759614
20	5	0.38	1.620657	0.814559

From this table we see that the cost increases with c_1 . i.e., cost varies directly with c_1 .

For fixed α and C_1 :

C_2	Optimized m	Optimized S	$\mu = m^\alpha$	Min C(s,m)
2	5	0.38	1.620657	0.759614
3	5	0.38	1.620657	0.670329
6	5	0.38	1.620657	0.499874
9	5	0.38	1.620657	0.402472
12	5	0.38	1.620657	0.339446
15	5	0.38	1.620657	0.295329

From this table we see that the cost decreases with the increasing values of c_2 . i.e., cost varies indirectly with c_2 .

For fixed C_1 and C_2 :

α	Optimized m	Optimized S	$\mu = m^\alpha$	Min C(s,m)
0.1	2	0.32	1.0711773	0.553118
0.2	4	0.36	1.319508	0.52942
0.3	5	0.38	1.620657	0.499874
0.4	6	0.40	2.047673	0.469934
0.5	7	0.42	2.645751	0.442182

Thus, the cost decreases with the increasing values of α . i.e., cost varies indirectly with α .

Secondly, considering the mean demand function as $\mu=A-B^{*\eta^m}$. Then,

For fixed η and C_2 :

C_1	Optimized m	Optimized S	$\mu=A-B^{*\eta^m}$	Min C(s,m)
2	2	0.549318	1.8942	0.386668
5	2	0.549318	1.8942	0.591214
10	2	0.549318	1.8942	0.761668
15	2	0.549318	1.8942	0.850954
20	2	0.549318	1.8942	0.905899

From this table we see that the cost increases with c_1 . i.e., cost varies directly with c_1 .

For fixed η and C_1 :

C_2	Optimized m	Optimized S	$\mu=A-B^{*\eta^m}$	Min C(s,m)
2	2	0.549318	1.8942	0.850954
3	2	0.549318	1.8942	0.761668
6	2	0.549318	1.8942	0.591214
9	2	0.549318	1.8942	0.493811
12	2	0.549318	1.8942	0.430786
15	2	0.549318	1.8942	0.386668

From this table we see that the cost decreases with the increasing values of c_2 . i.e., cost varies indirectly with c_2 .

For fixed C_1 and C_2 :

η	Optimized m	Optimized S	$\mu=A-B^{*\eta^m}$	Min C(s,m)
0.23	2	0.549318	1.8942	0.591214
0.3	2	0.549318	1.82	0.581256
0.4	2	0.549318	1.68	0.565116
0.5	2	0.549318	1.5	0.550099
0.6	2	0.549318	1.28	0.542321

Thus, the cost decreases with the increasing values of η . i.e., cost varies indirectly with η .

V. Concluding Remark :

In this paper, we consider a dynamic risk inventory model allowing shortage, where mean demand is assumed to be an increasing concave function of the frequency of advertisement, but with a diminishing rate. Demand is assumed to have a one-parameter exponential distribution. A marketing policy is expected to induce increase in demand and sale. As such, it is likely that the inventory maintained by a business will be affected by the marketing policy adopted. Till date, few researchers have investigated inventory models with demand dependent on the marketing strategy. Thereby, this helps us to take decision about marketing straight or marketing policies. From this model, we can say that number of advertisement is one of the most important things and different costs have impact on total cost of the model. In future, researchers can do more work about several parts of marketing, like demand, communication, selling etc.

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References

- [1]. Bhunia, A.K. and Maiti, M. (1997): An inventory model for decaying items with selling price, frequency of advertisement and linearly dependent demand with shortages, *IAPQR Transactions*, 22, 1-49.
- [2]. Eliashberg, J. and Steinberg, M.S. (1993): Marketing production joint decision making, *Handbook in OR and MS*, Vol. 5 (Eds. J. Eliashberg and G.L. Lilien). Elsevier, Amsterdam.
- [3]. Jucker, J.K. and Rosenblatt, M.J. (1985): A single period inventory model with demand uncertainty and quantity discounts: behavioural implications and a new solution procedure, *Naval Research Logistic Quarterly*, 32, 537-550.
- [4]. Ladany, S. and Sternlieb, A (1974): The interaction of economic ordering quantities and marketing policies, *AIIE Transactions*, 6, 35-40.
- [5]. Lal, R. and Staelin, R. (1984): An approach for developing an optimal discount pricing policy, *Management Science*, 30, 1524-1539.
- [6]. Pal and Manna (2003): A marketing decision problem in a single period stochastic inventory model, *Opsearch*, 40(3), 230-240.
- [7]. Shah, Y.K. and Jha, P.J. (1991): A single-period stochastic inventory model under the influence of marketing policies, *Journal of the Operational Research Society*, 42, 173-176.