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Modal Analysis of Rectangular Plate with Lap Joints to Find Natural Frequencies and Mode Shapes

M. Bhusnar[1], S. S. Sarawade[2]
(Department of Mechanical Engineering, M.E.S. College of Engineering, S.P.Pune University, India)

Abstract : *The aim of this study is to analyze the dynamic behavior of structures like a rectangular plate, bolted lap joint, welded lap joint and single lap epoxy adhesive joint subjected to impact or shock loads using Finite Element Analysis (FEA) and analytical methods. The various factors that affect the response of bolted, welded lap joint and adhesive joint structures are studied, such as natural frequencies, mode shapes, damping ratio etc. In this work the modal analysis of rectangular plates with different lap joints are investigated. The four different specimens are made of aluminum material. The finite element analysis software is used for modal analysis of all joints. The initial case study is focused on a simple rectangular plate of cantilever beam subjected to impact force. The second case study is focused on bolted lap joint, welded lap joint and single lap adhesive joint. The main objective of this work is to determine the natural frequency and mode shape of all four specimens at cantilever beam condition and to compare the result of all joints with the single rectangular plate and to find the error between software analysis and analytical solution. In practical application this kind of modal analysis can be used to analyze some structures such as cantilever bridge, frame of bicycle, automobile product, Industrial robots (manipulator), building structures, heavy machineries and aircraft industry etc. From these studies it is found that the analyzed results by above method are compared with finite element analysis and analytical solution and found to be satisfactory.*

Keywords**—** *Modal Analysis, Rectangular Plate, Lap Joints, Natural Frequency, Mode Shapes, Cantilever, FEA*

I. INTRODUCTION

Vibration problems are often occurred in mechanical structure. It is important to prevent such problems because it can cause structures damage by fatigue. The structure itself has a certain properties so it is necessary to understand its characteristics. In this work a modal analysis by finite element method is used. The main purpose of modal analysis is to study the dynamic properties of structures like natural frequency, damping and mode shapes. This can also be used for some purposes such as, troubleshooting i.e. direct insight into the root cause of vibration problems, find structural flexibility properties quickly to monitor incremental structural changes, design optimization-design according to noise and vibration targets, enhance performance and reduce component and overall vibration & fast, test based evaluation of redesign for dynamics etc. So it is important to determine dynamic properties of a cantilever beam rectangular plate, bolted lap joint; welded lap joint and single lap adhesive joints of an aluminum material to study the structural response of these joints after loading or impact. First the modal analysis of a rectangular plate & bolted lap joint is done and after that the modal analysis of welded lap joints and single lap adhesive bonded lap joint is done by analytical and finite element analysis software method and considered as a cantilever beam. The araldite epoxy resin adhesive is used as bonded material for single lap adhesive joints. In this work the modal analysis of a bolted, welded lap joint and single lap adhesive joint is done in ansys software and then it is compared with theoretical results. Then the result of these three components can be compared with each other to find the error between software and experimental modal analysis.

The main objective of this work is to determine the natural frequency and mode shape of a bolted lap joint, welded lap joint and single lap adhesive joint at cantilever beam condition. The first step of the work is to do the modal analysis using Ansys 13.0 software for determining the dynamic properties of the beam. The modal analysis is used to understand the dynamic properties of structure such as natural frequency (resonant frequency), damping ratio and mode shape. The next step is to find natural frequencies of a rectangular plate by theoretical method using Euler's Bernoulli's beam theory. The finite element analysis results are compared with the obtained theoretical results to verify the modal analysis. With modal analysis, we can extract the modal parameters (dynamic properties) of a structure. The modal parameters, including natural frequency, damping ratio, and mode shape, are the fundamental elements that describe the movement and response of a structure to ambient excitation as well as forced excitation. Knowing these modal parameters help to understand the structure's response to ambient conditions as well as perform design validation.

The adhesive bonding becomes important in structural bonding in aircraft industry. The subject of adhesives became even more interesting to scientists when the application of synthetic resins as adhesives for wood, rubber, glass and metals were discovered. Adhesive bonding as an alternative method of joining materials together has many advantages over the more conventional joining methods such as fusion and spot welding, bolting and riveting. Adhesive bonding is gaining more and more interest due to the increasing demand for joining similar or dissimilar structural components, mostly within the framework of designing light weight structures. The current trends are to use visco-elastic material in the joints for passive vibration control in the structures subjected to dynamic loading. These components are often subjected to dynamic loading, which may cause initiation and propagation of failure in the joint. In order to ensure the reliability of these structures, their dynamic response and its variation in the bonded area must be understood. In adhesive joint the major function of adhesive is to transmit loads from one member of joint to another. It allows a more uniform stress distribution than is obtained by another mechanical joining process such as welding, bolting, riveting, etc. Thus, adhesive often permit the fabrication of structures that are mechanical equivalent or superior to conventional assembles and furthermore cost and weight benefits. The conventional joining process increase the weight of the structure by adding extra material such as bolt, screws, extra filler material. If it is needed to joint two plate by bolting then hole is created in the plate which result in stress concentration or if you joint by weld then there is localized heating of the component take place which alter its mechanical properties. In adhesive joining process you do not need to create the hole in the plate or there is no localized heating take place. Thus adhesive bonding gaining more importance in joining process where it avoids stress concentration and localized heating. In addition adhesive can produce joints with high strength, rigidity, dimensional precision in the light metals, such as aluminum and magnesium, which may be weakened or distorted by welding. Adhesive can also prevent electrochemical corrosion between dissimilar metals.

Adhesive Bonding: Adhesive bonding is a material joining process in which an adhesive, placed between the adhered surfaces, solidifies to produce an adhesive bond (Figure1). When we bond components together the adhesive first thoroughly wets the surface and fills the gap between, then it solidifies. When solidification is completed the bond can withstand the stresses. The strongest adhesives solidify through chemical reaction and have a pronounced affinity for the joint surfaces. Adhesives come in several forms thin liquids, thick pastes, films, powders, pre-applied on tapes, or solids that must be melted. Adhesive can be designed with a wide range of strengths, all the way from weak temporary adhesives for holding papers in place to high strength structural systems that bond cars and airplanes. Now a day’s adhesive compete with mechanical fastening systems such as nuts, bolts, and rivets, or welding and soldering. In the practical application this kind of modal analysis can be used to analyze some structures such as cantilever bridge, frame of bicycle, automobile product, Industrial robots (manipulator), building structures, heavy machineries etc that can be simplified as beam and so on.

II. THEORETICAL METHOD

For cantilever beam subjected to free vibration, and the system is continuous system in which the beam mass is considered as distributed along with the stiffness of the plate.

Where, E is the modulus of elasticity of beam material, I is moment of inertia of a rectangular cross section, A is the cross sectional area of the beam and w is natural frequency of the beam in rad/sec.

We know,

$l = 1000\ mm, b = 50\ mm, h = 5\ mm etc.$

$E = 70x10^3\ N/mm^2 (Aluminum)$

$I = \frac{bh^3}{12} in mm^4, \quad A = bxh in mm^2$

$\rho = 2700x10^{-9} Kg/mm^3$

K= constant, (1.875, 4.694, 7.855, 10.995, 14.137, 17.279 etc.)

Now, to find the natural frequencies for different modes, here we are going to find the natural frequencies for first three modes which are calculated below by analytical methods.

a. First Natural Frequency

$$\omega_n = K^2\sqrt{\frac{EI}{\rho A l^4}}$$

$$= 1.875^2\sqrt{\frac{70{,}000x50x5^3}{12x50x5x2700x10^{-9}x1000^4}}$$

$$= 0.8170\ \frac{rad}{sec}.$$

We know,

$$f_n = \frac{\omega_n}{2\pi}$$

f_n = 0.135 Hz

b. Second Natural Frequency

$$\omega_n = K^2\sqrt{\frac{EI}{\rho A l^4}} \quad = 4.694^2\sqrt{\frac{70{,}000x50x5^3}{12x50x5x2700x10^{-9}x1000^4}}$$

$$= 5.1206\ \frac{rad}{sec}.$$

We know,

$$f_n = \frac{\omega_n}{2\pi}$$

f_n = 0.8149 Hz

c. Third Natural Frequency

$$\omega_n = K^2\sqrt{\frac{EI}{\rho A l^4}} \quad = 7.855^2\sqrt{\frac{70{,}000x50x5^3}{12x50x5x2700x10^{-9}x1000^4}} = 13.24\ \frac{rad}{sec}.$$

We know,

$$f_n = \frac{\omega_n}{2\pi}$$

f_n = 2.1083 Hz

Similarly, we can calculate natural frequencies for all modes by this method and also for all kinds of lap joints by calculating the moment of inertia and cross sectional area of joints. The natural frequencies which are obtained will be compared with the finite element analysis.

III. FINITE ELEMENT ANALYSIS

The finite element method (FEM) is a computational technique used to obtain approximate solution of boundary value problems in engineering. Simply stated, a boundary value problem is mathematical problem in which one or more dependent variables must satisfy a differential equation everywhere within a known domain of independent variables and satisfy specific conditions on the boundary of the domain.

The modal analysis of aluminum material rectangular plate, bolted, welded and single lap epoxy adhesive joint is done by finite element method using ansys 13.0 software. The material properties used are same with the properties used in theoretical analysis except Poisson's ratio. Which is mentioned in below table 1. The analysis is done for 3D element and the type of element used in this analysis is solid 185. The required properties of the aluminum and adhesive for the finite element analysis are given in table 1 and table 2.

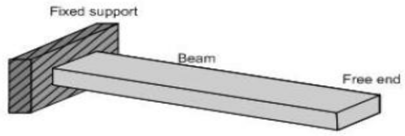
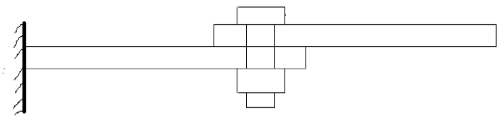
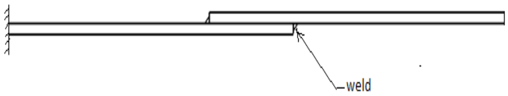
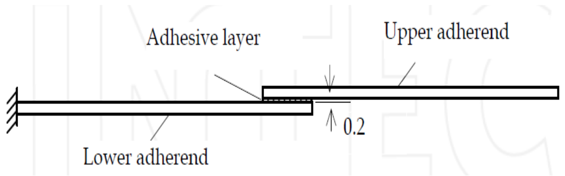
Sr. No.	Specimen	Specification
1	Single Rectangular Plate 	L=1000mm, b=50mm, h=5mm
2	Bolted Lap Joint 	L1= 500 mm, L2=500mm, b=50mm, h=5mm Lap dimensions = 50mmx 50mm
3	Welded Lap Joint 	L1= 500 mm, L2=500mm, b=50mm, h=5mm Lap dimensions = 50mmx 50mm
4	Single Lap Adhesive Joint 	L1= 500 mm, L2=500mm, b=50mm, h=5mm Lap dimensions = 50mmx 50mmx0.2mm

Figure 1: Specifications of Specimen used for analysis

Table 3: Comparative table of all joints

Mode Shapes	**Natural Frequencies in Hertz**			
	Single Rectangular Plate	Bolted Lap Joint	Welded Lap Joint	Single lap Adhesive joint
First	0.29774	0.28115	0.18885	0.20229
Second	1.3296	1.4450	1.2458	1.3820
Third	1.8632	1.7527	1.4421	1.4432
Fourth	5.2034	4.9879	3.3478	4.1426
Fifth	8.2374	8.6378	6.7290	7.9265

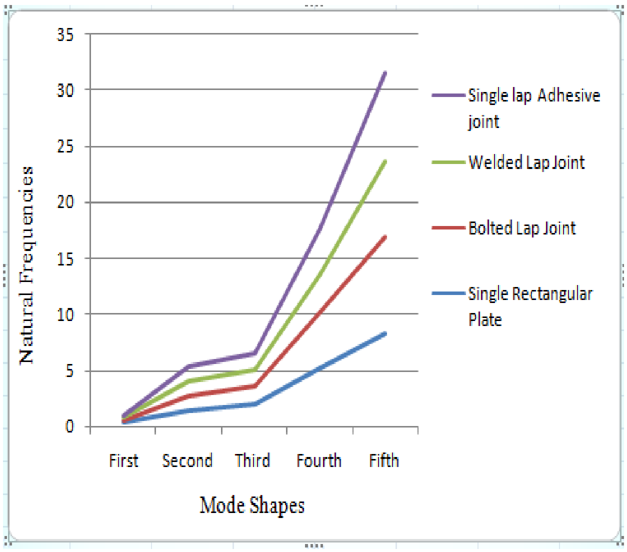


Figure 2: Comparison of natural frequencies with respect to modes

Table 4:Comparison of theoretical and software analysis

	Natural Frequencies	
Modes	**Theoretical Analysis**	**Software Analysis**
First	0.130	0.29774
Second	0.815	1.3296
Third	2.108	1.8632

Fourth	4.471	5.2034
Fifth	7.390	8.2374

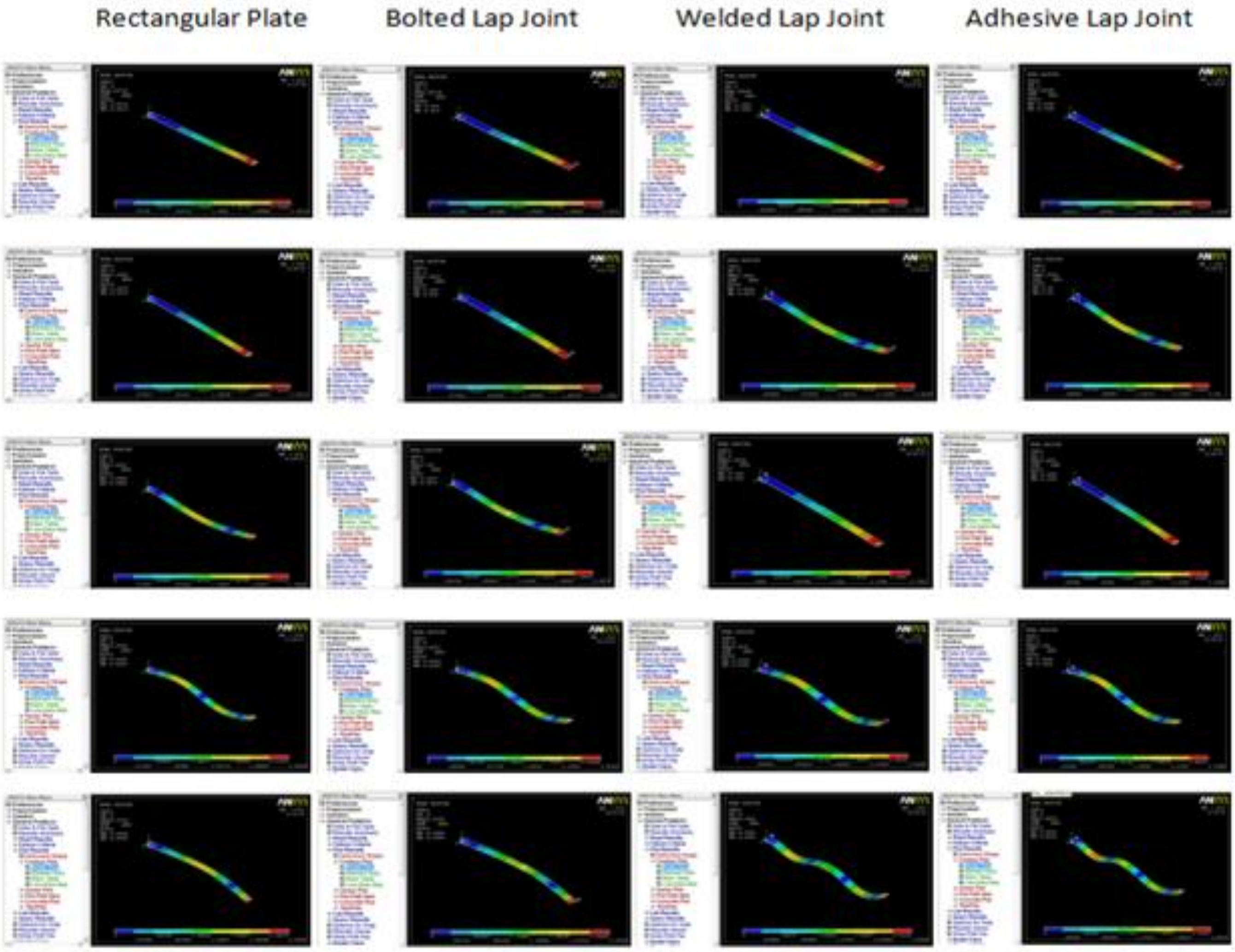


Figure 3: Mode shapes of rectangular plate

Figure 4: Mode shapes of bolted lap joint

Figure 5: Mode shapes of welded lap joint

Figure 6: Mode shapes of Single lap joint

Table 1: Material Properties of Aluminum Material

Material Properties	Notation	Aluminum	Unit
Modulus of Elasticity	E	0.7E11	N/m2
Poisson's Ratio	v	0.3	-
Density	ρ	2700	kg/m^3
Length	l	1.00	m
Width	b	0.050	m
Thickness	h	0.005	m

Table 2: Properties of Epoxy Adhesive Material

Material Properties of Adhesive Used			
Material Properties	Notation	Adhesive	Unit
Modulus of elasticity	E	4.00E+04	N/mm2
Poissons Ratio	v	0.4	-
Length	l	50	mm
Width	b	50	mm
Thickness	t	0.2	mm

1

There are certain common steps in formulating a finite element analysis of a physical problem, whether structural, fluid flow, heat transfer, vibration and some other problem. These steps are usually embodied in commercial finite element software packages. There are three main steps, namely: preprocessing, solution and post processing. The preprocessing (model definition) step is critical. This step includes; define the geometric domain of the problem, the element types to be used, the material properties of the elements, the geometric properties of the elements (length, area and the like), the element connectivity (mesh the model), the physical constraints (boundary conditions) and the loadings. The next step is solution, in this step the governing algebraic equations in matrix form and computes the unknown value of the primary field variables is assembled. Actually the features in this step such as matrix manipulation, numerical integration and equation solving are carried out automatically by commercial software. The final step is post processing, the analysis and evaluation of the result is conducted in this step. The specimens used for this analysis are given in below fig.1

1. RESULT AND DISCUSSION

In this work the modal analysis of rectangular plate, bolted, welded and single lap adhesive joint of plates are carried out. The modal analysis has been done by using Ansys 13.0 FEA software. The first five natural frequencies corresponding to different lap joints are shown in table 3. The comparison of frequencies with respect to mode shapes is shown in figure 2.

The result show that the natural frequencies of all lap joints are almost close to each other and if we compare it with a single rectangular plate that also comes to be satisfactory. The result show that the natural frequencies are depends on Young's modulus, Poisson's ration and density of the material. It is observed that the material of the lap joint is same but the kind of joint is only different but then also the natural frequencies and mode shapes are nearby same which is shown by table 3. In table 4 we also compared the theoretical solution and software analysis solution with each other and found that the natural frequencies at first five modes are nearby same that means the results are satisfactory as per the expectation. Figure 3-6 shows the mode shapes for same material and for different lap joints. The inspection of mode shapes show that the most of the mode shapes are similar for different lap joints and are found to be satisfactory.

1. CONCLUSION

In this work the modal analysis of all joints and rectangular plate is done by software as well as analytical methods and found to be satisfactory. From this work it can be concluded that the adhesive bonding of joints is an alternative method of joining materials together which has many advantages over the conventional joining methods such as welding, bolting and riveting. Adhesives can be used to bond dissimilar materials or plates, adhesive joints have a high stiffness to weight ratio and the stress distribution within the joint will be improved and there will not be any vibration also. The corrosion and vibration stress associated with mechanical fasteners and welds can be reduced or eliminated by forming adhesive joint. It is important to study the modal analysis i.e. natural frequency and mode shapes of the single lap joint to understand the dynamic nature of the systems and also in design control.

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