IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)
e-ISSN: 2278-1684,p-ISSN: 2320-334X, Volume 13, Issue 6 Ver. I (Nov. - Dec. 2016), PP 85-90
www.iosrjournals.org

Effect of permeability on mixed convective flow past an inclined porous plate in porous medium

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Abstract: *This analysis is investigated to influence of momentum, heat and mass transfer characteristics in the basis of boundary layer approximations on mixed convective flow past an inclined porous plate in porous medium. A set of non-dimensional transformations is used to transform the momentum, energy and concentration equations under consideration into nonlinear boundary layer equations which are then solved numerically using the Runge-Kutta sixth-order integration scheme together with Nachtsheim-Swigert shooting iteration technique. The validity of the numerical result is checked by comparing the results obtained for some specific cases with those available in the literature which was published, and a comparatively good agreement is reached. The numerical results are obtained and presented graphically illustrating the effect of the Prandtl number Pr, Schmidt number Sc and permeability parameter K on the flow field, and analyzed thereafter.*
Keywords: *Inclined porous plate, mixed convection, permeability, porous medium*

I. Introduction

An analysis of heat and mass transfer in MHD flow by natural convection from a permeable, inclined surface with variable wall temperature and concentration, taking into consideration the effects of ohmic heating and viscous dissipation is investigated by reference [1]. In this paper, the results are presented for the major governing parameters including the magnetic parameter, and in presence of magnetic field, the velocity is found to decrease whereas the temperature and concentration increase. In reference [2], the author's studied the problem of combined free-forced convection and mass transfer flow over a vertical porous flat plate, in presence of heat generation and thermal diffusion. In this paper, the author showed the effects of suction parameter, heat generation parameter and Soret number on the flow field of a hydrogen-air mixture as a non-chemical reacting fluid pair, and observed that the flow field is significantly influenced by these parameters. In reference [2], the author's also investigated the effects of thermophoresis and the homogeneous chemical reactions of first order on magnetohydrodynamics mixed convective flow past a heated inclined permeable flat plate in the presence of heat generation or absorption considering the viscous dissipation and Joule heating. MHD mixed convective heat transfer about a semi-infinite inclined plate in the presence of magneto and thermal radiation effects has been examined by reference [4]. The effects of the mixed convection parameter, the angle of inclination, the magnetic parameter and the radiation conduction parameter are discussed on the velocity and the temperature profiles as well as the local skin friction and the local heat transfer parameters by reference [4]. From this numerical investigation, the author reported that, an increase in the angle of inclination, decrease the local skin friction and the local heat transfer parameters. Effect of permeability on double diffusive mixed convective flow past an inclined porous plate investigated with MHD by reference [5]. The author considered MHD effects but an opportunity is to work by neglecting the effect of MHD. In reference [6], the author's analyzed a steady two-dimensional MHD free convection and mass transfer flow past an inclined semi-infinite vertical surface in the presence of heat generation in a porous medium. The author considered the porous medium but an opportunity is to extend this work by including the fluid suction effects for a porous plate. Magnetohydrodynamic mixed convective slip flow over an inclined porous plate with viscous dissipation and joule heating is investigated by reference [7]. As mentioned above, it is more reasonable to include the porous plate to explore the impact of the momentum, heat and mass transfer characteristics. Therefore, in the light of above literatures, the aim of the present work is to investigate the mixed convective flow past an inclined porous plate in porous medium.

II. Mathematical Analysis

Two-dimensional MHD mixed convective flow of a steady, viscous and incompressible electrically conducting fluid past inclined porous plate with an acute angle α to the vertical is considered. The physical coordinates (x,y) are chosen such that x is measured from the leading edge in the streamwise direction and y is measured normal to the inclined porous plate. The velocity components in the direction of flow and normal to the flow are u and v respectively. Acceleration due to gravity is g. The external flow with a uniform velocity U_∞ takes place in the direction parallel to the inclined porous plate.

It is assumed that T and C are the temperature and concentration of the fluid. The surface is maintained at a wall temperature T_w which is higher than the ambient temperature T_∞ of the surrounding fluid and the wall concentration C_w is greater than the ambient concentration C_∞. The schematic view of flow configuration and coordinates system is shown below in Fig. 1.1.

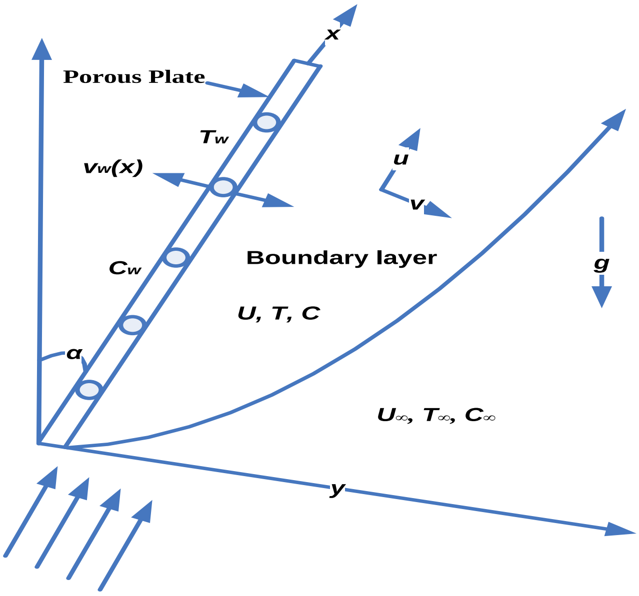


Fig. 1.1: Schematic view of flow configuration and coordinates system

Under the above assumptions, the governing equations for steady, two-dimensional, laminar boundary layer flow are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta_T(T-T_\infty)\cos\alpha+g\beta_C(C-C_\infty)\cos\alpha-\frac{\nu}{k^*}(u-U_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho C_p}(T-T_\infty) \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_M\frac{\partial^2 C}{\partial y^2}-\frac{\partial}{\partial y}[V_T(C-C_\infty)] \tag{4}$$

In the above equations, ν is the kinematic viscosity, β_T is the volumetric coefficient of thermal expansion, β_C is the volumetric coefficient of expansion with mass expansion, σ is the electrical conductivity of the fluid, ρ is the density of the fluid, k^* is permeability of the porous medium, k is the thermal conductivity of the fluid, C_p is the specific heat at constant pressure, Q_0 is the heat generation constant, D_M is the mass diffusivity, V_T is the thermophoretic velocity which can be expressed in the following form as:

$$V_T=-\frac{\kappa\upsilon}{T_{ref}}\frac{\partial T}{\partial y}$$

where T_{ref} is some reference temperature and κ is the thermophoretic coefficient which defined by reference [8] as:

$$\kappa=\frac{2C_s\left(\frac{k_g}{k_p}+C_t k_n\right)\left[1+k_n\left(C_1+C_2 e^{-\frac{C_3}{k_n}}\right)\right]}{\left(1+3C_m k_n\right)\left(1+2\frac{k_g}{k_p}+2C_t k_n\right)}$$

where C_1, C_2, C_3, C_m, C_s and C_t are constants, k_g and k_p are the thermal conductivities of the fluid and diffused particle, respectively and k_n is the Knudsen number.

The appropriate boundary conditions for the velocity, temperature and concentration of this consideration are as follows:

$$u = 0,\ v = \pm V_w(x),\ T = T_w,\ \text{and}\ C = C_w\ \text{at}\ y = 0$$
$$u = U_\infty,\ T = T_\infty,\ \text{and}\ C = C_\infty\ \text{at}\ y \to \infty$$

In addition, U_∞ is the free stream velocity and $V_w(x)$ is the permeability of the porous plate where its sign indicates suction (< 0) or blowing (> 0), subscripts w and ∞ refer to the wall and boundary layer edge, respectively. To facilitate the analysis, the governing differential equations are to be made nondimensional with suitable transformations and the following dimensionless variables defined by reference [9] are introduced:

$$\eta = y\sqrt{\frac{U_\infty}{\nu x}},\ \psi = \sqrt{\nu x U_\infty}\ f(\eta),\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \text{and}\ s(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

where, $\psi(x, y)$ is the stream function defined by $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$, such that the continuity equation (1) is satisfied automatically. In terms of these new variables, the velocity components can be expressed as:

$$u = U_\infty f'(\eta)\ \text{and}\ v = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\left[\eta f'(\eta) - f(\eta)\right]$$

Here, the prime stands for ordinary differentiation with respect to similarity variable η. Using dimensionless variables, the transformed momentum, energy, and concentration equations together with the boundary conditions can be written as:

$$f''' + \frac{1}{2} ff'' + R_{it}\,\theta\cos\alpha + R_{ic}\,s\cos\alpha - K\left(f' - 1\right) = 0 \tag{5}$$

$$\theta'' + \frac{1}{2}\Pr f\theta' + \Pr Q\theta = 0 \tag{6}$$

$$s'' + \frac{1}{2} Sc\ fs' - \tau\, Sc\left[s\theta'' + s'\theta'\right] = 0 \tag{7}$$

with the boundary conditions:

$$f = f_w,\ \ f' = 0,\ \ \theta = 1,\ \text{and}\ \ s = 1\ \text{at}\ \eta = 0$$
$$f' \to 1,\ \ \theta \to 0,\ \text{and}\ \ s \to 0\ \text{at}\ \eta \to \infty$$

where, $f_w = -V_w(x)\sqrt{\frac{x}{\nu U_\infty}}$ is the wall mass transfer coefficient such that $f_w > 0$ indicates wall suction and $f_w > 0$ indicates wall injection or blowing.

The corresponding dimensionless groups that appear in the nondimensional form of governing equations are defined as:

$$R_{it} = \frac{Gr_t}{(\mathrm{Re})^2},\ R_{ic} == \frac{Gr_c}{(\mathrm{Re})^2}, K = \frac{\nu x}{k^* U_\infty}, \Pr = \frac{\nu\rho C_p}{k}, Q = \frac{xQ_0}{\rho C_p U_\infty}, Sc = \frac{\nu}{D_M}, \tau = -\frac{\kappa}{T_{ref}}\left(T_w - T_\infty\right)$$

where, R_{it} is the local thermal Richardson number, R_{ic} is the local mass Richardson number, Gr_t is the local thermal Grashof number, Gr_c is the local mass Grashof number, Re is the local Reynolds number, K is the permeability parameter, Pr is the Prandtl number, Q is the heat generation parameter, Sc is the Schmidt number and τ is the thermophoretic parameter.

By employing the definition of wall shear stress $\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$ along with Fourier's law $q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$, and Fick's law $q_m = -D_M\left(\frac{\partial C}{\partial y}\right)_{y=0}$ the nondimensional forms of local skin-friction coefficient is $C_f = 2\left(\mathrm{Re}\right)^{-\frac{1}{2}} f''(0)$, local Nusselt number (rate of heat transfer) is $N_u = -\left(\mathrm{Re}\right)^{\frac{1}{2}} \theta'(0)$ and local Sherwood number (rate of mass transfer) is $S_h = -\left(\mathrm{Re}\right)^{\frac{1}{2}} s'(0)$ where $\mathrm{Re} = \frac{xU_\infty}{\nu}$ is denoting the local Reynolds number.

III. Solution Procedures

First of all, non-linear governing equations are transformed into simultaneous ordinary differential equations using suitable similarity transformations and these equations are further transformed the boundary value problem into initial value problem by using the shooting technique which is defined by reference [10]. The resultant initial value problem will be solved by employing Runge-Kutta sixth order technique. From the numerical process of computation, the velocity, temperature and concentration of the flow fields are computed and their numerical values are presented in graphically. The numerical methods are described in details, referring to reference [10].

IV. Comparison

For the accuracy of the numerical results, the present work is compared with the previous published work Reddy et al. [8] as shown below in Fig. 2. In Fig. 2, it is observed that the present numerical results are in good agreement with the velocity distribution of reference [6]. This favorable comparison leads to present and analysis the numerical results in the next sections.

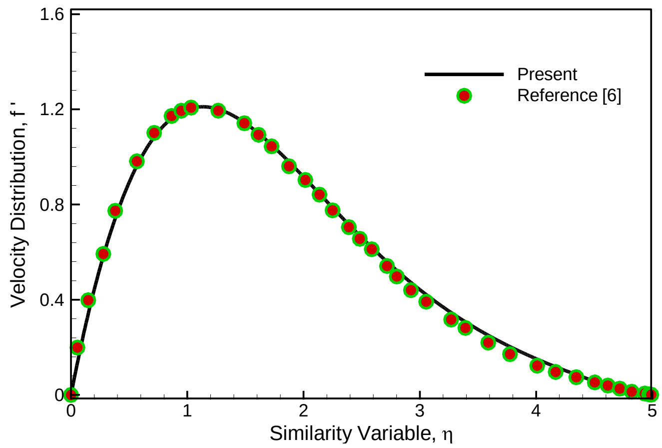
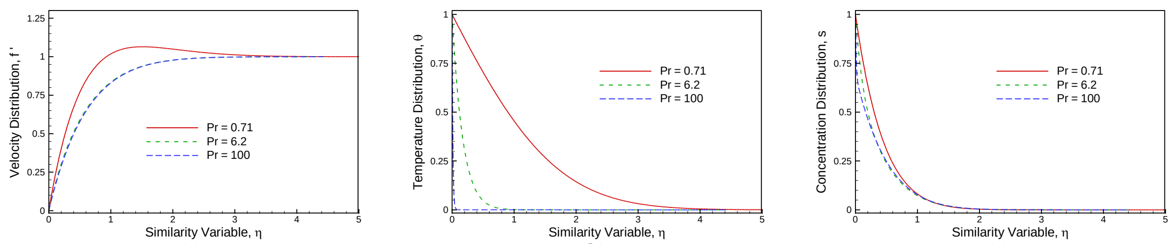


Fig. 2: Comparison of velocity distribution for $Gr_t = R_{it} = 2$, $Gr_m = R_{ic} = 2$, $M = 0.5$ (in present work is 0), $Q = 0.5$, K (coefficient of f' otherwise 0) = 0.5, $\alpha = 30^0$, Pr = 0.71, $f_w = 0$ and $Sc = 0.6$

V. Results and Discussion

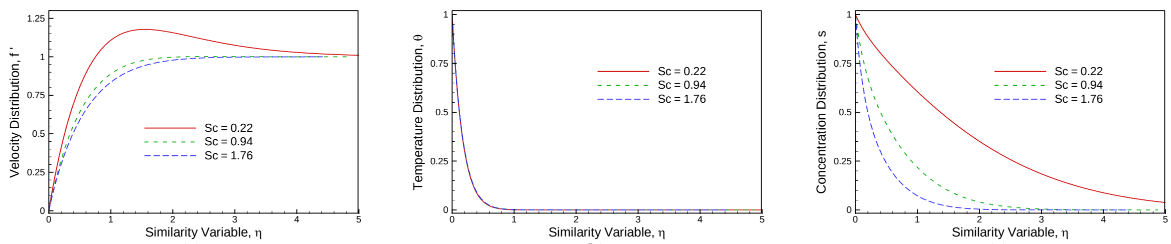
A spacious set of numerical results which is obtained by solving nonlinear ordinary differential equations is presented graphically. The numerical results for velocity, temperature and concentration distributions as well as the wall shear stress, the rate of heat transfer and the rate of mass transfer have been carried out using different values of the various physical parameters. The set of considered values of the various physical parameters occurring into the problem for numerical investigation are $R_{it} = 1$, $R_{ic} = 1$, $K = 0.5$, $Q = 0.5$, $\alpha = 30^0$, Pr = 6.2, $f_w = 2$, $Sc = 1.76$ and $\tau = 0.25$ unless otherwise specified. The value of Prandtl number Pr is taken to be 6.2 which is corresponds physically to water and the values of Schmidt number Sc are taken 1.76 for Benzene (C_6H_6) respectively.

Figures 3(a) – 3(c) describe the behavior of velocity, temperature and concentration on the flow field for different values of Prandtl number Pr (Pr = 0.71, 6.2, and 100) keeping other parameters of the flow field constant. The Prandtl number is the ratio of momentum diffusivity (kinematic viscosity) to thermal diffusivity. As the Prandtl number increases, the momentum diffusivity of the flow field increases and as a results the velocity as well as the hydrodynamic boundary layer thickness decrease which are observed in Fig. 3(a). Inspection of the temperature and concentration distributions in Figs. 3(b) – 3(c) reveal that increase in the values of Prandtl number lead to decrease the temperature of the flow field as well as the thermal boundary layer thickness and the concentration of the flow field decreases. Keeping all other parameters of the flow field constant except Schmidt number *Sc*, Figs. 4(a) – 4(c) display the impact of Schmidt number *Sc* (*Sc* = 0.22, 0.94 and 1.76) on the velocity, temperature and concentration distributions. The values of Schmidt number *Sc* are taken to be 0.22, 0.94 and 1.76 which are corresponding physically to hydrogen, carbon dioxide and benzene. The Schmidt number embodies the ratio of the momentum to the mass diffusivity. The Schmidt number therefore quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic and concentration boundary layer. In Fig. 4(a), it is observed that when the Schmidt number increases, the velocity of the flow field decreases because in presence of heavier diffusing species. On the other hand the temperature distribution which is presented in Fig. 4(b) changes insignificantly compared to the velocity of the flow field. The concentration of the flow field in Fig. 4(c) decreases as the Schmidt number increases i.e. in presences of heavier species. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reductions in the velocity and concentration distributions are accompanied by simultaneous reductions in the momentum and concentration boundary layer thickness and this is the analogous to the effect of increasing the Prandtl number on the thickness of a thermal boundary layer.



(a) (b) (c)

Fig. 3: Representative (a) velocity (b) temperature and (c) concentration distributions for different values of Prandtl number Pr

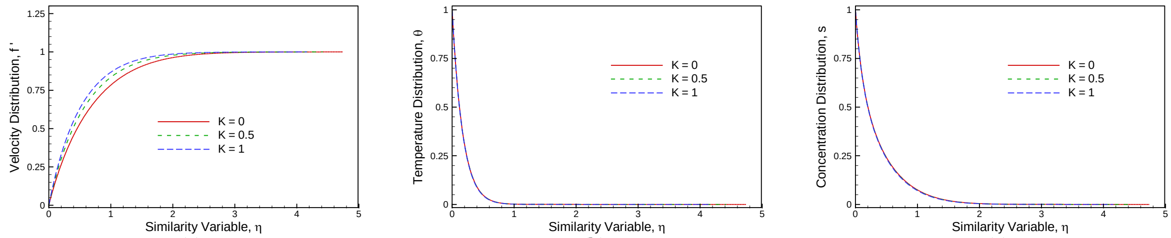


(a) (b) (c)

Fig. 4: Representative (a) velocity (b) temperature and (c) concentration distributions for different values of Schmidt number *Sc*

Figures 5(a) – 5(c) present the typical dimensionless velocity, temperature and concentration distributions for various values of the permeability parameter *K* (*K* = 0, 0.5 and 1) keeping other parameters of the flow field constant. The increase of the permeability of the porous medium has the tendency to increase the velocity of the fluid in the boundary layer which is observed in Fig. 5(a).

In addition, the velocity boundary layer thickness increases due to rising in the permeability of the porous medium parameter. However, the temperature and the concentration of the flow field changes insignificantly as permeability of the porous medium parameter increases which observed in Fig. 5(b) and Figs. 5(c).



(a) (b) (c)

Fig. 5: Representative (a) velocity (b) temperature and (c) concentration distributions for different values of permeability parameter *K*

Table 1: The wall shear stress $f''(0)$, the rate of heat transfer $\theta'(0)$ and the rate of mass transfer is $s'(0)$ at the plate $\eta = 0$ for R_{it} = 1, R_{ic} = 1, K = 0.5, Q = 0.5, $\alpha = 30^0$, Pr = 6.2, f_w = 2, Sc = 1.76 and τ = 0.25 unless otherwise specified

Pr		$2f''(0)$	$-\theta'(0)$	$-s'(0)$
Pr	0.71	4.85199013	0.62085343	2.22362533
	6.20	3.69654714	5.80002741	4.43303363
	100.0	3.40769797	99.4976585	43.9725872
Sc		$2f''(0)$	$-\theta'(0)$	$-s'(0)$
Sc	0.22	4.92226377	5.84547604	0.71809313
	0.94	4.01863605	5.81243078	2.47085559
	1.76	3.69654714	5.80002741	4.43303363
K		$2f''(0)$	$-\theta'(0)$	$-s'(0)$
K	0.0	3.19771188	5.78478730	4.41714086
	0.5	3.69654714	5.80002741	4.43303363
	1.0	4.09941505	5.81141986	4.44461713

VI. Conclusion

In this paper mixed convective flow past an inclined porous plate in porous medium has been investigated and the following conclusions can be drawn for the effect of the Prandtl number Pr, Schmidt number *Sc* and permeability parameter *K* on the flow field as: as increase in Pr and *Sc*, the velocity of the flow field decreases while as increase in *K* the velocity of the flow field increase; as the Pr increase, the temperature of the flow field tends to decrease whereas an increase in *Sc* and *K*, the temperature of the flow field changes insignificantly; due to increase in Pr and *Sc*, the concentration of the flow field decreases but an increase in *K*, the concentration of the flow field changes insignificantly.

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