IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)
e-ISSN: 2278-1684,p-ISSN: 2320-334X, Volume 18, Issue 1 Ser. I (Jan. –Feb. 2021), PP 59-67
www.iosrjournals.org

Analysis of a Two-Way Continuous Plate Based On Beam Analogy

Adah, E.I[1]., Ibearugbulem, O. M[2].,Ubi, S. E[3].&[4]Idagu, F.O
[1]*Lecturer, Department of Civil & Environmental Engineering,University ofCalabar, Nigeria*
[2]*Lecturer, Department of Civil Engineering, Federal University of Technology, Owerri, Nigeria*
[3]*Lecturer, Department of Civil Engineering, Cross River University of Technology, Calabar, Nigeria*
[4]*Researcher, Department of Civil Engineering, Federal University of Technology, Owerri, Nigeria*

Abstract:*The use of trigonometric shape functions in analysis of continuous plates in two directions by earlier researchers have been with a lot of difficulties of complex and cumbersome equations. In this study, polynomial displacement functions are used toanalyze a two-way continuous plate. A continuous plate spanning in both x and y-directionsis divided into twelve (12) panels, four(4) panels each along the x-direction and three(3) panels each along the y-direction. The external edges are assumed simply supported while the internal edges of each panel are assumed clamped. Edge moments of the clamped edges are calculated for each panel using appropriate boundary conditions which formed the fixed end moments(FEMs). Beam analogy is use to analyze the continuous plate using stiffness method in both directions to obtain the support and span moments. The support moments obtainedare comparedwith the fixed end moment. These showed a good distribution of the FEMs with percentage difference of 6.1% maximum in both directions. Simple equations are proposed based on the obtained result to quicken the analysis and design. Therefore, we conclude that analysis of continuous plate in two directions is adequate and easier with the use of polynomial displacement shape functions.*

Key Words: *Two-wayContinuous Plate, Polynomial Displacement Functions, Fixed Edge Moment, Support Moment, Span Moment, Beam Analogy, Element Stiffness Method.*

Date of Submission: 05-01-2021 Date of Acceptance: 21-01-2021

I. Introduction

The use of trigonometric shape functions in classical and approximate methods of analysis of continuous plates in two ways by earlier scholars such as [1, 2, 3]had been with a lot of difficulties due to complex and cumbersome equations. A plate by definition, is a structural element whose one dimension, the thickness, is smaller than the other two dimensions[2,3,4,5]. A continuous plate is that plate spanning over several supports into several equal or unequal panels. It is called one-way continuous plate if it spans only in one direction for example along x-axis, and if it spans along both x- and y- directions, then, it is called a two–way continuous plate. Application of the rigorous classical methods to the design of continuous floor slabs often leads to cumbersome calculations, and some illusory results based on indeterminable factors affecting the magnitude of the moments of the plate. This may be as a result of the fact that, continuous plates are externally statically indeterminate. The general analysis methods are force and deformation methods. Earlier scholars have used trigonometric displacement functions in analyzing continuous plate both in one-way and two ways[1,2,3,]. Dallam[6]used slope deflection method in his analysis. In finding an easy way of analyzing continuous plate in general and atwo-way continuous plate in particular, some scholars applied numerical and optimization techniques in theanalysesof continuous plates[7,8,9,10,11,12,13,14,15,16,17].

The present study employ the use of polynomial displacement functions and beam analogy. This is because of their simplicity in mathematical manipulations. Several scholars[18,19,20,21,22], have applied them to analyze single panel plates for bending, buckling and vibration. [23]applied them to analyzeda one-way continuous plate. However, there is indeed no evidence of their use in analyzing a two-way continuous plate. Also, there is limited literatures, at least to the authors' knowledge, for a two-way continuous plate analysis. According to[24], beam analogy method makes use of characteristic orthogonal polynomials to obtain meaningful displacement functions for each beam strip and plate. In view of [25,26], even though, a plate is a two dimensional element, the assumptions of beam theory still applies to plates.Trigonometric functions have been the dominantfunctions in analysis of continuous plates. None at least to the authors' knowledge, uses polynomial displacement functions in a two–way continuous plate analysis. Therefore, this research work is aimed at the application of polynomial displacement functions using beam analogy to analyze a two-way continuous plate with the view of obtaining fixed edge moments, support moments and span moments. Also, we aimed to obtain simple equations for calculating these moments based on the results obtained.

II. Methodology

Figure 1.0, showed a two-way continuous plate (that is, a 3 by 4 span). The plate is divided into twelve single panels of rectangular plates. It is assumed that all external edges (i.e. edges 1-2, 2-3, 3-4, 4-5, 5-6, 6-7, 7-5, 1-6, 7-1) are simply supported and internal edges(i.e. edges 2-2, 3-3, 4-4, 6-6, 7-7) are clamped.

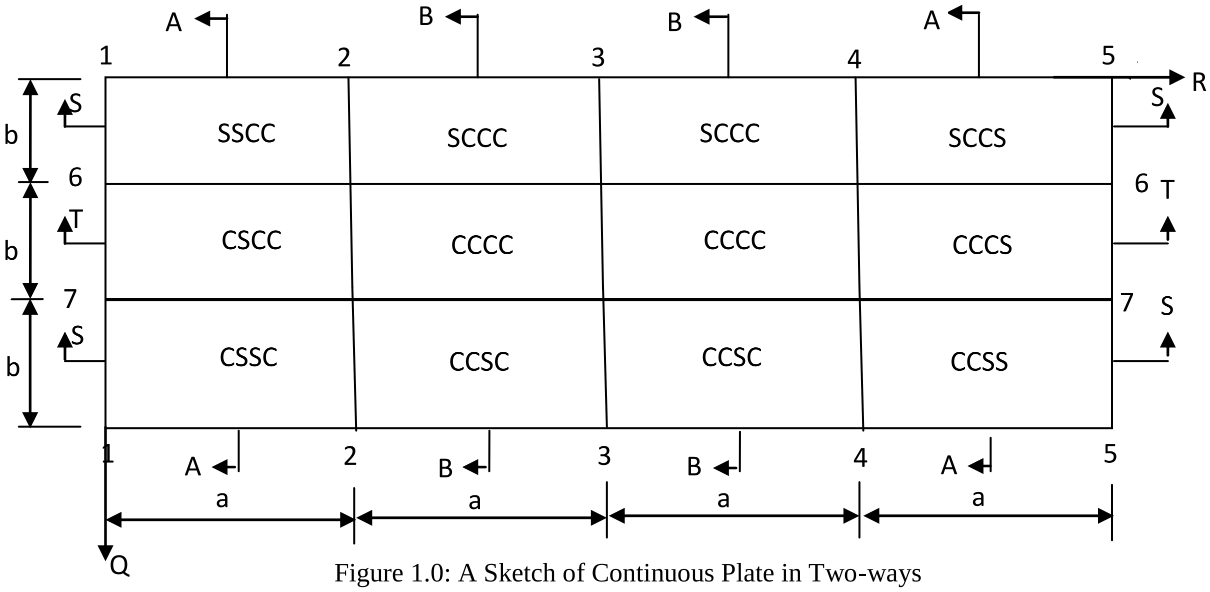

Figure 1.0: A Sketch of Continuous Plate in Two-ways

The naming of the plate panels is from the top moving anticlockwise. Hence,
SSCC plate means a plate with the first two edges simply supported and the third and four edges clamped.
SCCC plate means a plate simply supported at the first edge and clamped at the second, third and fourth edges.
The rest of the panels follows the same pattern as above.
The displacement shape profile, h, for the nine different plate types are given by Adah[27] as presented in Table no 1.

Table no 1: Polynomial DisplacementShape Functions for the Nine Rectangular Plates Types

S/N	Types of Plates	Displacement Shape Profile, h = R*Q
1	SSCC = SC * SC	$(0.5R-1.5R^3+R^4)(0.5Q-1.5Q^3+Q^4)$
2	SCCC = CC * SC	$(R^2-2R^3+R^4)(0.5Q-1.5Q^3+Q^4)$
3	SCCS = CS * SC	$(1.5R-2.5R^3+R^4)(0.5Q-1.5Q^3+Q^4)$
4	CSCC = SC * CC	$(0.5R-1.5R^3+R^4)(Q^2-2Q^3+Q^4)$
5	CCCC = CC * CC	$(R^2-2R^3+R^4)(Q^2-2Q^3+Q^4)$
6	CCCS = CS * CC	$(1.5R^2-2.5R^3+R^4)(Q^2-2Q^3+Q^4)$
7	CSSC = SC * CS	$(0.5R-1.5R^3+R^4)\ (1.5Q^2-2.5Q^3+Q^4)$
8	CCSC = CC * CS	$(R^2-2R^3+R^4)\ (1.5Q^2-2.5Q^3+Q^4)$
9	CCSS = CS * CS	$(1.5R^2-2.5R^3+R^4)\ (1.5Q^2-2.5Q^3+Q^4)$

In non-dimensional parameters, R and Q; for X = aR, Y = bQ, for $0 \le R \le 1$, $0 \le Q \le 1$. Where 'a' and 'b' are the plate dimensions along x- and y- axes respectively.
The total potential energy functional of a rectangular plate under pure bending is given by [23] as

$$\Pi = \frac{DA^2}{2\text{Ƨ}^3 a^2} \int_0^1 . \int_0^1 \left[\text{Ƨ}^4 \frac{\partial^2 h}{\partial R^2} + 2\text{Ƨ}^2 \frac{\partial^2 h}{\partial R\, \partial Q} + \frac{\partial^2 h}{\partial Q^2} \right] \partial R\, \partial Q - a^2 \text{Ƨ} A q \int_0^1 . \int_0^1 h\, \partial R\, \partial Q \ (1)$$

Where: h is the general displacement shape profile, and is given as equation (2)

$$h = \frac{w}{A} \qquad (2)$$

D, the flexural rigidity of the plate material and is given as equation (3)

$$D = \frac{Et^3}{12(1-v^2)} \qquad (3)$$

and A is the amplitude of displacement, t is the plate thickness, $\boldsymbol{\nu}$ is the Poison ratio. While Ƨ is the aspect ratio b/a.

If equation (1) is minimize with respect to A, and after making the amplitude, A, the subject of the formula we have,

$$A = u_s \frac{qa^4}{\mathrm{D}} \qquad (4)$$

$$\text{Where, } \mathrm{u_s} = \frac{\int_0^1 \cdot \int_0^1 h\, \partial \mathrm{R}\, \partial \mathrm{Q}}{\int_0^1 \cdot \int_0^1 \left[\frac{\partial^2 \mathrm{h}}{\partial \mathrm{R}^2} + \frac{2}{Ƨ^2}\frac{\partial^2 \mathrm{h}}{\partial \mathrm{R}\, \partial \mathrm{Q}} + \frac{1}{Ƨ^4}\frac{\partial^2 \mathrm{h}}{\partial \mathrm{Q}^2}\right] \partial \mathrm{R}\, \partial \mathrm{Q}} \ (5)$$

The bending moment-curvature of a plate is given as

$$M_x = -D\left(\frac{\partial^2 \mathrm{w}}{\partial x^2} + \mathrm{v}\frac{\partial^2 \mathrm{w}}{\partial y^2}\right) (6a)$$

$$M_y = -D\left(v\frac{\partial^2 \mathrm{w}}{\partial x^2} + \frac{\partial^2 \mathrm{w}}{\partial y^2}\right) (6b)$$

Where $\boldsymbol{\nu}$ is the Poisson ratio of the plate material

By substituting equations (2) and (4) into equation (6), the bending moment-Curvatures, M_R and M_Q of a rectangular isotropic plate in R- and Q- directions in non-dimensional form are[27]

$$\mathrm{M}_R = -u_s \mathrm{q}a^2\left(\frac{\partial^2 \mathrm{h}}{\partial R^2} + v\frac{\partial^2 \mathrm{h}}{Ƨ^2\, \partial Q^2}\right)(7a)$$

$$\mathrm{M}_Q = -u_s \mathrm{q}a^2\left(v\frac{\partial^2 \mathrm{h}}{\partial R^2} + \frac{\partial^2 \mathrm{h}}{Ƨ^2\, \partial Q^2}\right)(7b)$$

Equations (7) becomes

$$\mathrm{M}_R = \beta_x \mathrm{q}a^2 = FEM \qquad (8a)$$

$$\mathrm{M}_Q = \beta_y \mathrm{q}a^2 = FEM \qquad (8b)$$

$$\text{where } \beta_x = -u_s\left(\frac{\partial^2 \mathrm{h}}{\partial R^2} + v\frac{\partial^2 \mathrm{h}}{Ƨ^2\, \partial Q^2}\right);\ \beta_y = -u_s\left(v\frac{\partial^2 \mathrm{h}}{\partial R^2} + \frac{\partial^2 \mathrm{h}}{Ƨ^2\, \partial Q^2}\right)(9)$$

Assuming the individual plate panels are square (that is a = b). The coefficient of the fixed end moment of each span is obtained by differentiating, the displacement shape profile, h, of each panel with respect to R and Q. The resulting expressions is substituted into equation (9). For instance,

Now for SSCC plate panel, the fixed edge moment ($FEM_R = M_R$) is

$$FEM_R = -u_s \mathrm{q}a^2\left((12R^2 - 9\mathrm{R})(0.5Q - 1.5Q^3 + 4^4) + \frac{v}{Ƨ^2}(0.5R - 1.5R^3 + R^4)(12Q^2 - 12\mathrm{R})\right)(10)$$

At the midpoint of Support 2, R = 1, Q = ½, hence, equation (10) becomes,

$$FEM_R = -0.375 u_s \mathrm{q}a^2 (11a)$$

Similarly,

$$FEM_Q = \frac{-0.375 u_s}{Ƨ^2}\mathrm{q}a^2 (11b)$$

Where u_s from Equation (5) for SSCC plate and b/a = 1, is 0.134453581, resulting in

$$FEM_R = -0.375 * 0.134453581\mathrm{q}a^2 = -0.05042\mathrm{q}a^2 \ (12a)$$

$$FEM_Q = -0.375 * 0.134453581\mathrm{q}a^2 = -0.05042\mathrm{q}a^2 (12b)$$

Similarly, the values of the coefficients of fixed endmoments of the individual plate panels were obtained and are presented in Figure 2.0 and Table 2.0. The external edges have zero FEMs because a simply supported edge has no fixed end moment.

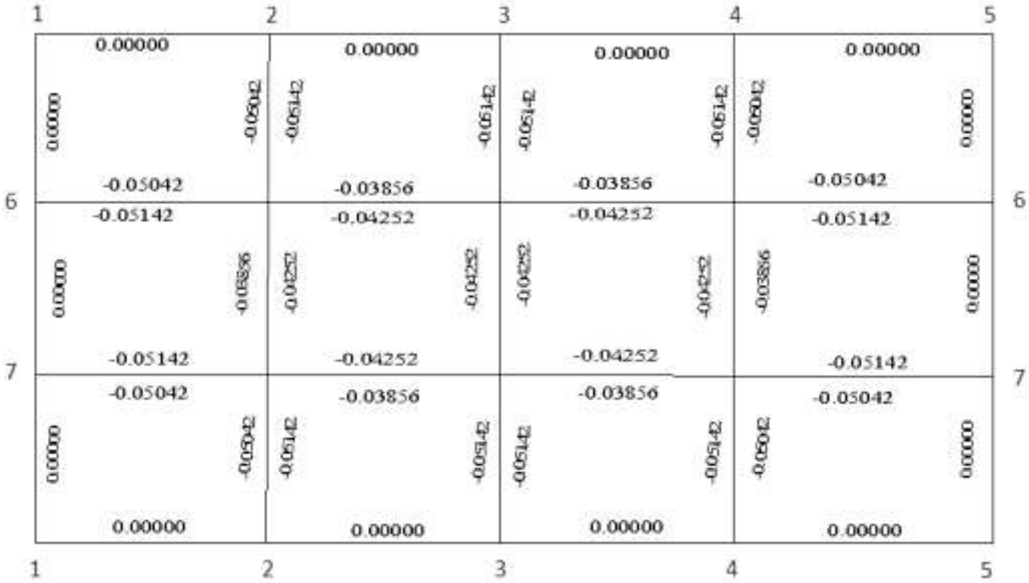

Figure 2.0: FEMs Coefficients at the edges of the individual plate panels of a Two-Way Continuous plate

Supports Moments Calculation Using Element Stiffness Method

Three sections along x-axis and four sections along y-axis are taken through the center of each panel(Figures 1.0). The strips taken are presented as a beam loaded uniformly as shown in Figure 3.0a and c along x- and y-axes respectively.

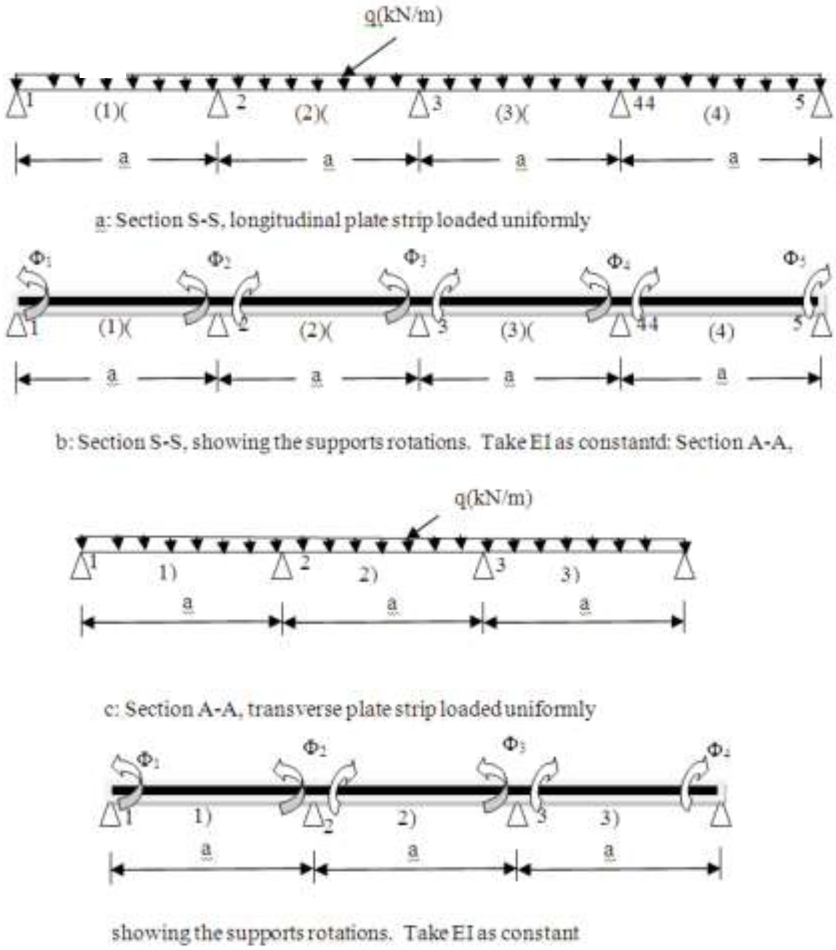

Figure 3.0: Sections from a 3 by 4 span Continuous plate in Figure 1.0;

The rotations or deformations, Φ, at the supports are as shown in Figure 3b and d. These beams are analyzed using element stiffness method for support and span moments. Note that, (1), (2), (3) and (4) denote the spans 1, 2, …n; and 1,2,3,4, 5, denote supports 1, 2, …n; a is the plate dimension along x-axis, b is along y-axis and q is the uniformly distributed transverse load on the plate. Assuming equal spans i.e. (1) = (2,) = (3),= (4) , that is , aspect ratio Ƨ = b/a = 1.
Considering Section S-S, due to symmetry, the element stiffness (K_e) for members 1,2; 2,3; 3,4; and 4,5; are the same and given as

$$k_e = \begin{vmatrix} 4EI/L_i & 2EI/L_i \\ 2EI/L_i & 4EI/L_i \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} \quad (13)$$

Where i indicates the span under consideration.
Combining all the elements stiffness for the section, to obtain the global stiffness K,

$$K = EI/a \begin{vmatrix} 4 & 2 & 0 & 0 & 0 \\ 2 & 8 & 2 & 0 & 0 \\ 0 & 2 & 8 & 2 & 0 \\ 0 & 0 & 2 & 8 & 2 \\ 0 & 0 & 0 & 2 & 4 \end{vmatrix} \quad (14)$$

The fixed end moment ($FEM_{i,j}$)
Considering symmetry, the fixed end moments at the supports of the continuous plates are as follows:
$FEM_{1,1} = FEM_{5,4} =$ 0kNm;
$FEM_{2,1} = FEM_{4,5} = -0.05042qa^2$kNm;
$FEM_{2,2} = FEM_{3,2} = FEM_{3,3} = FEM_{4,3} = -0.05142qa^2$kNm;
Presenting the result in matrix form we obtain Equation (15)

$$FEM = qa^2 \begin{vmatrix} 0.00000 \\ -0.05042 \\ -0.05142 \\ -0.05142 \\ -0.05142 \\ -0.05142 \\ -0.05042 \\ 0.00000 \end{vmatrix} \begin{matrix} FEM_{1,2} \\ FEM_{2,1} \\ FEM_{2,3} \\ FEM_{3,2} \\ FEM_{3,4} \\ FEM_{4,3} \\ FEM_{4,5} \\ FEM_{5,4} \end{matrix} \ \text{kNm} \quad (15)$$

The total fixed end moment at each support are obtained by adding the FEMs at each support for the adjoining plate edges, and presenting the result in matrix form we have

$$; \begin{vmatrix} FEM1 \\ FEM2 \\ FEM3 \\ FEM4 \\ FEM5 \end{vmatrix} = qa^2 \begin{vmatrix} 0 \\ -0.05042+0.05142 \\ -0.05142+0.05142 \\ -0.05142+0.05142 \\ 0 \end{vmatrix} = qa^2 \begin{vmatrix} 0.00000 \\ 0.001 \\ 0.00000 \\ -0.001 \\ 0.00000 \end{vmatrix} \ \text{kNm} \quad (16)$$

Force is stiffness multiply by deformation or rotation
That is $|FEM| = |K| * |\Phi|$ (17)
Substituting Equations (14) and (16) into Equation (17) yields

$$\begin{vmatrix} 0.00000 \\ 0.001 \\ 0.00000 \\ k\text{-}0.001 \\ 0.00000 \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 & 0 & 0 & 0 \\ 2 & 8 & 2 & 0 & 0 \\ 0 & 2 & 8 & 2 & 0 \\ 0 & 0 & 2 & 8 & 2 \\ 0 & 0 & 0 & 2 & 4 \end{vmatrix} * \begin{vmatrix} \Phi1 \\ \Phi2 \\ \Phi3 \\ \Phi4 \\ \Phi5 \end{vmatrix} \quad (18)$$

Making deformation the subject of the equation by taken the inverse of the global stiffness matrix we obtain equation (19)

$$\begin{vmatrix} \Phi 1 \\ \Phi 2 \\ \Phi 3 \\ \Phi 4 \\ \Phi 5 \end{vmatrix} = a/EI \begin{vmatrix} 0.28869 & -0.07738 & 0.020833 & -0.00595 & 0.002976 \\ -0.07738 & 0.154762 & -0.04167 & 0.011905 & -0.00595 \\ 0.020833 & -0.04167 & 0.145833 & -0.04167 & 0.020833 \\ -0.00595 & 0.011905 & -0.04167 & 0.154762 & -0.07738 \\ 0.002976 & -0.00595 & 0.020833 & -0.07738 & 0.28869 \end{vmatrix} * \begin{vmatrix} 0.00000 \\ 0.001 \\ 0.00000 \\ -0.001 \\ 0.00000 \end{vmatrix} qa^2$$

Resolving we have

$$\begin{vmatrix} \Phi 1 \\ \Phi 2 \\ \Phi 3 \\ \Phi 4 \\ \Phi 5 \end{vmatrix} = qa^3/EI \begin{vmatrix} -0.00007 \\ 0.000143 \\ 0.00000 \\ -0.00014 \\ 0.00007 \end{vmatrix} \qquad (19)$$

Member moment (MD) force calculation

This is obtained from the product of element stiffness of each member and the corresponding deformation

$$MD = k * \Phi \qquad (20)$$

Hence, for member 1,2

$$\begin{vmatrix} MD1,2 \\ MD2,1 \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} * qa^3/EI \begin{vmatrix} -0.00007 \\ 0.000143 \end{vmatrix} = qa^2 \begin{vmatrix} 0.00000 \\ 0.000429 \end{vmatrix} \qquad (21a)$$

For member 2,3

$$\begin{vmatrix} MD2,3 \\ MD3,2 \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} * qa^3/EI \begin{vmatrix} 0.000143 \\ 0.00000 \end{vmatrix} = qa^2 \begin{vmatrix} 0.000571 \\ 0.000286 \end{vmatrix} \qquad (21b)$$

For member 3,4

$$\begin{vmatrix} MD3,4 \\ MD4,3 \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} * qa^3/EI \begin{vmatrix} 0.00000 \\ -0.00014 \end{vmatrix} = qa^2 \begin{vmatrix} -0.00029 \\ -0.00057 \end{vmatrix} \qquad (21c)$$

For member 4,5

$$\begin{vmatrix} MD4,5 \\ MD5,4 \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} * qa^3/EI \begin{vmatrix} -0.00014 \\ 0.00007 \end{vmatrix} = qa^2 \begin{vmatrix} -0.00043 \\ 0.00000 \end{vmatrix} \qquad (21d)$$

Final Support Moments (SM)

The moment at the supports are obtained from the difference between the FEM and the MD that is Equations (15) and (21).

$$SM = FEM - MD \qquad (22)$$

Hence, for member 1,2

$$\begin{vmatrix} M1,2 \\ M2,1 \end{vmatrix} = qa^2 \begin{vmatrix} 0.00000 \\ -0.05042 \end{vmatrix} - qa^2 \begin{vmatrix} 0.00000 \\ 0.000429 \end{vmatrix} = qa^2 \begin{vmatrix} 0.00000 \\ -0.05085 \end{vmatrix} \qquad (23a)$$

For member 2,3

$$\begin{vmatrix} SM2,3 \\ SM3,2 \end{vmatrix} = qa^2 \begin{vmatrix} -0.05142 \\ -0.05142 \end{vmatrix} - qa^2 \begin{vmatrix} 0.000571 \\ 0.000286 \end{vmatrix} = qa^2 \begin{vmatrix} -0.05199 \\ -0.05171 \end{vmatrix} \qquad (23b)$$

For member 3,4

$$\begin{vmatrix} SM3,4 \\ SM4,3 \end{vmatrix} = qa^2 \begin{vmatrix} -0.05142 \\ -0.05142 \end{vmatrix} - qa^2 \begin{vmatrix} -0.00029 \\ -0.00057 \end{vmatrix} = qa^2 \begin{vmatrix} -0.05113 \\ -0.05085 \end{vmatrix} \qquad (23c)$$

For member 4,5

$$\begin{vmatrix} SM4,5 \\ SM5,4 \end{vmatrix} = qa^2 \begin{vmatrix} -0.05042 \\ 0.0000 \end{vmatrix} - qa^2 \begin{vmatrix} -0.00043 \\ 0.00000 \end{vmatrix} = qa^2 \begin{vmatrix} -0.04999 \\ 0.00000 \end{vmatrix} \qquad (23d)$$

Span Moment (M_{spn})

Oyenuga[28] expressed the span moment as

$M_{spn} = 0.125qa^2 - 0.5(FEMi,j+ FEMj,i)\ qa^2$ (24)

Therefore, substituting FEMs values in equation (15) into equation (24) gives

$M_{spn(1)}= 0.09929qa$ (25a)

$M_{spn(2)} = 0.07358qa^2$ (25b)

$M_{spn(3)}= 0.07408qa^2$ (25c)

$M_{spn(4)}= 0.09979qa^2$ (25d)

In a similar way, other sections, that is, T-T, A-A and B-B, were analyzed for FEMs, SM and M_{spn}.

III. Results

The numerical results obtained for fixed edge moments, support moments from this work are presented in Tables no 2 and 3 for x- and y- directions respectively. FEMs and SMs of Section S-S are presented Columns 2 and 3 respectively, and those for Section T-T along x-direction are presented in columns 5 and 6 respectively. The numerical values in the Tables are the coefficients only. Also, results obtained for span moments along x- and y- directions are presented in Table no 4.

Table no 2: Values of Coefficients of FEM and SM,of a Two-way Continuous Plate in x-direction obtained from Present Study. (Ƨ = 1, 4-Spans).

	Section S-S			Section T-T		
Support	$FEM_{i,j} = β_1qa^2$ $β_1$	$SM_{i,j} = β_2qa^2$ $β_2$	Percentage Difference $100(β_2- β_1)/ β_1$	$FEM_{i,j} = β_3qa^2$ $β_3$	$SM_{i,j} = β_4qa^2$ $β_4$	Percentage Difference $100(β_4- β_3)/ β_3$
1	0.0000	0.00000	0.00	0.0000	0.00000	0.00
2	-0.05042	-0.05085	-0.85	-0.03856	-0.04026	-4.41
	-0.05142	-0.05199	-1.11	-0.04252	-0.04478	-5.32
3	-0.05142	-0.05171	-0.56	-0.04252	-0.04365	-2.66
	-0.05142	-0.05113	0.03	-0.04252	-0.04139	2.66
4	-0.05142	-0.05085	1.11	-0.04252	-0.04024	5.37
	-0.05042	-0.04999	0.85	-0.03856	-0.03686	4.41
5	0.0000	0.00000	0.00	0.0000	0.00000	0.00
	$FEM_E = -qa^2/18$,	$SM_E = -qa^2/18$,		$FEM_I = -qa^2/22$	$SM_I = -qa^2/22$	

Table no 3: Coefficients of FEM and SM, of a Two-way Continuous Plate along y-direction obtained from Present Study. (Ƨ = 1, 3-Spans).

	Section A-A			Section B-B		
Support	$FEM_{i,j} = βqa^2$ $β_1$	$SM_{i,j} =βqa^2$ $β_2$	Percentage Difference $100(β_2- β_1)/ β_1$	$FEM_{i,j}=β_3qa^2$ $β_3$	$SM_{i,j} =β_4qa^2$ $β_4$	Percentage Difference $100(β_4- β_3)/ β_3$
1	0.0000	0.00000	0.00	0.0000	0.00000	0.00
2	-0.05042	-0.05102	-1.20	-0.03856	-0.04094	-6.17
	-0.05142	-0.05182	-0.78	-0.04252	-0.04410	-3.72
3	-0.05142	-0.05102	0.78	-0.04252	-0.04094	3.72
	-0.05042	-0.04982	1.19	-0.03856	-0.03618	6.17
4	0.00000	0.00000	0.00	0.00000	0.00000	0.00
Equation	$FEM_E = -qa^2/18$,	$SM_E = -qa^2/18$,		$FEM_I = -qa^2/22$,	$SM_I = -qa^2/22$,	

(Noted: Subscript, E is External strip, I is Internal Strip)

Table no 4: Coefficients of Span moment (M_{spn}), of a Two-way Continuous Plate along x- and y-directions obtained from Present Study.

Span Moment	$M_{spn(1)} = \beta qb^2$ B	$M_{spn(2)} = \beta qb^2$ β	$M_{spn(3)} = \beta qb^2$ β	$M_{spn(4)} = \beta qb^2$ β
Section S-S	0.09929	0.07358	0.07408	0.09979
Section T-T	0.1037	0.08248	0.08447	0.10573
Section A-A	0.09929	0.07408	0.09979	-
Section B-B	0.10374	0.08446	0.10572	-
Equation	$M_{spn} = qa^2/10$			

IV. Discussions

Comparison was made between the fixed end moments and support moments obtained from this work as show incolumns 4 and 7 of Table no 2 and 3 for x- and y- directions respectively. This is to validate the results of this present study.For sections S-S and T-T along x-axis, the maximum percentage differences are1.11% and 5.37% respectively. This shows that the values of the FEM and SMare very close and implies that there is a good distribution of load at the supports. Also, for sections A-A and B-B, Table no 3 columns 4 and7 showed that the percentage differences are all less than 5% for FEM and have a maximum of 6.17% for support moment(SM). This value even though greater than 5% is acceptable in statistics. These calculations were based on aspect ratio of one which has simplify the manual calculation but these calculationscan be done for any aspect ratio. In addition, from the results of the study, it is evidenced that the FEM and SM can be calculated easily using $-qa^2/18$ and $-qa^2/22$ for external strips and internal strips of a continuous plate respectively. While from Table no 4, the span moments can be calculated using $qa^2/10$. Where 'a' is the length of the plate along x-axis and 'q' is the uniformly distributed load on the plate. With this equations, the present study has simplified continuous plate analysis and offer an easy way to design.

V. Conclusion

The present study, presents analysis of a 3 by 4 spans continuous plate using polynomial displacement functions. This approach used is based on beam analogy. It is evidenced from the discussion above that the results of this present study using polynomial shape functions for analyzing 3x4-spans continuous plates in two directions are satisfactory. Also, the present study has formulated equations to simplify the analysis. Hence, the conclusion that polynomial displacement shape functions are simpler approximations of the deflected shape function of rectangular plate suitable for continuous plate analysis in two directions, and the equations offer an easy approach to design which safe time.

Reference

[1]. Timoshenko, S.P &Woinowsky-Krieger. Theory of plates and shells. 2nd ed., New York; McGraw Hills, 1959
[2]. Szilard, R.. Theories and application of plate analysis. New Jersey, John Wiley & sons, 2004.
[3]. Ventsel, E. &KrauthamerT.. Thin plates and shells: Theory, analysis and applications, New York, Marcel Dekker, 2001.
[4]. Enem, J. I.. Geometrically nonlinear analysis of isotropic rectangular thin plates using Ritz method, PhD Thesis: department of civil engineering Federal University of Technology, Owerri Nigeria, 2018.
[5]. Njoku, C. F. Dynamic Analysis of rectangular thin isotropic plates using beam analogy method, PhD Thesis: department of civil engineering Federal University of Technology, Owerri Nigeria, 2018.
[6]. Dallam, N.L. Analysis of continuous rectangular plates systems by the slope deflection method. A PhD thesis submitted to Oklahoma State University, 1966.
[7]. Golley, B.W., &Petrolito, J. Methods for analyzing tanks and continuous plates, Computers & Structures, 1984, 18(6), 1141-1151.
[8]. Lau, W. W.S. Analysis of straight, Continuous plate-girder bridges, M.Sc Thesis submitted to School of Graduate studies, University of Ottawa, Canada, 1988.
[9]. Moshaiou, A. &Vitooraporn, C. Application of boundary integral method to continuous plates, Journal of Ship research, 1990, 34(3).
[10]. Ohga, M. and Shigematsu, T. Analysis of continuous plates by a combined boundary element-transfer matrix method, Computers & Structures, 1990, 36(1), 81-89.
[11]. Memani, A. M, West, H.H., &Caralier, T. M. Optimization of continuous steel plate girder bridges, Structural Optimization, 1991, 3(4), 231-239,.
[12]. Chalal, F. Z. Optimum design of continuous plate girder with variable depth, 2017, 8(3), 494-509.
[13]. Han-Tawchen and Jyh-Tyng L. Optimum dimensions of continuous plate fin with various tube arrays, Journal of numerical heat transfer, Part A: Application, 1998, 34(2).
[14]. Bakhaty, A.A, Govindjee, S, &Mosalam, K,M. Theoretical evaluation of hybrid simulation applied to continuous plate structures, ASCE Journal of Eng. Mech., 2016, 142(12).
[15]. Yang, J., Ouyang, H., &Stancioiu, D. Numerical studies of vibration of four-span continuous plate with rails excited by car with experimental validation, International journal of Structural Stability and Dynamics, 2017, 17(10).
[16]. Yang, J., Ouyang, H., Stancioiu, D., Cao, S., &He, X. Dynamic response of a four-span continuous plate structure subjected to moving cars with time-varying speeds, Journal of vibration and acoustic, 140(6), 2018.

[17]. Najib, N., Bachok, N., &Md.Afrfin, N. Stability of dual solutions of mass transfer on a continuous flat plate moving in parallel or reversely to a free stream in presence of chemical reaction with second order clip, AIP Conference proceedings, 1830(1), 2017.

[18]. Ibearugbulem, O. M. Application of direct variational principles in elastic stability of rectangular flat thin plates. Ph.D thesis submitted to Postgraduate School, Federal University of Technology, Owerri, 2012.

[19]. Ibearugbulem, O. M., Ezeh J. C. &Ettu, L. O. Energy methods in theory of rectangular plates: Use of polynomial Shape Functions, Owerri, Liu House of excellence ventures, 2014.

[20]. Adah, E. I., Onwuka, D. O., &Ibearugbulem, O. M.Development of polynomial-based program for pure bending analysis of SSSS rectangular thin isotropic plate, International Journal of science, engineering and technological research, 2016, 5(7), 2290-2295.

[21]. Onwuka, D. O. Ibearugbulem, O. M., & Adah, E. I. Stability analysis of axially compressed SSSS & CSCS plates using MATLAB programming. International journal of science and technoledge, 2016, 4(1), 66-71.

[22]. Ibearugbulem, O. M., Adah, E. I., Onwuka, D. O., &Anyadiegu, C. P. Computer based free vibration analysis of isotropic thin rectangular flat CCCC plate. The International journal of engineering and science, 2016, 5(7), 38-41.

[23]. Adah, E. I., Ibearugbulem, O. M. &Ubi, P. A. Analysis of continuous plate using polynomial displacement functions based on beam analogy. Umudike Journal of Engineering and Technology, 2020, 6 (1), 111-119.

[24]. Chakraverty, S. Vibration of plates, London, Taylor & Francis Group, 2009.

[25]. Gere, J. M. Mechanics of materials, 6th ed., New York, Thomson Learning Inc.2004.

[26]. Bansal, R. K. A textbook of strength of materials, 4th ed., New Delhi, Laxmi Publishers (P) Ltd.2009.

[27]. Adah, E. I. Development of computer programs for analysis of single panel and continuous rectangular plate. M. Eng. Thesis submitted to Postgraduate School, Federal University of Technology, Owerri, 2016.

[28]. Oyenuga, V. O. Simplified reinforced concrete design, 2nded, Lagos, Asros Ltd.2008.

.